Impact Analysis of Wind Power Injection on Time-Scale Separation of Power System Oscillations

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Abstract—In this work, we present an analytical relationship between the two time-scale behavior of coherent power system networks with increasing levels of wind penetration. We first derive a mathematical model coupling the electro-mechanical swing dynamics of the grid integrated with the dynamics of a doubly-fed induction machine via power flow. We then consider the system to be comprised of r coherent areas and apply a similarity transformation to explicitly show that the integrated system may exhibit a three time-scale behavior depending on the amount of wind power injection. This effect is illustrated using simulations of a model of the US Western Interconnection.

I. INTRODUCTION

Wind energy is one of the fastest growing sources of renewable energy. In the US, the rapid increase in wind farm installations is being accelerated by government mandates and renewable energy goals, such as US Department of Energy’s goal of 20% wind by 2015 [1]. Tremendous amount of research has been devoted to this topic over the past decade, with the focus being on overcoming the operational challenges regarding wind installation [2], particularly in maintaining frequency regulation and maximum power output [3]. Some recent papers have also started investigating how wind penetration, beyond simply affecting steady-state power flow dispatch, may also impact dynamic electro-mechanical oscillations, especially inter-area oscillations. For example, in conventional power systems groups of synchronous generators are allowed to swing freely with respect to one another, thereby leading to fast and slow oscillations [4]. However, a wind power system can only be interfaced to the grid via a DFIG whose speed can vary based on wind speed to maximize the wind turbine power output. This, in turn, may significantly change the time-scale separation of the oscillations in the grid. With increasing levels of wind penetration, or equivalently with more and more synchronous generators being replaced by wind generators, the impact on time-scale separation, and by extension on coherence, is likely to become significant. Several papers in the wind power system literature have alluded to this fact by means of simulations and modal analysis [5], but a detailed, rigorous, analysis of how time-scale separation properties of a coherent group of synchronous generators may get affected by various model parameters of a wind power system, is still missing. In this paper we bridge this gap by first deriving a dynamic model of a multi-machine power system integrated with a wind power system via a DFIG, and thereafter casting this model in terms of coherent clusters. Using the similarity transform discussed in [4], we obtain the time-scale separated form of the wind-integrated system model. An interesting observation following from this model is that certain levels of wind power injection lead to a dominant three time-scale behavior for the phase angle and frequency responses of the grid rather than the two time-scale behavior reported for conventional synchronous generators [4]. We illustrate this result by simulating wind penetrations in several areas of a reduced-order model of the WECC power system.

II. DYNAMIC MODEL OF WIND INJECTION

We first derive the dynamic model for a group of wind turbines injecting power to the grid at a point of common coupling. The model is based on a single representative turbine with a parameter $\varepsilon_w$ that scales the output power level to represent multiple turbines. We next describe the two subsystems that comprise the wind power system model.

A. Mechanical subsystem

The mechanical subsystem model consists of a two-shaft drive train connecting the turbine and the generator with respective inertias $J_r$ and $J_g$, and friction coefficients $B_r$ and $B_g$ [6]. The transmission gear connecting the shafts has a gear ratio of $N_g$, a torsion stiffness $K_{dt}$ and damping of $B_{dt}$. The model, in terms of the turbine rotor speed $\omega_r(t)$, the generator torsion angle $\theta_T(t)$ and generator speed $\omega_g(t)$ is given by

$$J_r\ddot{\omega}_r(t) = \frac{B_{dt}}{N_g}\omega_g(t) - K_{dt}\theta_T(t) - (B_{dt} + B_r)\omega_r(t) + T_a(t)$$

(1a)

$$J_g\ddot{\omega}_g(t) = \frac{B_{dt}}{N_g}\omega_r(t) + \frac{K_{dt}}{N_g}\theta_T(t) - \left(\frac{B_{dt}}{N_g^2} + B_g\right)\omega_g(t) - T_g(t)$$

(1b)

$$\dot{\theta}_T(t) = \omega_r(t) - \frac{1}{N_g}\omega_g(t),$$

(1c)

where

$$T_a(t) = \frac{\rho A_s \omega^3(t)C_p(t)}{2\omega_r(t)}$$

(2)

is the aerodynamic torque input for air density $\rho$ at wind speed $\omega_r(t)$ for a turbine with swept area $A_s$ and power coefficient $C_p(t)$. The generator torque $T_g(t)$ based on the DFIG model is derived in the next subsection.
B. Electrical subsystem

The dynamics of the electrical subsystem are given by [7],

\[ v_{qs}(t) = (R_s + sL_s) i_{qs}(t) + \omega_e L_i i_{ds}(t) + sL_m i_{qr}(t) + \omega_e L_i i_{dr}(t) \]

\[ v_{ds}(t) = -\omega_e L_i i_{qs}(t) + (R_s + sL_s) i_{ds}(t) - \omega_e L_m i_{qr}(t) + sL_m i_{dr}(t) \]

\[ v_{qr}(t) = sL_m i_{qs}(t) + (\omega_e - \omega_{g1})L_r i_{dr}(t) + (R_r + sL_r) i_{dr}(t) \]

\[ v_{dr}(t) = -sL_m i_{qs}(t) + (\omega_e - \omega_{g1})L_r i_{qr}(t) + sL_m i_{ds}(t) - (\omega_e - \omega_{g1})L_r i_{qr}(t) + (R_r + sL_r) i_{dr}(t) \]

where \( s \) represents the differential operator. Here, \( v, i, R \) and \( L \) respectively denote the voltage, current, resistance and inductance of the DFIG stator and rotor circuits, which are respectively indicated with subscripts \( s \) and \( r \). The subscripts \( d \) and \( q \) represent the direct and quadrature axis of the reference frame of a machine with speed \( \omega_e \). The electrical speed and the magnetizing inductance of the generator are \( \omega_{g1} \) and \( L_m \), respectively. The electromagnetic torque of the DFIG is

\[ T_g(t) = -\frac{3p}{4 L_m} (i_{qs}(t) i_{dr}(t) - i_{ds}(t) i_{qr}(t)) . \]

Linearizing the combined system (1)-(4) about an operating point \((\omega_{e0}, \omega_{g0}, \theta_{f0}, i_{q0}, i_{d0}, i_{qr}, i_{dr})\) yields

\[ \Delta \ddot{Z} = A \Delta Z + B_1 U_1 + B_2 U_2 \]

with states

\[ Z := [\Delta \omega_r \ \Delta \theta_r \ \Delta i_{qs} \ \Delta i_{ds} \ \Delta i_{qr} \ \Delta i_{dr}]^T \]

and inputs \( U_1 := [\Delta v_{qs} \ \Delta v_{ds}]^T \) and \( U_2 := [\Delta v_{qr} \ \Delta v_{dr}]^T \).

We assume a constant value for \( v_{dr}(t) \). The electrical and mechanical variables in the system can be decoupled such that

\[ A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \]

where \( A_{11} \) follows from the linearization of (1) and (2); \( A_{22} \), \( B_1 \) and \( B_2 \) follow from the linearization of (3). The connection matrices are defined from (3)-(4) as,

\[ A_{12} = \left( \frac{3p}{4 L_m} \right) \begin{bmatrix} 0 & 0 & 0 & 0 \\ i_{d0} & -i_{q0} & -i_{ds} & i_{qs} \\ 0 & 0 & 0 & 0 \end{bmatrix} , \]

\[ A_{21} = \left( \frac{2}{K_1 p} \right) \begin{bmatrix} 0 & -K_{21}i_{d0} & 0 & 0 \\ 0 & K_{21}i_{q0} & 0 & 0 \\ 0 & 0 & -K_{31}i_{d0} & 0 \\ 0 & 0 & 0 & -K_{31}i_{q0} \end{bmatrix} . \]

The respective active and reactive power outputs of the DFIG are given by

\[ P_e = v_{qs} i_{qs} + v_{ds} i_{ds} + v_{dr} i_{dr} + v_{qr} i_{qr} \]

\[ Q_e = v_{ds} i_{qs} - v_{ds} i_{ds} + v_{dr} i_{qr} - v_{qr} i_{dr} \]

with a corresponding linearized output equation given by,

\[ \begin{bmatrix} \Delta P_e \\ \Delta Q_e \end{bmatrix} = \begin{bmatrix} C_1 Z \\ C_2 Z \end{bmatrix} + \begin{bmatrix} D_{11} U_1 \\ D_{21} U_1 \end{bmatrix} + \begin{bmatrix} D_{31} U_2 \\ D_{41} U_2 \end{bmatrix} \]

In what follows, we neglect the contribution of the input \( U_2 \) as it is not directly related to the power flowing to the grid but note that it can be used as a control input for DFIG operation.

III. DYNAMICS OF A POWER SYSTEM NETWORK

We next consider a power system with \( n \) generators and \( N \) buses. We assume \( n-1 \) synchronous generators and one wind generator with dynamics given by (5). The \( i \)th synchronous generator in the network connected to the \( i \)th bus is modeled as a constant voltage \( E_i \) behind a transient reactance \( x_{di} \). The corresponding swing equation is given as,

\[ m_i \ddot{\delta}_i = P_{mi} - \frac{E_i}{x_{di}} (V_{i_{re}} \sin \delta_i - V_{i_{im}} \cos \delta_i) \]

where \( \delta_i \) is the angle of the \( i \)th generator, \( V_{i_{re}} + jV_{i_{im}} \) represents the voltage \( V \) at the \( i \)th bus, \( m_i \) and \( P_{mi} \) are the inertia and the mechanical input to the generator. The linearized model for the system of generators is given by,

\[ M \ddot{\delta} = k_{11} \dot{\delta} + k_{12} \Delta V + \Delta P_m \]

where \( \delta_0 \) in front of each variable implies a small signal change of that variable. The coefficients \( k_{11} \) and \( k_{12} \) are determined from machine parameters and steady-state voltages of the buses. \( M \) represents a diagonal matrix of \( m_i \). The linearized active and reactive power outputs of the synchronous machines are given as,

\[ \Delta P_e = -k_{11} \dot{\delta} - k_{12} \Delta V \]

\[ \Delta Q_e = -k_{21} \dot{\delta} - k_{22} \Delta V \]

where \( k_{21} \) and \( k_{22} \) are determined from machine parameters and steady-state voltages of the buses.

We consider the power flow among the \( N \) buses in the power system with active and reactive power flow for the \( j \)th bus given by,

\[ g_{2j-1} = P_{ej} - \text{Re} \left\{ \sum_{k=1,k \neq j}^{N} V_{jk} \left( \frac{V_j}{Z_{jk}} \right)^* \right\} - V_j^2 G_j \]

\[ g_{2j} = Q_{ej} - \text{Im} \left\{ \sum_{k=1,k \neq j}^{N} V_{jk} \left( \frac{V_j}{Z_{jk}} \right)^* \right\} - V_j^2 B_j \]

where \( P_{ej}, Q_{ej}, G_j \) and \( B_j \) respectively denote the active and reactive power flow from the generators, load conductance and load susceptance with line charging in bus \( j \). \( Z_{jk} \) denotes the impedance of the lines connecting buses \( j \) and \( k \), and

\[ V_{jk} = (V_{j_{re}} - V_{j_{im}}) + j(V_{j_{re}} - V_{k_{im}}) \]

We consider the loads to be of constant impedance and purely reactive line. We linearize (11a) and (11b) for all synchronous generator buses to obtain

\[ 0 = \Delta P_{ej} - k_2 \Delta V \]

\[ 0 = \Delta Q_{ej} - k_4 \Delta V \]

where \( \Delta P_{ej} \) and \( \Delta Q_{ej} \) are the small signal active and reactive power input as shown in (10). Thus, it follows that,

\[ 0 = -k_{11} \dot{\delta} - (k_2 + k_{12}) \Delta V \]

\[ 0 = -k_{21} \dot{\delta} - (k_4 + k_{22}) \Delta V, \]
where $k_2$ is a $n \times 2N$ matrix whose $j^{th}$ row is given by
\[
k_2(j, j) = \sum_{k=1, k \neq j}^{N} V_{k1m}^0 Y_{jk} + 2V_{jR_e}^0 G_j,
k_2(j, k) = -V_{jR_e}^0 Y_{jk}, \quad k_2(j, N + k) = V_{jR_e}^0 Y_{jk},
k_2(j, N + j) = -\sum_{k=1, k \neq j}^{N} V_{k1m}^0 Y_{jk} + V_{j1m}^0 G_j,
\]
$k_4$ is a $n \times 2N$ matrix whose $j^{th}$ row is given by
\[
k_4(j, j) = \sum_{k=1, k \neq j}^{N} (2V_{jR_e}^0 - V_{kR_e}^0) Y_{jk} + 2V_{jR_e}^0 B_j
nk_4(j, k) = -V_{jR_e}^0 Y_{jk}, \quad k_4(j, N + k) = -V_{jR_e}^0 Y_{jk},
k_4(j, N + j) = \sum_{k=1, k \neq j}^{N} (2V_{j1m}^0 - V_{k1m}^0) Y_{jk} + V_{j1m}^0 B_j
\]
and $Y_{jk} := 1/Z_{jk}$.

The stator of the DFIG is directly connected to a power system bus. So $U_1$ is a component $\Delta V$. Using (7) the linearized power flow for the wind injection bus can be shown to be
\[
0 = C_1 \Delta Z + (D_{1n} + k_5) \Delta V + D_3 U_2 \quad (14a)
0 = C_2 \Delta Z + (D_{2n} + k_6) \Delta V + D_4 U_2 \quad (14b)
\]
where $D_{1n} \Delta V = D_1 U_1, \quad D_{2n} \Delta V = D_2 U_1, \quad$ and $k_5$ and $k_6$ respectively have the row structure of $k_2$ and $k_4$. The power flow at buses without generators is given by
\[
k_7 \Delta V = 0 \quad k_8 \Delta V = 0 \quad (15)
\]
where $k_7$ and $k_8$ respectively have the row structure of $k_5$ and $k_6$. Combining (13)-(15) the full system power flow is
\[
0 = A_1 \Delta \delta + A_2 Z + A_3 U_2 + A_4 \Delta V \quad (16)
\]
and $A_1 = [k_{11} \ 0 \ 0 \ 0 \ k_{21} \ 0 \ 0 \ 0 \ 0]^T, \quad A_2 = [0 \ C_1 \ 0 \ 0 \ C_2 \ 0 \ 0 \ 0 \ 0]^T$, $A_3 = [0 \ D_2 \ 0 \ 0 \ D_4 \ 0 \ 0 \ 0 \ 0]^T$, and $A_4 = [(k_2 + k_12) \ (D_{1n} + k_5) \ k_7 \ (k_4 + k_22) \ (D_{2n} + k_6) \ k_8]^T$ where $A_4$ is a weighted admittance matrix of the network. From (16) we obtain,
\[
\Delta V = -A_4^{-1} (A_1 \delta + A_2 Z + A_3 U_2).
\]
Substituting this expression into (5) and (9), the Kron reduced form of the wind integrated power system can be written as,
\[
\begin{bmatrix}
  M \Delta \delta \\
  Z
\end{bmatrix} = \begin{bmatrix}
  A_{M11} & A_{M12} \\
  A_{M21} & A_{M22}
\end{bmatrix} \begin{bmatrix}
  \Delta \delta \\
  Z
\end{bmatrix}
+ \begin{bmatrix}
  I \\
  0
\end{bmatrix} \begin{bmatrix}
  -k_{12} A_{41}^{-1} \\
  (B_2 - B_1 A_{41}^{-1} A_{32})
\end{bmatrix} \begin{bmatrix}
  \Delta P_m \\
  U_2
\end{bmatrix} \quad (18)
\]
where $A_{M11} := (k_{11} - k_{12} A_{41}^{-1} A_{11}), \quad A_{M12} := -k_{12} A_{41}^{-1} A_{22}, \quad A_{M21} := -B_1 A_{41}^{-1} A_{12}$ and $A_{M22} := (A - B_1 A_{41}^{-1} A_{22})$.

\[\text{IV. TWO-TIME-SCALE POWER SYSTEM MODEL}\]

We next study the time-scale separation of the wind integrated model (18). Here, we assume a network with $r$ coherent areas with $r - 1$ of the areas having only synchronous generators and the remaining area having both synchronous and wind generators. In what follows, the parameters and variables of each synchronous generator in an area $\alpha$ is indicated by a superscript $\alpha$. The machines are numbered such that $\Delta \delta_\alpha^0$ from the same coherent groups appear consecutively in $\Delta \delta$. It is well known that aggregation and slow coherency in a power system containing synchronous generators is primarily due to the fact that the connection between the machines within a specific coherent area is stiffer than that between machines in two different areas. This additional stiffness is attributed to the following two phenomena:

1. The maximum admittance of the external connections $B_{ij}^E$ is lower than the minimum admittance of the connections $B_{ij}^I$ within an area. Thus, we define a parameter $\varepsilon_1 = \frac{B_{ij}^E}{B_{ij}^I}$ which should be small to ensure slow coherency between the areas.

2. The number of the external connections is much less than the number of internal connections in a certain area. So we define a parameter $\varepsilon_2 = \frac{\gamma_1^E}{\gamma_1^I}, \quad \gamma_1^E = \max \{\gamma_\alpha^E\}, \quad \gamma_1^I = \min \{\gamma_\alpha^I\}, \alpha = 1, \ldots, r$. We respectively denote the ratios of external and internal connections for an area $\alpha$ to the number of buses in that area ($N_\alpha$) as $\gamma_\alpha^E$ and $\gamma_\alpha^I$.

For a large power system, we can partition $A_4$ as
\[
A_4 = A_4^l + \varepsilon A_4^l,
\]
where $\varepsilon := \varepsilon_1 \varepsilon_2$, the superscript $I$ indicates a matrix comprised of internal connections within an area and superscript $E$ indicates a matrix of external connections between areas.

In presence of a wind generator, the matrix $A_4^l$ can be further sub-divided as
\[
A_4^l = A_4^{l,sg} + \varepsilon_w A_4^{l,wg}.
\]
where $A_4^{l,sg}$ is the matrix of the internal connections between synchronous generators, $A_4^{l,wg}$ is a matrix of internal connections that include the wind generator, $\varepsilon_w = \frac{i_{\text{smax}}}{B_{ij}^I}$. Here, $i_{\text{smax}}$ denotes the maximum stator current of the DFIG, which indicates the level of wind power injection.

It should be noted that $\varepsilon_w$, unlike $\varepsilon$ is a small parameter. The various system matrices of the power system model (18), e.g., $A_{M11}, \ A_{M12}, \ A_{M21}$, can be similarly partitioned such that
\[
A_{M11} = A_{M11}^l + \varepsilon A_{M11}^E,
\]
where $A_{M11}^l = k_{11} - k_{12}(A_{41}^{-1} A_{11}), \quad A_{M11}^E = -k_{12} A_{41}^{-1} A_{11}$ and
\[
A_{4e}^E := -(A_{4}^{-1} A_{4}^E + \varepsilon ((A_{4}^{-1} A_{4}^E)^2 + \cdots) (A_{4}^{-1}).
\]
The internal admittance matrix $A_4^I$ can be further subdivided as

$$
(A_4^I)^{-1} = (A_4^{I,sg})^{-1} + \varepsilon_w A_4^{I,wgp} 
$$

(22)

where

$$
A_4^{I,wgp} := -\left( A_4^{I,sg} \right)^{-1} A_4^{I,sg} \left( I + \varepsilon_w \left( A_4^{I,sg} \right)^{-1} A_4^{I,wug} \right)^{-1} \left( A_4^{I,sg} \right)^{-1}
$$

Thus, $A_{11}^{I}$ can be decomposed into,

$$
A_{11}^{I,sg} := k_{11} - k_{12} \left( A_4^{I,sg} \right)^{-1} A_1 \text{ and } A_{11}^{I,wug} := -k_{12} \left( A_4^{I,wgp} \right) A_1.
$$

A similar procedure can be followed to separate all of the internal matrices in the system, e.g., $A_{12}^{I}$ and $A_{21}^{I}$.

In order to show the slow coherency and clustering in the power system, we use a transformation for the synchronous generator angles ($\Delta \delta$) to obtain an aggregate angle for an area and difference variables within an area. In order to describe the slow motion of the $r - 1$ areas with only synchronous generators we define,

$$
\delta_\alpha^a = \sum_{i=1}^{n_a} m_i^a \Delta \delta_i^a / m^a
$$

(23)

where $n_a$ is the number of machines in area $\alpha$ and $m^a = \sum_{i=1}^{n_a} m_i^a$ is the aggregate inertia of area $\alpha$. Similarly, to describe the slow motion of the area with wind injection we define,

$$
\delta_{s,wg}^a = \sum_{i=1}^{n_{s,wg}} m_i^{s,wg} \Delta \delta_i^{s,wg} / m^a
$$

(25)

where $m_{s,wg}$ is the aggregate inertia of $n_{s,wg} - 1$ synchronous generators following from (24). The aggregate variables of the system are shown as,

$$
\begin{bmatrix}
\delta_{s,sg}^a \\
\delta_{s,wg}^a
\end{bmatrix}
= C \Delta \delta = M_a^{-1} U^T M \Delta \delta
$$

(26)

where $U := \text{blockdiag}(u_1, u_2, \ldots, u_r)$ is the grouping matrix of column vectors $u_i$, whose length is equal to the number of synchronous generators in area $\alpha$ and $M_a := U^T M$ is the aggregate inertia matrix. For the fast dynamics, we define the motions of the synchronous generators in area $\alpha$ relative to a reference machine as

$$
\delta_{f,i} = \Delta \delta_i - \Delta \delta_1^i, \quad i = 2, 3, \ldots, n_a - n_{w,\alpha}
$$

(27)

where $n_{w,\alpha}$ is the number of wind generators in area $\alpha$. Putting all the fast variables from the $r$ areas into matrix form leads to

$$
\delta_f = G \Delta \delta = \text{blockdiag} \left( G^1, G^2, \ldots, G^r \right) \Delta \delta
$$

(28)

where $G^\alpha := \begin{bmatrix}
-1 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 \\
\vdots & \vdots & \vdots & \vdots \\
-1 & 0 & 0 & 1
\end{bmatrix}_{n_a-1 \times n_a}$

Fig. 1: WECC Power system with wind generation

We use the relations in (26) and (28) to transform (18) into a time-scale separated form as shown below,

$$
\begin{bmatrix}
M_a,sg \delta_{s,sg}^a \\
M_a,wg \delta_{s,wg}^a \\
M_{\delta_f} \\
\varepsilon K_a,sg a,sg \\
\varepsilon K_a,wg a,wg \\
\varepsilon K_a,sg a,sg \\
\varepsilon K_a,wg a,wg \\
\varepsilon K_a,sg a,sg \\
\varepsilon K_a,wg a,wg \\
K_{b,sg} b,sg \\
K_{b,wg} b,wg
\end{bmatrix}
= \tilde{A}
\begin{bmatrix}
\delta_{s,sg}^a \\
\delta_{s,wg}^a \\
\delta_f \\
\varepsilon w K_a,sg a,sg \\
\varepsilon w K_a,wg a,wg \\
\varepsilon w K_a,sg a,sg \\
\varepsilon w K_a,wg a,wg \\
\varepsilon w K_a,sg a,sg \\
\varepsilon w K_a,wg a,wg \\
K_d b,sg \\
K_d b,wg
\end{bmatrix}
$$

(29)

where

$$
\tilde{A} := \begin{bmatrix}
\varepsilon K_a,sg a,sg & \varepsilon K_a,wg a,wg & \varepsilon K_a,sg a,sg & \varepsilon K_a,wg a,wg \\
\varepsilon w K_a,sg a,sg & \varepsilon w K_a,wg a,wg & \varepsilon w K_a,sg a,sg & \varepsilon w K_a,wg a,wg \\
\varepsilon K_a,sg a,sg & \varepsilon K_a,wg a,wg & \varepsilon K_d b,sg & \varepsilon K_d b,wg \\
\varepsilon w K_a,sg a,sg & \varepsilon w K_a,wg a,wg & \varepsilon w K_a,sg a,sg & \varepsilon w K_a,wg a,wg \\
K_{b,sg} b,sg & K_{b,wg} b,wg & K_{b,sg} b,sg & K_{b,wg} b,wg
\end{bmatrix}
$$

The various matrices shown in (29) are given as

$$
K_a,sg = U^T A_{M11}^E U, \quad K_a,wd = U^T A_{M12}^E M^{-1} G^T M \quad K_{b,sg} = A_{M12}^E, \quad K_{b,wg} = A_{M21}^E U, \quad G^+ = G^T (G G^T)^{-1},
$$

and $K_d = M_d G M^{-1} A_{M11}^E + \varepsilon M_d G M^{-1} A_{M11}^E$.

We specify the first $r - 1$ rows of the matrices with subscript $sg$ and the last row with subscript $wg$. From (29) we see that the time constant for $\delta_{s,sg}$ is proportional to $\varepsilon^{-1/2}$ while that for $\delta_{s,wg}$ is proportional to $(\varepsilon_w)^{-1/2}$. Since depending on the level of wind power injection, $\varepsilon_w \gg \varepsilon$, this indicates that $\delta_{s,wg}$ as a function of time will evolve over a faster time scale than that of $\delta_f$. In other words, the model in (29) exhibits a three time-scale behavior. In the next section we provide a case study to illustrate this result.

V. Simulation Results

In order to check our analysis we have chosen the grid model for the Western Interconnection in the US, popularly known as the WECC model. The geographic size and diversity of the Western Interconnection leads to a special network...
topology with separate and well-defined generation and load centers, with long tie lines interconnecting these various regions. This results in a well-defined time scale separation of slow and fast modes of oscillation within WECCs 500kV network. A full order model of a real power system can be reduced to an equivalent lower order model which produces the same inter-area modes as those produced by the real power system [8]. This reduced order model collapses coherent portions of the network into aggregated synchronous generators (ASG) having equivalent inertia, damping and power generation of the collapsed portion. We simulate such a model of the WECC system in RSCAD software and introduce a wind injection in one of the areas. WECC is divided into five separate areas which are connected in a cyclic topology through long 500kV transmission lines [8]. Thus, the five areas of WECC can be represented by five ASG with the interconnecting 500kV lines between any two areas reduced to a single equivalent transmission line between the two ASGs as shown in Figure 1. The angle difference between the buses are recorded and passed through a modal decomposition tool called ERA to capture the slow modes of the system.

Figure 2a and 2b show that the time response of the inter area angle between area 1 and 2 is faster than that between area 2 and 3. The inter area modes identified between area 1 and 2 is also faster than that between area 2 and 3 as shown in Table I. The inter-area angles between area 1 and 2, unlike that between 2 and 3 become considerably faster as the wind penetration is increased. Particularly when the wind penetration increases from 30% to 60% in area 1, there is a jump in the modal frequency of the inter area mode between area 1 and 2 which follows our derivations in Section IV.

<table>
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<tr>
<th></th>
<th>no wind</th>
<th>30% wind</th>
<th>60% wind</th>
<th>90% wind</th>
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<td>0.33</td>
<td>0.67</td>
<td>0.66</td>
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TABLE I: Inter-area modes in Hz between different areas

VI. CONCLUSION

In this paper we developed an analytical relationship between the inter-area oscillations of a large wind-integrated power system and the level of wind penetration. We first derived the dynamic model of the integrated system using coupled power flow equations of synchronous generators and a DFIG, and thereafter cast this model in a time-scale separated form. Our derivations show that depending on the amount of wind power injection, the time-scale of the aggregate motion of the phase angles in the area containing the DFIG may be significantly faster than that with only synchronous generators. This indicates how increasing deployment of wind generators in conventional power grids may change the inter-area oscillation spectrum, necessitating more rigorous control.

REFERENCES