New Methods for Model Identification of Large-scale Electrical Networks

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Network Dynamic Systems (NDS)

- **NDS**: A collection of *dynamic systems* that can collect or exchange information over a *connection graph* $G$

- **Graph**: $G = (V, E)$

- **Representation of a linear NDS**: $(A, B, C, D, G)$

  In general, $\dot{x}_i = f_i(x, G), \quad i = 1, 2, \ldots, n$

- **Examples**: Social networks
  - Formation of aircrafts, space vehicles
  - Communications and sensor networks
  
  *(Fax & Murray, Wen & Beard, Tanner & Jadbabaie, Zelazo & Mesbahi, Olfati-Saber, Paley & Leonard etc.)*

  **Electric power system networks**
Identification of NDS

Synthesis procedures for the purpose of:

- Controller design (eg: PSS, FACTS)
- State Estimation
- Transient performance (LQR, $H_\infty/H_2$)
- Robustness and fault-detection
- Simulation

Need an accurate mathematical model

- Impractical for a large-scale system
- Depend on dynamic measurements from specific parts of the network

Identification: The process of finding the model of a dynamic system from a known data set.
Two-Machine Single Line System

\[ \tilde{I} = I \angle \theta_1 \]

\[ \tilde{V}_1 = V_1 \angle \theta_1 \quad \tilde{V}_2 = V_2 \angle \theta_2 \]
Two-Machine Single Line System

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**Problem:**
How to estimate all parameters?

\( x_1, x_2, x_e, H_1, H_2 \)

\[ \dot{\delta} = \omega \]

\[ 2 \frac{H_1 H_2}{H_1 + H_2} \dot{\omega} = \frac{H_2 P_{m1} - H_1 P_{m2}}{H_1 + H_2} - \frac{E_1 E_2}{(x_1 + x_e + x_2)} \sin \delta \]

*Swing Equation*
Graphical Interpretation

\[ \tilde{I} = I \angle \theta_1 \]

\[ \tilde{V}_1 = V_1 \angle \theta_1 \quad \tilde{V}_2 = V_2 \angle \theta_2 \]

**Inverse Problem**: Find edge weights from measurements
Graphical Interpretation

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\[ \widetilde{V}_1 = V_1 \angle \theta_1 \quad \widetilde{V}_2 = V_2 \angle \theta_2 \]

**Inverse Problem:** Find edge weights from measurements

Recall AC Ohm’s Law:

\[ \widetilde{V}_a - \widetilde{V}_b = (jx_{ab})\widetilde{I}_{ab} \]

How to find \( x_1 \) and \( x_2 \)?

\[ x_e = \frac{\widetilde{V}_1 - \widetilde{V}_2}{j\widetilde{I}} \]

Allows to go to DN from MN
IME: Method \textit{(Reactance Extrapolation)}

- **Key idea**: Amplitude of voltage oscillation at any point is a function of its electrical distance from the two fixed voltage sources.

\[
\tilde{V}(x) = [E_2(1-a) + E_1a \cos(\delta)] + j E_1a \sin(\delta), \quad a = \frac{x}{x_1 + x_e + x_2}
\]
**IME: Method (Reactance Extrapolation)**

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\[ \tilde{V}(x) = [E_2(1-a) + E_1a \cos(\delta)] + j E_1a \sin(\delta), \quad a = \frac{x}{x_1 + x_e + x_2} \]

- Voltage magnitude: \( V = |\tilde{V}(x)| = \sqrt{c + 2E_1E_2(a-a^2)\cos(\delta)}, \quad c = (1-a)^2E_2^2 + a^2E_1^2 \)
IME: **Method (Reactance Extrapolation)**

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- Assume the system is initially in an equilibrium \((\delta_0, \omega_0 = 0, V_{ss})\):

\[
\Delta V(x) = J(a, \delta_0)\Delta\delta
\]

\[
J(a, \delta_0) := \frac{\partial V(a, \delta_0)}{\partial \delta} \bigg|_{\delta = \delta_0} = \frac{-E_1E_2}{V(a, \delta_0)}(a-a^2)\sin(\delta_0)
\]
Reactance Extrapolation

\[ \Delta V(x,t)V(a,\delta_0) = -E_1E_2 \sin(\delta_0) (a - a^2) \Delta \delta(t) \]

can be computed from measurements at \( x \)
Reactance Extrapolation

\[ \Delta V(x,t)V(a,\delta_0) = -E_1 E_2 \sin(\delta_0) (a - a^2) \Delta \delta(t) \]

Note: Spatial and temporal dependence are separated

can be computed from measurements at \( x \)

\[ V_n(x,t) = A (a - a^2) \Delta \delta(t) \]
\[ \Delta V(x,t)V(a,\delta_0) = -E_1E_2 \sin(\delta_0)(a-a^2)\Delta\delta(t) \]

- Can be computed from measurements at \( x \)
- \( A \)
- \( V_n(x,t) = A(a-a^2)\Delta\delta(t) \)

Note: Spatial and temporal dependence are separated

• Fix time: \( t=t^* \)

\[ V_n(x,t^*) = A(a-a^2)\Delta\delta(t^*) \]

How can we use this relation to solve our problem?
Reactance Extrapolation

\[ V_n(x, t^*) = A(a - a^2)\Delta\delta(t^*) \]
**Reactance Extrapolation**

\[ V_n(x, t^*) = A (a - a^2) \Delta \delta(t^*) \]

At Bus 2, \( a_2 = \frac{x_2}{x_1 + x_e + x_2} \) \( \rightarrow \) \( V_{n, Bus2} = A (a_2 - a_2^2) \Delta \delta(t^*) \)
Reactance Extrapolation

\[ V_n(x, t^*) = A \left( a - a_1^2 \right) \Delta \delta(t^*) \]

\[ \begin{align*}
E_1 & \angle \delta \\
1 & \\\n2 & \\\nE_2 & \angle 0
\end{align*} \]

At Bus 2, \( a_2 = \frac{x_2}{x_1 + x_e + x_2} \) \quad \Rightarrow \quad V_{n, Bus2} = A \left( a_2 - a_2^2 \right) \Delta \delta(t^*)

At Bus 1, \( a_1 = \frac{x_e + x_2}{x_1 + x_e + x_2} \) \quad \Rightarrow \quad V_{n, Bus1} = A \left( a_1 - a_1^2 \right) \Delta \delta(t^*)

\[ \frac{V_{n, Bus2}}{V_{n, Bus1}} = \frac{a_2(1-a_2)}{a_1(1-a_1)} \]

- Need one more equation
- hence, need one more measurement at a known distance
**Reactance Extrapolation**

\[ V_n(x, t^*) = A(a - a^2) \Delta \delta(t^*) \]

At Bus 1,
\[ \delta \Delta - \frac{a a A}{V_{\text{Bus} n}} \]

- Need one more equation
  - hence, need one more measurement at a known distance

At Bus 2,
\[ a_2 = \frac{x_2}{x_1 + x_e + x_2} \quad \rightarrow \quad V_{n, Bus2} = A(a_2 - a_2^2) \Delta \delta(t^*) \]

\[ \frac{V_{n, Bus2}}{V_{n, Bus1}} = \frac{a_2(1 - a_2)}{a_1(1 - a_1)} \]

At Bus 1,
\[ a_1 = \frac{x_e + x_2}{x_1 + x_e + x_2} \quad \rightarrow \quad V_{n, Bus1} = A(a_1 - a_1^2) \Delta \delta(t^*) \]

\[ \frac{V_{n, Bus3}}{V_{n, Bus1}} = \frac{a_3(1 - a_3)}{a_1(1 - a_1)} \]
Reactance Extrapolation

\[ V_n(a) = A a (1 - a) \]

Key idea: Exploit the spatial variation of phasor outputs
IME: Method (*Inertia Estimation*)

- From linearized model

\[
f_s = \frac{1}{2\pi} \sqrt{\frac{E_1 E_2 \cos(\delta_0) \Omega}{2H(x_e + x_1 + x_2)}}
\]

where \( f_s \) is the *measured* swing frequency and \( H = \frac{H_1 H_2}{H_1 + H_2} \)
IME: Method (Inertia Estimation)

• From linearized model

\[ f_s = \frac{1}{2\pi} \sqrt{\frac{E_E E_2 \cos(\delta_0)\Omega}{2H(x_e + x_1 + x_2)}} \]

where \( f_s \) is the measured swing frequency and \( H = \frac{H_1H_2}{H_1 + H_2} \)

• For a second equation in \( H_1 \) and \( H_2 \), use law of conservation of angular momentum

\[ 2H_1\omega_1 + 2H_2\omega_2 = 2\int (H_1\dot{\omega}_1 + H_2\dot{\omega}_2)dt = \int (P_{m1} - P_{e1} + P_{m2} - P_{e2})dt = 0 \]

\[ \Rightarrow \quad \frac{H_1}{H_2} = -\frac{\omega_2}{\omega_1} \]

• However, \( \omega_1 \) and \( \omega_2 \) are not available from PMU data,

→ Estimate \( \omega_1 \) and \( \omega_2 \) from the measured frequencies \( \xi_1 \) and \( \xi_2 \) at Buses 1 and 2
IME: Method (*Inertia Estimation*)

- Express *voltage angle* $\theta$ as a function of $\delta$, and differentiate wrt time to obtain a relation between the machine speeds and bus frequencies:

$$\dot{\xi}_1 = \frac{a_1 \omega_1 + b_1 (\omega_1 + \omega_2) \cos(\delta_1 - \delta_2) + c_1 \omega_2}{a_1 + 2b_1 \cos(\delta_1 - \delta_2) + c_1}$$

$$\dot{\xi}_2 = \frac{a_2 \omega_1 + b_2 (\omega_1 + \omega_2) \cos(\delta_1 - \delta_2) + c_2 \omega_2}{a_2 + 2b_2 \cos(\delta_1 - \delta_2) + c_2}$$

- $\dot{\xi}_1$ and $\dot{\xi}_2$ are measured, and $a_i$, $b_i$, $c_i$ are known from reactance extrapolation.

- Hence, we calculate $\omega_1/\omega_2$ to solve for $H_1$ and $H_2$.

where,

$$a_i = E_i^2 (1-r_i)^2, \quad b_i = E_i E_2 r_i (1-r_i),$$

$$c_i = E_2^2 r_i^2$$
Illustration: 2-Machine Example

- Illustrate IME on classical 2-machine model \((r_e = 0)\)
- Disturbance is applied to the system and the response simulated in MATLAB

\[
V_{1m} = 0.0292 \quad V_{2m} = 0.0316 \quad V_{3m} = 0.0371 \\
V_{1ss} = 1.0320 \quad V_{2ss} = 1.0317 \quad V_{3ss} = 1.0136 \\
V_{1n} = 0.0301 \quad V_{2n} = 0.0326 \quad V_{3n} = 0.0376
\]

IME Algorithm

\[
\begin{align*}
x_1 &= 0.3382 \text{ pu} \\
x_2 &= 0.3880 \text{ pu}
\end{align*}
\]

Exact values:
\[
x_1 = 0.34 \text{ pu}, \quad x_2 = 0.39 \text{ pu}
\]

\[
G(s) = \frac{s}{sT+1}
\]

Bus angle oscillations

\[
H_1 = 6.48 \text{ pu} \\
H_2 = 9.49 \text{ pu}
\]

Exact values:
\[
H_1 = 6.5 \text{ pu}, \quad H_2 = 9.5 \text{ pu}
\]
IME for Complex Topologies

Star-graph with 3 Generators

- 6 Unknowns: \( x_1, x_2, x_3, x_e1, x_e2, x_e3 \)
IME for Complex Topologies

Star-graph with 3 Generators

- Current in each branch is different
- No single spatial variable $a$
- Derivations need to be done piecewise (each edge of the star)

Two relative states: $\delta_1, \delta_2$

Steps for one piece (say Branch 2):

1. Express $\vec{V}_g = f(E_1, E_2, E_3, \delta_1, \delta_2, a_1, a_2, a_3)$
IME for Complex Topologies

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Steps for one piece (say Branch 2):

1. Express $\vec{V}_g = f(E_1, E_2, E_3, \delta_1, \delta_2, a_1, a_2, a_3)$
2. $\vec{V} = (1 - a_2)\vec{E}_2 + a_2\vec{V}_g$
   $\quad \quad \quad = \Psi(E_1, E_2, E_3, \delta_1, \delta_2, a_1, a_2, a_3)$
IME for Complex Topologies

Star-graph with 3 Generators

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Steps for one piece (say Branch 2):

1. Express $\tilde{V}_g = f(E_1, E_2, E_3, \delta_1, \delta_2, a_1, a_2, a_3)$
2. $\tilde{V} = (1 - a_2)\tilde{E}_2 + a_2\tilde{V}_g$
   \[= \Psi(E_1, E_2, E_3, \delta_1, \delta_2, a_1, a_2, a_3)\]
3. $\Delta V_{ss} = J_1\Delta \delta_1(t) + J_2\Delta \delta_2(t)$

Time-space separation destroyed!
IME for Complex Topologies

Star-graph with 3 Generators

Time-space separation destroyed!

3. \( V_n = J_1 \Delta \delta_1(t) + J_2 \Delta \delta_2(t) \)
IME for Complex Topologies

Star-graph with 3 Generators

Time-space separation destroyed!

3. \( V_n = J_1 \Delta \delta_1(t) + J_2 \Delta \delta_2(t) \)

- Only voltage magnitude has been used
- Voltage phase is another degree of freedom

4. \( \theta_n = J_3 \Delta \delta_1(t) + J_4 \Delta \delta_2(t) \)
IME for Complex Topologies

Star-graph with 3 Generators

Time-space separation destroyed!

3. \( V_n = J_1 \Delta \delta_1(t) + J_2 \Delta \delta_2(t) \)

- Only voltage magnitude has been used
- Voltage phase is another degree of freedom

4. \( \theta_n = J_3 \Delta \delta_1(t) + J_4 \Delta \delta_2(t) \)

At \( t = t^* \), \( V_n - \frac{J_2}{J_4} \theta_n = \left( J_1 - \frac{J_2 J_3}{J_4} \right) \Delta \delta_1(t^*) \)

known from measurement at that point

→ 4. Write the variant of this equation for all 4 \( M \)-nodes

→ 5. Get rid of \( \Delta \delta(t^*) \) by ratioing – have 3 equations for 3 unknowns
IME for Complex Topologies

At $t = t^*$, $V_n - \frac{J_2}{J_4} \theta_n = \left( J_1 - \frac{J_2 J_3}{J_4} \right) \Delta \delta_1(t^*)$

known from measurement at that point

4. Write the variant of this equation for all 4 $M$-nodes

5. Get rid of $\Delta \delta(t^*)$ by ratioing – have 3 equations for 3 unknowns
IME for Generalized Topology

\[ \delta \text{ wrt reference (lineage 1)} \]

The two levels of hierarchy must be joined by a tree graph

One measurement in each edge is sufficient

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Lineage of the reference node

Tree-graph allows for the state-variable \( \delta \) to be the phase of the 1\text{st} child-MN
Application to WECC Data

Needs processing to get usable data
- Sudden change/jump
- Oscillations
- Slowly varying steady-state (governer effects)
WECC Data

Oscillations

Bus Voltage (pu)

Time (sec)

Fast Oscillations (pu)

Time (sec)

Quasi-steady State

Slow Voltage (pu)

Time (sec)

Band-pass Filter

Choose pass-band covering typical swing mode range
Can use modal identification methods such as: **ERA, Prony, Steiglitz-McBride**
IME Applications

• Dynamic Security Assessment in Radial Transfer Paths

  — Measurement Based Energy Functions

• Use the estimation results for better assessment of transient and damping stability in two-area power systems following a disturbance

• PMU data-based Energy Functions as a stability metric

\[ S = S_1 + S_2 = \sum_{j=1}^{n(n-1)/2} \int_{\delta_j^*}^{z_j} \psi_j(k) \, dk + \sum_{j=1}^{n} \frac{M_j}{2} \xi_j^2 \]

\[ \dot{\delta} = \Omega \delta, \quad 2H \dot{\omega} = P_m - \frac{E_1 E_2 \sin(\delta)}{x_e} \]

\[ P = \frac{E_1 E_2 \sin(\delta)}{x_e} \]
Energy Functions for Two-machine System

\[ S = \frac{E_1 E_2}{x_e'} \left[ \cos(\delta_{op}) - \cos(\delta) + \sin(\delta_{op})(\delta_{op} - \delta) \right] + H\omega^2 \]

Using IME algorithm: \( x_e' \), \( E_1 \), \( E_2 \), \( \delta = \delta_1 - \delta_2 \), \( \delta_{op} \), \( \omega = \omega_1 - \omega_2 \) & \( H \) are computable from:

\[ x_e, \ V_1, \ V_2, \ \theta = \theta_1 - \theta_2, \ \theta_{op}, \ v=v_1-v_2 \ & \omega_s \]

• Note: \( \theta_{op} = \text{pre-disturbance angular separation} \)
Energy Functions for WECC Disturbance Event

Sending End and Receiving End Bus Angles

IME

Angle difference between machine internal nodes

Machine speed difference
Energy Functions for WECC Disturbance Event

Total Energy = Kinetic Energy + Potential Energy

- Total energy decays exponentially – *damping stability*
- Total energy does not oscillate – *Out-of-phase osc.*
  - Damped pendulum
Phasor Measurement Units

1 PMUs digitally record 3-φ voltages and currents.
2. High internal sampling rate (eg: 2.88 kHz).
3. Writes/exports data at **10-60 samples/s**.
4. Time stamped with GPS signals.
5. Send data via the internet to PDC.
Conclusions

- We developed novel methods for model identification and reduction of two-area power systems to represent interarea dynamics
  - spatial variation patterns of phasor variables are exploited

- Fast sampled dynamic phasor measurements are used for building these tools

- Both with and without voltage support cases are considered

- Appropriate signal processing tools are developed

- The method enables better estimation of energy margins, better estimation of wave speeds, easier design of PSS, etc.
Thank You