Distributed Control of Power System Oscillations under Communication and Actuation Constraints

Abhishek Jain\textsuperscript{a,*}, Aranya Chakrabortty\textsuperscript{a}, Emrah Biyik\textsuperscript{b}

\textsuperscript{a}Electrical & Computer Engineering Department, North Carolina State University, Raleigh
\textsuperscript{b}Department of Energy Systems Engineering, Yasar University, Izmir, Turkey

Abstract

A distributed Model Predictive Control design is presented for inter-area oscillation damping in power systems under two critical cyber-physical constraints - namely, communication constraints that lead to sparsification of the underlying communication network, and actuation constraints on excitation controllers. Currently damping controllers are usually executed over fixed communication topologies that are agnostic of the magnitude and location of the incoming disturbance. In contrast, the communication topology for our controller is selected in real-time after a disturbance event, based on event-specific correlations of the generator states with the dominant oscillation modes. The design is validated using a 140-bus power system model.

Keywords: Distributed control, wide-area control, predictive optimization, oscillation damping, participation factors.

1. Introduction

Over the past decade, significant increase in transmission expansion and renewable integration in the US power grid have forced power system operators to look beyond the traditional mindset of controlling the grid using local control methods, and transition to wide-area control using synchronized phasor measurements available from Phasor Measurement Units (PMUs). One of the most commonly known application of wide-area control is to improve damping of power flow oscillations in small-signal models of power systems by employing state exchange between distant generators through a wide-area communication network. An enormous literature already exists for damping control of synchronous generators [1, 2, 3] using local output feedback via power system stabilizers (PSS) and FACTS devices. These controllers are known to damp fast oscillation modes quite satisfactorily, but they often fail to improve the damping of low-frequency inter-area oscillations [4]. Recent papers such as [5, 6, 7, 8] have shown that wide-area control can be a promising solution to this problem.

Ideally, wide-area controllers can be designed using standard pole placement techniques and state-feedback controllers such as linear quadratic regulators (LQR) [9, 10]. However, such control schemes suffer from two major drawbacks. First, they lead to a dense all-to-all communication strategy between the generators amounting to a centralized implementation, and second, they are designed offline based on nominal models of the power system that are most often agnostic of where a disturbance may occur, or how this disturbance may impact the inter-area oscillations. In our recent paper [11] a sparse LQR controller was designed that is devoid of both of these drawbacks. In this paper that design is extended to a completely online model predictive control strategy that accommodates additional constraints on actuation. A sparse state-feedback controller is developed using excitation control of synchronous generators where the sparsity pattern of the underlying communication network is decided according to the modal residues of the inter-area oscillation modes excited by that disturbance. Generators that share high values of residues corresponding to a certain inter-area mode in open-loop are encouraged to communicate with each other for enhancing the damping of that mode in the closed-loop system. Note that these residues depend on the location and magnitude of the disturbance, and therefore can be different for different events. The choice of the sets of influential generators, and of the resulting communication topology thus becomes completely aware of the disturbance rather than being agnostic. The control signals are computed over this sparse topology using MPC in a distributed way, and implemented as a supplementary excitation control on top of existing PSS at selected sets of generators.

Compared to other online optimal control methods, MPC exhibits more robustness to load fluctuations and parametric uncertainties in the grid model as it evaluates the control inputs based on the current state of the system at every time-step [12]. It also allows us to explicitly incorporate actuator constraints, which is important in this case as the margin of variation for excitation voltages in sup-
plemepory controllers is severely limited. Several other recent works such as [13, 14, 15] have used MPC for power system control. In [13], a centralized MPC controller is designed for HVDC, while in [14, 15] MPC is used for load-frequency control.

The basic steps of our design can be summarized as follows. It is assumed that the physical power system network is divided into the so-called ‘utility areas’ which are owned and operated by different utility companies. Each of these companies has a computational platform in their local cloud-computing network, where PMU measurements from the utility area of that company are gathered.

Step 1: As soon as a fault happens in the system, a central coordinator (CCO) sends a contingency signal to all local clouds. Each cloud then uses voltage and current measurements from PMUs to estimate the phasors for all generator buses in their respective utility areas, and uses these phasors to estimate the generator states through a decentralized Kalman Filter [16]. The state vector is communicated to the CCO immediately after the fault clearing (Section 2).

Step 2: CCO then uses the small-signal model and the estimated $x_0$ to estimate the residues of the inter-area eigenvalues of the open-loop system, as reflected in the predicted outputs of every generator. Note that these residues will depend on the magnitude and location of the fault, and, therefore, cannot be computed prior to the fault.

Step 3: The CCO identifies dominant inter-area modes based on the relative magnitudes of the residues, and, the corresponding set of influential generators for that event (Section 3).

Step 4: A distributed modal cost function is constructed using the concept of selective discrete Fourier transform (SDFT) for minimizing the energy content of the identified dominant inter-area modes (Section 4).

Step 5: A communication architecture is formed on the basis of the influential, and a set of distributed controllers are implemented over this communication topology. Each controller minimizes a modal cost corresponding to a specific set of dominant modes following from Step 3 (Section 5).

Step 6: Distributed model predictive control (dMPC) is used to solve the minimization problem in Step 4, with added actuator constraints for every generator (Section 6). Effectiveness of control design is shown with simulation results on a 48-machine Northeast Power Coordinating Council (NPCC) power system model (Section 7).

2. System Modeling and State Estimation

Consider a power system network with $m$ synchronous generators. The network is assumed to be divided into $M$ number of utility areas. All assets including generators, loads, PMUs, controllers, etc. in each area are owned and maintained by the utility company in charge of that area. Each company can choose to install multiple PMUs in its own area, but may or may not be willing to share its data with the other areas. Keeping in mind the above restrictions on modeling and data, an area-wise state estimation strategy is formulated in this section for feedback control of the generators.

2.1. Nonlinear Power System Model

Each generator is modeled by a transient model assuming that the time constants of the $d$- and $q$-axis flux are fast enough to neglect their dynamics, that the rotor frequency is around the normalized constant synchronous speed, and that the amortisseur effects are negligible. The model of the $i^{th}$ generator is written as [17]:

\[
\begin{align*}
\dot{\delta}_i &= \omega_i - \omega_s \quad (1a) \\
M_i \dot{\omega}_i &= P_{mi} - P_{ci} - d_i(\omega_i - \omega_s) \quad (1b) \\
T_{qi} \dot{E}_{qi} &= -E'_{qi} + (x_{di} - x_{di}^r)I_{di} + E_{fdi} \quad (1c) \\
T_{di} \dot{E}_{di} &= -E_{di} + (x_{qi} - x_{qi}^r)I_{qi} \quad (1d) \\
T_{Ai} \dot{E}_{fdi} &= -E_{fdi} + K_{Ai}(V_{ref,i} - V_i) + u_i(t), \quad (1e)
\end{align*}
\]

where $P_{ci} = V_i I_{qi} \cos(\theta_i - \delta_i) + V_i I_{di} \sin(\delta_i - \theta_i)$, and for all $i = 1, \ldots, m$. Equations (1a)-(1b) are referred to as the swing equations while (1c)-(1e) as the excitation equations. The states $\delta_i$, $\omega_i$, $E_{qi}^r$, $E_{di}^r$, and $E_{fdi}$ respectively denote the generator phase angle, rotor velocity, the quadrature-axis internal emf, the direct-axis internal emf, and the field excitation voltage. The voltage at the generator terminal bus is denoted in the polar representation as $V_i(t) = V_i(t)\angle \theta_i(t)$. $V_{ref,i}$ is the constant setpoint for $V_i$. The generator current in complex phasor form is written as $I_{di} + jI_{qi} \equiv I_i \angle \phi_i$. $\omega_s$ is the synchronous frequency, which is equal to $120\pi$ rad/sec for a 60-Hz power system. $M_i$ is the generator inertia, $d_i$ is the generator damping, and $P_{mi}$ is the mechanical power input from the $i^{th}$ turbine, all of which are considered to be constant. $T_{di}$, $T_{qi}$, and $T_{Ai}$ are the excitation time constants; $K_{Ai}$ is the constant voltage regulator gain; $x_{di}^r$, $x_{qi}^r$, and $x_{di}^r$ are the direct-axis and quadrature-axis salient reactances and transient reactances, respectively. It is assumed that all of these constant model parameters are known. All variables, except for the phase angles (radians), are expressed in per unit. Equations (1a)-(1e) can be written in a compact form as:

\[
\begin{align*}
\dot{x}_i(t) &= f(x_i(t), z_i(t), u_i(t), \alpha_i), \quad (2)
\end{align*}
\]

where $x_i = [\delta_i, \omega_i, E_{qi}^r, E_{di}^r, E_{fdi}]^T \in \mathbb{R}^5$ denotes the vector of state variables, $z_i = [V_i, \delta_i, I_i, \phi_i]^T \in \mathbb{R}^5$ denotes the vector of algebraic variables, and $\alpha_i$ is the vector of the constant parameters $P_{mi}$, $\omega_s$, $d_i$, $T_{qi}$, $T_{di}$, $T_{Ai}$, $M_i$, $K_{Ai}$, $V_{ref,i}$, $x_{di}^r$, $x_{qi}^r$, $x_{di}^r$, and $x_{di}^r$, all of which are assumed to be known. The control input $u_i$ is usually constrained as
2.3. Generator Bus Phasor Estimation

The model (2) is a completely decentralized model since it is driven by variables belonging to the $i^{th}$ generator only. It is, however, not a state-space model as it contains the auxiliary variables $z_i$. The states $x_i$ can be estimated for this model in a completely decentralized way if one has access to $z_i(t)$ at every instant of time. This can be assured by placing PMUs within each utility area such that the generator buses inside that area becomes geometrically observable, measuring the voltage and currents at the PMU buses, and thereafter computing the generator bus voltage $V_i \angle \theta_i$ and current $I_i \angle \phi_i$ from those measurements. These steps are described next.

2.2. PMU Placement for Observability

In this section an area-wise optimal PMU placement (OPP) algorithm is provided, where the objective is to identify PMU bus locations (inside any area) such that the observability of generator buses is assured [18]. Our method differs from the classical OPP method [19] since our method does not require observability of all buses in the power network, thereby leading to a potentially lesser number of PMUs. The method is given as follows.

Let $N_\kappa$ be the total number of buses in the utility area $\kappa$, and $\mathcal{Y}_\kappa \in \mathbb{R}^{N_\kappa \times N_\kappa}$ is the indicator matrix for the admittance matrix of all buses in area $\kappa$. Also, let $P_{\kappa} \in \mathbb{R}^{N_\kappa}$ be the binary vector indicating the presence or absence of a PMU on a bus, i.e. for a vector element $P_{\kappa,i} = 1$ if bus $i$ has a PMU installed, $P_{\kappa,i} = 0$ otherwise. Then, for all areas $\kappa = 1, \ldots, M$, the following integer programming problem is solved:

$$
\mathcal{P}_{\kappa}^{\text{opp}} : \quad \min_{P_{\kappa}} c_{\kappa}^T P_{\kappa}
$$

s.t. $\mathcal{Y}_\kappa P_{\kappa} \geq \bar{I}_{N_\kappa}$ and $P_{\kappa,i} \in \{0, 1\}$,

where $c_{\kappa} \in \mathbb{R}^{N_\kappa}$ is the vector of relative costs for installing PMUs, and the elements of vector $\bar{I}_{N_\kappa}$ are given by:

$$
\bar{I}_{N_\kappa,i} = \begin{cases} 1 & \text{if bus } i \text{ is a generator bus} \\ 0 & \text{otherwise} \end{cases}
$$

The solution to the problem $\mathcal{P}_{\kappa}^{\text{opp}}$ will assure observability of all generator buses in the utility area $\kappa$, and also provides the optimal location of PMUs as the non-zero elements of the solution $P_{\kappa}^*$. It is noted that since this problem is non-convex, $P_{\kappa}^*$ is not unique but rather is one of the possible solutions with the minimum number of PMUs. It is assumed that the OPP problem is solved offline as part of the system planning.

2.3. Generator Bus Phasor Estimation

Next, using the voltage and current measurements from the PMUs located at buses identified from solving $\mathcal{P}_{\kappa}^{\text{opp}}$, the phasor voltages and currents for all generator buses inside the area $\kappa$ are estimated, described as follows.

For any area $\kappa$, let the magnitudes and phase angles of bus voltages and currents, recorded by the PMUs, be stacked in a vector $\chi^\kappa$. Let the covariance matrix for the measurement noise be $\Sigma_\kappa$. Let the voltage and current phasors at the generator buses in Area $\kappa$ be stacked in the vector $z_\kappa$. From Kirchhoff’s law:

$$
\bar{\chi}^\kappa = H_\kappa \bar{z}^\kappa + \epsilon^\kappa,
$$

where $\bar{z}^\kappa$ is obtained from a polar-to-rectangular coordinate transformation of $z^\kappa$; $H_\kappa$ is a known matrix of transmission line impedences; $\bar{\chi}^\kappa$ is obtained from polar to rectangular transformation of $\chi^\kappa$; and $\epsilon^\kappa$ is a noise vector with covariance matrix $\Sigma_\kappa = R \Sigma_\kappa R^T$ obtained from the linearization of the polar to rectangular co-ordinate transformation equations, as outlined in [20]. Please see the Appendix for details on construction of vectors $\chi^\kappa$, $\bar{\chi}^\kappa$, $z^\kappa$, $\bar{z}^\kappa$. $R$ is a rotation matrix. The vectors $\bar{z}^\kappa$ are obtained from the solution of the weighted least squares problem:

$$
\mathcal{P}_\kappa^{\text{ls}} : \quad \min_{\bar{z}^\kappa} \left[ (\bar{\chi}^\kappa - H_\kappa \bar{z}^\kappa)^T \Sigma_\kappa^{-1} (\bar{\chi}^\kappa - H_\kappa \bar{z}^\kappa) \right].
$$

The solution is given by $\bar{z}^\kappa_\kappa = G_\kappa \chi^\kappa$, where $G_\kappa = (H_\kappa^T \Sigma_\kappa^{-1} H_\kappa)^{-1} H_\kappa^T \Sigma_\kappa^{-1}$ is the estimation gain. The voltages and currents in $z^\kappa_\kappa$ are converted back to polar co-ordinates, and the resulting vector is denoted as $\bar{z}^\kappa$. The procedure is done for every utility area $\kappa$, and at every time instant $t$, thereby providing the estimate for all $z_i(t)$ as $\hat{z}_i(t), i = 1, \ldots, m$, in (2).

2.4. Generator Dynamics State Estimation

The continuous-time model (2) is next expressed in discrete-time using forward Euler transformation as:

$$
x_i(k+1) = x_i(k) + T g(x_i(k), z_i(k), u_i(k), \alpha_i),
$$

where $T$ is the sampling time, assumed to be the same for all generators. Using the estimate $\hat{z}_i(k)$, a Kalman-filter is next designed as:

$$
\hat{x}_i(k+1) = \hat{x}_i(k) + T g(\hat{x}_i(k), \hat{z}_i(k), u_i(k), \alpha_i),
$$

producing the state estimates $\hat{x}_i(k)$ for the $i^{th}$ generator at any instant $k$. For details of this state estimator $\hat{g}(\cdot)$, which represents the model of an Unscented Kalman Filter (UKF), please see [16].

3. Post-Disturbance Modal Participation

3.1. Linearized Model

As shown in [17], the variables $z_i(t)$ in (2) can be eliminated using the algebraic power flow equations for the network (skipped here for brevity) using a process called Kron reduction. The resulting state-space model is linearized about a given operating point, and is discretized using the sampling time $T$. The resulting model, with $n=5m$ number of total states, is written as:

$$
x(k+1) = Ax(k) + Bu(k), \quad \text{with } x_0 \triangleq x(0),
$$

(9a)
\[ y(k) = \Delta \omega(k) = C \hat{x}(k), \text{ and } u(k) \in U^m, \]  
(9b)

where \( x = [\Delta x_1, \ldots, \Delta x_n]' \), \( u = [\Delta u_1, \ldots, \Delta u_m]' \), \( A \in \mathbb{R}^{n \times n} \) is the state matrix, \( B \in \mathbb{R}^{n \times m} \) is the control input matrix, \( C \) is the block-diagonal binary matrix to select the small-signal rotor speeds \( \Delta \omega(k) = [\Delta \omega_1(k), \ldots, \Delta \omega_n(k)]' \), and \( U^m = U_1 \times \ldots \times U_m \) is the \( m \)-times cartesian product of the individual input constraint sets, as mentioned in Section 2.1. The model is assumed to be excited by an exogenous but vanishing disturbance such as a fault, whose effect is captured by the post-disturbance ‘initial’ state \( x_0 \). For this paper the rotor speeds \( \Delta \omega(k) \) are chosen as the performance variables \( y(k) \), which describe the kinetic energy of the system. Other electromechanical variables, such as paired difference of phase angles between generators describing the potential energy of the system can also be used in the design objective, if needed. It is assumed that \( A \) is bounded-input bounded-output stable, and \((A,B)\) is stabilizable.

3.2. Open-Loop Modal Analysis

The goal is to enhance the damping of the low-frequency oscillations of (9) in closed-loop. This objective is formulated using an online model predictive approach in terms of minimizing a quadratic energy function of \( y(k) \). The exact expression for the energy function will be provided in Section 4. The different steps of the design are described next, as listed in Section 1, over the rest of the paper.

The first step describes how the CCO, using \( x_0 \) estimated by various UKFs, estimates the modal residues of \( y(k) \), from which it can decide the sparsity structure of the wide-area communication network. From linear system theory it follows that \( x(k) = A^k x_0 \) for the unforced system (9). Equivalently, one may also write the state response in the modal decomposition form as:

\[ x(k) = \hat{M} \{\lambda_1, \ldots, \lambda_n\}', \]  
(10)

where \( \{\lambda_i\} \) are the eigenvalues of \( A \) (assumed to be distinct); \( \hat{M} = \text{col} (\alpha_1 p_1, \ldots, \alpha_n p_n) \) where \( \text{col}() \) denotes a matrix of column vectors; \( \{p_i\} \) are the right eigenvectors of \( A \); and \( \{\alpha_i\} \) are scalar weights. Let \( M = \text{col}(\rho_1, \ldots, \rho_n) \) be the modal matrix, \( \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_n) \), and \( \alpha = (\alpha_1, \ldots, \alpha_n)' \). From (10): \( x(k) = \hat{M} \Lambda^k \alpha \). Comparing this equation with the state equation in initial value form, and using the identity \( A^k = \hat{M} \Lambda^k M^{-1} \):

\[ \alpha = \hat{M}^{-1} x_0. \]  
(11)

Also from (9b), \( y(k) = C M \Lambda^k \alpha \), which means that the individual speeds can be written in the modal form as:

\[ y_i(k) = C_i \sum_{j=1}^n \alpha_j \rho_j \lambda_j^k = \sum_{j=1}^n \bar{\rho}_{ij} \lambda_j^k, \]  
(12)

The above expression (12) shows that once the CCO knows \( \hat{x}_0 \) (which depends on the magnitude and location of the unknown exogenous disturbance), it can estimate \( \bar{a} = \hat{M}^{-1} x_0 \) from (11), and therefore, the modal coefficients \( \{\bar{\rho}_{ij}\} \). Note that the implicit assumption here is that the CCO has full knowledge of the matrix \( A \). For damping control our design will be focused only on the inter-area modes (i.e. oscillation modes with frequencies between 0.1 and 2 Hz). However, dominance of these modes is defined not based on their frequencies, but on their residues. For example, consider a power system with five generators, with each generator considered as a coherent area in itself yielding 4 inter-area modes. The impulse response of the small-signal frequencies of the five generators can be written as:

\[ G_1 : y_1(k) = \bar{\rho}_{11} \lambda_1^k + \bar{\rho}_{12} \lambda_2^k + \bar{\rho}_{13} \lambda_3^k + \bar{\rho}_{14} \lambda_4^k + b_1(k), \]  
\[ G_2 : y_2(k) = \bar{\rho}_{21} \lambda_1^k + \bar{\rho}_{22} \lambda_2^k + \bar{\rho}_{23} \lambda_3^k + \bar{\rho}_{24} \lambda_4^k + b_2(k), \]  
\[ G_3 : y_3(k) = \bar{\rho}_{31} \lambda_1^k + \bar{\rho}_{32} \lambda_2^k + \bar{\rho}_{33} \lambda_3^k + \bar{\rho}_{34} \lambda_4^k + b_3(k), \]  
\[ G_4 : y_4(k) = \bar{\rho}_{41} \lambda_1^k + \bar{\rho}_{42} \lambda_2^k + \bar{\rho}_{43} \lambda_3^k + \bar{\rho}_{44} \lambda_4^k + b_4(k), \]  
\[ G_5 : y_5(k) = \bar{\rho}_{51} \lambda_1^k + \bar{\rho}_{52} \lambda_2^k + \bar{\rho}_{53} \lambda_3^k + \bar{\rho}_{54} \lambda_4^k + b_5(k), \]  
(13)

where \( b_i(k) = y_i^{dc} + \sum_{j=1}^4 \bar{\rho}_{ij} y_j^* \) with (*) denoting complex conjugation since \( \{\rho_{ij}\} \) and \( \{\lambda_j\} \) are, in general, complex numbers; \( y_i^{dc} \) represents the DC mode (equal to zero for small-signal rotor speeds); \( y_i^k \) is the high-frequency modal component; and \( \bar{\rho}_{ij} \) is the residue of the \( i \)-th mode in the \( j \)-th output. Let the residues \( \bar{\rho}_{11}, \bar{\rho}_{21}, \bar{\rho}_{22}, \bar{\rho}_{31}, \) and \( \bar{\rho}_{32}, \bar{\rho}_{33}, \bar{\rho}_{34}, \bar{\rho}_{41}, \bar{\rho}_{42}, \bar{\rho}_{43}, \bar{\rho}_{44}, \bar{\rho}_{51}, \bar{\rho}_{52}, \bar{\rho}_{53}, \bar{\rho}_{54} \), marked in boldface, be termed as dominant residues. Dominance is defined such that all \( |\bar{\rho}_{ij}| \geq \mu \), where \( \mu \) is a pre-specified threshold. This threshold can be chosen by the CCO in different ways, one possible choice being the arithmetic mean of all residues:

\[ \mu \triangleq \frac{1}{m(m-1)} \sum_{i=1}^m \sum_{j=1}^{m-1} |\bar{\rho}_{ij}|. \]  
(14)

In other words, it is assumed that only the inter-area modes \( \lambda_1, \lambda_2 \) are substantially excited by the incoming disturbance while the other inter-area modes have poorer participation in the rotor speed responses. The residue magnitudes are then collected in a so-called modal participation (MP) matrix that shows which generators contribute most to the excitation of which dominant mode.

For this example, generators \( G_1, G_2, G_5 \) contribute significantly to mode \( \lambda_1 \), and generators \( G_2, G_3 \) to mode \( \lambda_2 \). Information about this grouping is used to decide the topology of communication. Detailed description of this will be given in Section 5.

Note that two different disturbance events can result in two significantly different \( x_0 \), and hence, two significantly different MP matrices (see Figs. 4(a)-(c) for an example), indicating different sets of generators influencing different combinations of the inter-area modes. It is important for a controller to be aware of this dominance property instead of an offline controller that is agnostic to it. This is why our controller is designed in real-time after the fault.
4. Control Objective

Next, the control objective of our problem is formulated by deriving a cost function that reflects all the dominant modes identified by the CCO using the method described in Section 3.2. The concept of SDFT is introduced, first used in [4] for damping of selected modes in a power system. Let the output trajectory of the th generator, starting at time-step , be , , , , . The -point DFT of , , is proportional to the steepness of the peaks of (15). From left and right window edges respectively. For us the choice containing the corresponding rows.

The matrices , contain the real and imaginary parts of respectively. It is easy to show that for and , i.e. if the selected window encompasses the full frequency spectrum, then becomes an identity matrix, and therefore (19) becomes the usual quadratic energy of the outputs with unity weighting.

Remark 1. Note that the choice for the center of SDFT window is based on the frequencies of the open-loop eigenvalues of . Minimizing (19), however, would require an estimate of frequencies of the dominant eigenvalues of the closed-loop model. Hence, an implicit assumption behind the cost function (19) is that the applied control does not alter the frequency of closed-loop modes from their open-loop values to a significant extent. This assumption has been used in previous literature such as in [2], and is verified with simulation results in Section 7. If the frequencies of the inter-area modes change from open- to closed-loop, the width of SDFT window can accommodate this shift to some extent.

5. Communication Architecture for Control

The communication architecture for our controllers is developed next, based on the choice of influential generators for the dominant modes. A few notations are introduced first.

5.1. Notation

For an arbitrary set , its individual elements are denoted by . A power set is defined as the set of all its subsets, e.g. for , , and the power set is the collection (set of sets) . A mapping : is defined such that:

\[
\mathcal{P}_i = \begin{cases} 
\bigcap_{j \in \mathcal{P}_i} \mathcal{P}_i, & \text{if } \mathcal{P}_i \text{ is a collection}, \\
\mathcal{P}_i, & \text{if } \mathcal{P}_i \text{ is not a collection},
\end{cases}
\]

where is the th subset of the collection , which is itself the th subset of . Hence, after applying , no subset of is a collection. Also, let . Additional mappings , are defined, where maps a collection of intersecting elements to its symbolic form, i.e. , and maps any symbolic form to its set unions, i.e. .

5.2. Sparse Communication Architecture

Let the set of generators that exhibit the greatest influence on the excitation of the th dominant mode be denoted as , referred to as the th modal area. For the 5-machine example in Section 3.2, the dominant modes are , and the corresponding modal areas are , . Two or more modal areas can be overlapping iff any mode in the set of dominant modes are influenced by more than one generator. This is the case for generator in the example.
The proposed distributed control strategy then involves designing \( r = p + \hat{p} \) number of controllers, where \( p \) is the number of dominant modes, and \( \hat{p} \) is the number of overlappings between modal areas, if any. A single dedicated controller is assigned to each modal area, whereas generators in overlapping modal areas will have an additional controller of their own. The objective of each controller is to minimize the modal cost (19) formulated in Section 4. The underlying communication infrastructure is defined by:

(i) **Upstream links**: these links originate from the controller, and transmit control signals to its assigned generators, and

(ii) **Downstream links**: these links originate from the generators, and transmit their states to their respective controllers. Additionally, controllers may talk to a specific subset of other controllers, where this subset is decided by the overlapping modal areas. The intuition behind this communication strategy is that in order to design a controller \( C_i \), \( i = 1, \ldots, r \), that minimizes a modal cost corresponding to oscillation mode \( \lambda_i \), only those set of generators are important that show dominant participation in \( \lambda_i \) in their predicted output response.

The architecture can then be formalized as follows. Let the set of all identified \( p \) modal areas be: \( \mathcal{A} = \{ A_1, \ldots, A_p \} \). Since the modal areas can be overlapping, the non-empty intersecting subsets of \( \mathcal{A} \) are: a collection \( P(\mathcal{A}) = h(\mathcal{P}(\mathcal{A})) \) - \( \emptyset \). Let \( P(\mathcal{A}) = P^d(\mathcal{A}) + P^o(\mathcal{A}) \), where \( P^d(\mathcal{A}) \) contains the sets for dominant modal areas, and \( P^o(\mathcal{A}) \) contains the intersecting sets for overlapping modal areas. Then the set of generators which receive their control inputs from a distributed controller \( C_i \), \( i = 1, \ldots, r \), connected via the so-called upstream communication links, is given by:

\[
C^u_i = \begin{cases} \{ G_l \in P_i | G_l \notin P_j \} & \text{if } P_i \in P^d(\mathcal{A}) \\ \{ G_l \in P_i | g_1(P_l) \notin g_1(P_j) \} & \text{if } P_i \in P^o(\mathcal{A}) 
\end{cases} \tag{22}
\]

for all \( j = 1, \ldots, p \), \( j \neq i \), and where the generator index \( l \in \{ 1, \ldots, m \} \). \( P_i \) is the \( i^{th} \) element of the collection \( P(\mathcal{A}) \), and \( g_1(\cdot) \) is the symbolic mapping as defined in Section 5.1. The number of upstream links for the \( i^{th} \) controller is given by \( m_{u,i} = \text{card}(C^u_i) \). Expression (22) conveys assigning controllers to generators such that each controller is minimizing a unique modal cost specific to the generator’s modal area. Finally, the set of generators is constructed for downstream communication to each controller \( C_i \) as:

\[
C^d_i = \begin{cases} \{ G_l \in P_i \} & \text{if } P_i \in P^d(\mathcal{A}) \\ \{ G_l \in g_2(g_1(P_i)) \} & \text{if } P_i \in P^o(\mathcal{A}) 
\end{cases} \tag{23}
\]

where the mapping \( g_2(\cdot) \) is as defined in Section 5.1. The number of downstream links for the \( i^{th} \) controller is given by \( m_{d,i} = \text{card}(C^d_i) \). Expression (23) conveys assigning generators to the controller \( C_i \) from where state feedback is needed.

The steps for constructing this communication architecture over time are illustrated in Fig. 1 using the 5-machine example in (13). The physical states of the generators \( G_1 \sim G_5 \) are coupled with each other via kron reduction, and hence the leftmost figure shows all five generators to be connected to each other. Starting from time-step \( k = 0 \) (fault-clearing), the CCO uses a small number of time-steps, say \( k^* \), to compute the MP matrix, and determine that in this case the system needs to be divided into two modal areas \( A_1, A_2 \) with dominant frequency modes \( \lambda_1, \lambda_2 \), respectively, where \( A_1 = \{ G_1, G_2, G_5 \} \) and \( A_2 = \{ G_2, G_3 \} \). The power set is \( P(\mathcal{A}) = \{ A_1, A_2, \{ A_1, A_2 \}, \emptyset \} \), and \( h(P(\mathcal{A})) - \emptyset = P(\mathcal{A}) = \{ A_1, A_2, A_1 \cap A_2 = \{ A_1, A_2 \} + \{ A_1 \cap A_2 \} \approx P^d(\mathcal{A}) + P^o(\mathcal{A}) \). As shown, three controllers are then designed with respective sets of upstream and downstream links defined by the sets:

\[
C^u_i = \{ G_l \in P_i | G_l \notin P_j \} = \{ G_1, G_5 \} \tag{24a}
\]

\[
C^u_i = \{ G_l \in P_i | G_l \notin P_j \} = \{ G_3 \} \tag{24b}
\]

\[
C^u_i = \{ g_1(P_l) \notin A_1 \} = \{ G_2 \} \tag{24c}
\]

\[
C^d_i = \{ G_l \in P_j \} = \{ G_1, G_2, G_5 \} \tag{24d}
\]

\[
C^d_i = \{ G_l \in P_2 \} = \{ G_2, G_3 \} \tag{24e}
\]

\[
C^d_i = \{ G_l \in g_2(g_1(P_i)) \} = \{ G_1, G_2, G_3, G_5 \} \tag{24f}
\]

where \( g_1(P_1) = g_1(A_1 \cap A_2) = A_2 \), and \( g_2(g_1(P_3)) = A_1 \cup A_2 \). Sets (24a), (24b), (24d), (24e) correspond to the condition \( P_i \in P^d(\mathcal{A}) \) being true, and the sets (24c),(24f) correspond to the condition \( P_i \in P^o(\mathcal{A}) \) being true. These links can be seen in Fig. 1 for all the three controllers \( C_1 \sim C_3 \). The presence of a third controller is due to the overlap between the modal areas, resulting in controller-to-controller links denoted by \( U_{1,3} \) and \( U_{2,3} \). All controllers are triggered into action starting \( k = k^* \). It is noted that the system shown in Fig. 1 is a simple example used for illustrative purposes. For power systems with much larger size, the savings in communication from our distributed controller over a centralized controller will be significant, as will be shown in Section 7.

5.3. Cyber-Physical Architecture

It is noted that since the indices of generators associated with any controller \( C_i \) are not fixed *a priori* due to the unpredictable nature of disturbances, it is not advisable to install \( C_i \) at any fixed generator site, but rather in a shared computational platform such as a cloud network. Such a cloud-based cyber-physical architecture for wide-area control has been recently proposed in [10, 21], and can be a very promising medium for implementing our proposed distributed controller. This is explained as follows.

Each utility area is assumed to have a Phasor Data Concentrator (PDC). Each PDC is assumed to have access to a local cloud at its local control center, which is part of a larger cloud network referred to as the Internet of Clouds. PMUs in the utility area \( \kappa \) measure the phasor voltages and currents at designated buses, and transmit them to
their local PDC. The PDC in area $\kappa$ then transmits this information to the local cloud. State estimation for all generators in this utility area is done with an estimator $E_k$ using (5)-(8) in the local cloud.

This process goes on round the clock. At time $k=0^{-}$ a fault occurs in the system. At time $k=0^{-}+\Delta$, the UKF is assumed to reach steady-state, meaning onwards from $k=0^{-}+\Delta$, $x_k$ is regarded as a statistically close estimate of the true state $x_k$ for area $\kappa$. Let $k=0^{-}+\Delta$ be denoted as $k=0$, which is the starting point for the steps needed for computing our control inputs. In reality, the UKFs can be fast enough so that $\Delta$ is a very small time interval. The state vector $x_k(0)$ is communicated from the local clouds to the CCO. The CCO, after receiving the entire state vector $x_k(0)=x_0(0), \ldots, x_M(0)$, decides the communication topology following the steps outlined in Section 5.2. The CCO then informs the controller assignments to the local clouds, where controllers $C_i$, $\forall i=1, \ldots, r$, are created. At every time-step, after receiving the estimated states according to the set $C_i^d$, these controllers then solve the dMPC optimization problem (to be described in Section 6). Finally, the local clouds communicate the computed control inputs to generator actuators according to the set $C_i^a$. The inter-cloud information exchange is done via wide-area communication network. Please see Algorithm 1 for step-by-step description of this implementation.

The above procedure is illustrated with the 5-machine example (13), as shown in Fig. 2. For simplicity, it is assumed that each of the five machines belong to a distinct utility area. Since $G_4$ is not contained in any of the modal areas, only four clouds (corresponding to utility areas 1, 2, 3 and 5) are shown. To minimize communication inside the internet of clouds, $C_1$ is placed in the local cloud for utility area 1, $C_2$ in the local cloud for utility area 2, and $C_3$ in the local cloud for utility area 3. Cloud-to-cloud links $T_{ij}$, where $i, j$ are the indices for sending and receiving clouds respectively, are set up for the controllers to exchange necessary information. Their information content is given in the caption of Fig. 2. In this figure the previous time-step control trajectory for the $i^{th}$ generator is denoted by $\tilde{u}_i(k-1)=u_i(k|k-1), \ldots, u_i(k+N_c-1|k-1)$.

6. Distributed MPC

Using the communication architecture defined above, a distributed MPC control problem is formulated next.

6.1. Distributed Controller Prediction Modelling

To select a subset of states, outputs and control inputs, let $z_i=T_{x_i}x, \quad v_i=T_{v_i}u, \quad w_i=T_{w_i}u, \quad \eta_i=T_{\eta_i}y$, where $T_{x_i}\in\mathbb{R}^{n_{d,i}\times n}$, $T_{v_i}\in\mathbb{R}^{m_{a,i}\times n}$, $T_{w_i}\in\mathbb{R}^{m_{d,i}\times m}$ and $T_{\eta_i}\in\mathbb{R}^{m_{a,i}\times 1}$ are indicator matrices whose structures follow from the communication architecture for controller $C_i$. $n_{d,i}$ represents the total number of states for the generators associated with the downstream links set $C_i^d$, $z_i$ is the vector of states belonging to generators associated only with the downstream links; $v_i$ is the vector of optimized control inputs to actuators only associated with the upstream links; $\eta_i$ is the vector of outputs from generators only associated with the downstream links; $w_i$ is the vector of communicated control inputs computed at the previous time-step by other controllers, and communicated to $C_i$. The model for the set of generators associated with $C_i, \forall i=1, \ldots, r$, can then be written as:

$$z_i(k+1)=A_{z_i}z_i(k)+B_{v_i}v_i(k)+B_{w_i}w_i(k)+\hat{A}_{z_i}z_i^e(k),$$

$$\eta_i(k)=C_{\eta_i}z_i(k),$$

where $A_{z_i}=T_{z_i}A_{z_i}, \quad B_{v_i}=T_{v_i}B_{v_i}, \quad B_{w_i}=T_{w_i}B_{w_i}$ and $C_{\eta_i}=T_{\eta_i}C_{\eta_i}$. The term $A_{z_i}z_i^e(k)$ can be seen as the effect of generator dynamics not communicated to $C_i$, with the vector $z_i^e(k)$ containing the states for those generators. Moving (25) forward in time, the output prediction trajectory with horizon $N$ and control horizon $N_c$ can be written as:

$$\tilde{\eta}_i(k)=\Lambda_i z_i(k)+\Phi_{v_i}v_i(k)+\Phi_{w_i}w_i(k)+\Psi z_i^e(k),$$

where $\tilde{\eta}_i, v_i, w_i, z_i^e$ denote the trajectories:

$$\tilde{\eta}_i(k)=[\eta_i(k|k), \ldots, \eta_i(k+N-1|k)]',$$

$$v_i(k)=[v_i(k|k), \ldots, v_i(k+N_c-2|k)]',$n

$$w_i(k)=[w_i(k|k), \ldots, w_i(k+N_c-2|k)]',$n

$$z_i^e(k)=[z_i^e(k|k), \ldots, z_i^e(k+N-2|k)].$$

and $\Lambda_i, \Phi_{v_i}, \Phi_{w_i}, \Psi_i$ are block matrices easily constructed from $A_{z_i}, B_{v_i}, B_{w_i}, C_{\eta_i}, A_{z_i}$.

The expression in (26) represents the nominal prediction trajectory, i.e. assuming no unmeasured dynamics, (26) will provide accurate predictions. But due to the distributed nature of the communication architecture, the controller $C_i$ does not have access to $w_i(k)$ and $z_i^e(k)$. Hence, for our control design $w_i(k)$ is considered to be the control trajectory computed at the previous time-step by another controller $C_j$, and make sure that $C_i,C_j$ communicate so that $C_i$ can utilize this trajectory for its local predictions. For this reason the difference between the optimized control trajectory at the current and previous time-steps is penalized in (28). Also, because the states are not directly measurable, $C_i$ will only receive $\hat{z}_i(k)$ for feedback instead of $z_i(k)$. In the subsequent sections the notations for the variables defined in (27) are used, but it is assumed that the RHS follows from $\hat{z}_i(k)$ instead of $z_i(k)$.

6.2. dMPC with Modal Cost

The objective of each dMPC controller $C_i$ is to minimize the energy content of the SDFT spectrum of the generators assigned to it, while also respecting both actuation and communication constraints. For a controller $C_i, \forall i=1, \ldots, r$, a cost function using the feedback $\hat{z}_i(k)$ is formulated as:

$$J_i(k)=c_i\sum_{k=0^{-}+\Delta}^{T_{\text{end}} \Delta} e^{j\omega_t (k)}$$

where $\omega_t$ represents the total number of states for the generators associated with the downstream links set $C_i^d$. $c_i$ is the cost function using the feedback $\hat{z}_i(k)$.
Figure 1: Architecture of the proposed distributed control system, shown on a five-generator power system example, following (13). Subfigure (a) shows the physical interconnections between generators in the Kron-reduced form. CCO receives $\hat{x}_0$ from all generators and using the MP matrix decides the communication architecture, within the time-steps $k=0:k^*$. CCO then informs all generators about the communication topology. Subfigures (b) and (c) show state and control communications respectively for $k\geq k^*$, with the three dMPC controllers in feedback. The two identified modal areas are highlighted in red and blue.

Figure 2: Cyber-Physical Architecture for dMPC, shown on the five-generator example, following (13). For this example, since it is assumed that each generator bus has a PMU installed, the output of the $i^{th}$ PDC will be $\bar{V}_i$ as the generator bus voltage $|V_i|$ and bus angle $\theta_i$, and $\bar{I}_i$ as the vector of all phasor line currents $[|I_1|, \phi_1, |I_2|, \phi_2, \ldots]$ measured on all transmission lines connected to the generator bus. The cloud-to-cloud communication links are given as: $I_{21}(k)$=$I_{32}(k)$=$\{x_2(k), u_2(k-1)\}$, $I_{51}(k)$=$\{x_5(k)\}$, $I_{12}(k)$=$\{x_1(k), u_1(k-1)\}$, $I_{52}(k)$=$\{x_5(k), u_5(k-1)\}$, $I_{23}(k)$=$\{x_3(k), u_3(k-1)\}$ and $I_{13}(k)$=$\{u_3(k)\}$. 

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\[ J_i^{dist}(\tilde{z}_i(k)) = \eta_i(k)' Q_i \eta_i(k) + \tilde{v}_i(k)' R_i \tilde{v}_i(k) + \Delta \tilde{v}_i(k)' S_i \Delta \tilde{v}_i(k), \] (28)

where \( \Delta \tilde{v}_i(k) = \tilde{v}_i(k) - \tilde{v}_i(k-1) \) is the difference between the current and previous time-step optimized control trajectories; \( R_i \) and \( S_i \) are positive definite weighting matrices of size \( m_{a,i} \times N_c - 1 \); and \( Q_i \) is a semi-positive definite block diagonal output trajectory weighting matrix of size \( m_i \times N \). Section 2. The area-wise OPP problem \( P_{opp} \) is solved for all \( k=1, \ldots, 6 \), where the cost of installing PMUs is assumed to be unity for all buses. Locations for 39 PMUs are identified to assure observability of all generator buses. These buses are highlighted in red in Fig. 3. Voltage, current, and their phase angle measurements are recorded from these buses. The measurement noise is assumed to be zero-mean Gaussian noise. The noise variance values are obtained from the study in [24], where a signal-to-noise ratio of 45 dB was concluded to be a good approximation for PMU data. This data is used to estimate the phasor currents and voltages at the generator buses. Next, decentralized UKF is used to estimate generator states as detailed in Section 2.4. State estimation results for generator 1, in open-loop, are shown in Fig. 5. The results are seen to be consistent with the ones reported in [16].

7.2. Post-Disturbance Modal Analysis

The three scenarios considered for post-disturbance analysis on the NPCC model are:

(i) Case Study I: The system is perturbed with a three-phase fault on the transmission line connecting buses 10-11. Starting the simulation in steady-state at \( t=0 \) secs, the fault is applied at 1.1 secs, which is then cleared at 1.2 secs at bus 10, and at 1.25 secs at bus 11.

(ii) Case Study II: System is perturbed with a three-phase fault on the transmission line connecting buses 45-46. The fault is applied at 1.1 secs, cleared at 1.2 secs at bus 45, and at 1.3 secs at bus 46.

(iii) Case Study III: System is perturbed with a three-phase fault on the transmission line connecting buses 119-120. The fault is applied at 1.1 secs, cleared at 1.3 secs at bus 119, and at 1.45 secs at bus 120.

Note that the duration of the fault for Case Study I is smaller compared to other events. Since the eastern part of the power network is only connected via two buses (buses 29 and 35) to the rest of the grid, and due to the brief duration of this fault, it can be seen as a relatively ‘localized’ disturbance as compared to the other two events which occur on critical buses for longer durations. Once the fault is cleared at the remote end of the faulted line, the estimated post-disturbance system state \( x_0 \) is used to construct the MP matrix whose elements are shown pictorially in Figs. 4(a)-(c). The residues for these different events can clearly be seen to be different from each other. Hence, it is clear that different sets of generators are influencing different sets of inter-area modes depending on the magnitude and location of the fault.
Algorithm 1 Algorithm for Implementing Proposed dMPC Controller

1: Location of PMUs is determined offline by solving the OPP problem $\mathcal{P}_{\text{opp}}^\kappa$ described in Section 2.2 for all utility areas $\kappa = 1, \ldots, M$. PMUs are then installed at these locations.

2: At all times (before, during, after a fault), each local PDC continuously collects voltage and current measurements from all PMUs in area $\kappa$, and sends this data to its local cloud as shown in Fig. 2.

3: In each local cloud, this measurement data is fed to a local estimator $E_\kappa$ which first estimates the phasors for all generator buses by solving $\mathcal{P}_{ls}^\kappa$ in (6), and then estimates the generator states $\hat{x}_\kappa$ using the Kalman filter (8), for all generators in area $\kappa$. These states $\hat{x}_\kappa$ are used for the continuous monitoring of area dynamics.

4: At $k = 0$, a disturbance enters the grid, which is subsequently detected by all local clouds.

5: At $k = 0$, i.e. when $\hat{x}_\kappa(0) \approx x_\kappa(0)$, all local clouds send their estimated states to the CCO, thereby providing CCO with the full state vector $\hat{x}_0$.

6: Following the control design steps provided in Sections 3-6, the CCO informs the local clouds to set-up distributed MPC controllers $C_i$, $\forall i = 1, \ldots, r$.

7: For all utility areas $\kappa = 1, \ldots, M$, the control loop is then iterated as follows, for $k \geq 0$.

8: while any local cloud detects significant oscillations in $\hat{x}_\kappa$ do

9: States $\hat{x}_\kappa(k)$ are distributed among the controllers according to (23) using wide-area communication links, as shown in Fig. 2 for the 5-machine example system.

10: Using (25)-(30), every dMPC controller $C_i$ solves the problem $\mathcal{P}_i$, and then sends its control inputs to generators according to (22), as shown in Fig. 2(b).

11: Advance the control time-step $k \leftarrow k + 1$.

12: end while

13: All wide-area links are terminated, and the local clouds continue monitoring the steady-state state-estimates $\hat{x}_\kappa$.

Figure 3: The one-line diagram of the NPCC model shows the utility areas in background colors, the three faults considered for the three cases at lines connecting buses 10-11, 45-46 and 119-120. Modal areas for Case Study I are also shown, enclosed in dotted boundaries.
7.3. Distributed Control Design

For brevity, steps for our control design are provided next with respect to Case Study I only. Control designs for Case Studies II and III are done in a similar manner and their results are summarized in Table 1.

From the residues shown in Fig. 4(a), \( \mu = 1.03 \) is calculated for the MP matrix. Two modal areas are then constructed as \( A_1 = \{ G_1, \ldots, G_9 \} \) and \( A_2 = \{ G_{11}, G_{12}, G_{14} \} \), highlighted in Fig. 3 with dotted boundaries. It is noted that for this particular disturbance, only 12 out of 48 generators are influential in the dominant inter-area modes. The remaining 36 generators do not need to participate in wide-area control. Since for this case the intersecting sets between the non-overlapping modal areas are empty, \( P_i = A_i, \forall i = 1, 2 \). Design of \( r=2 \) distributed controllers is done for the two modal areas using the SDFT frequency windows (in Hz): \([0.53, 0.73]\) for \( C_1 \) and \([0.86, 1.2]\) for \( C_2 \). The open-loop frequency response for all rotor speeds is shown in Fig. 6. The windows are chosen by the CCO from the predicted open-loop frequency response. The input constraints enforced on the control signals are \(-0.1 \leq u_i(t) \leq 0.1, \forall t > 0\), allowing for a maximum of 10% supplementary control effort in the excitation voltage. The optimization problem \( P_i \) in (29) is solved for the two controllers, with control weightings \( R_1 = R_2 = \text{diag}(0.1, \ldots, 0.1) \) for less emphasis on the magnitude of control inputs. Additionally, \( S_1 = S_2 = \text{diag}(0.01, \ldots, 0.01) \) to obtain a less conservative control policy. The prediction horizon is chosen as \( N_c = 10 \) keeping in mind the trade-off for unmeasured dynamics and SDFT resolution, as discussed in Remark 2. The control horizon is kept small for lower execution times at \( N_r = 10 \). The optimization toolbox in Matlab is used to solve the constrained QP (29) with the interior-point convex algorithm.

7.4. Closed-Loop Results

Figs. 7(a)-(b) show the rotor speed output for the generators 7 and 8, in open- and closed-loop. Closed-loop results are shown for both a centralized MPC implementation and the dMPC implementation. The centralized MPC receives state estimates of all 48 generators as feedback, and sends control inputs to all 48 generators, and solves a single optimization problem at every time-step. In Figs. 7(a)-(b)
it is seen that the dMPC performs close to the optimal centralized performance. Fig. 7(c) shows the comparison between the centralized MPC and the dMPC control input voltages, for both generators 7 and 8. As shown, the control voltages lie in the constraint set $[-0.1, 0.1]$.

A comparison of open-loop versus dMPC closed-loop electrical power outputs for selected generators is shown in Fig. 8. It can be clearly seen that the dMPC controller suppresses the oscillation amplitudes successfully for power output of all generators, even the ones not included in the control design. From the closed-loop frequency responses, it is observed that the dominant mode 1 (around 0.6 Hz) show a 40.2% reduction in FFT peak, and dominant mode 2 (around 1 Hz) show a 14.6% reduction. It is also observed that the frequency of the two dominant modes are almost same in open- and closed-loop.

Average controller optimization times are reported in Table 1, and for the considered case studies, observed to be less than the sampling time of 0.1 secs. The optimization times increase with increasing the control horizon $N_c$ and also with increasing the number of generators to be controlled with a single controller. For systems with even larger number of generators per controller, the optimization times can scale up rapidly. In such a case, priorities can be given to only those generators with the highest modal residues in oscillations, or faster computational resources can be provided to the local clouds. It is observed that a longer control horizon does not result in a significant improvement in performance and hence is kept small. For the centralized MPC controller, the average optimization time is observed to be 0.3 secs, which is larger than the considered sampling time of 0.1secs. All simulations are performed on a 3.8 GHz quad-core Intel i7 processor with 16 GB RAM.

### 7.5. Performance vs. Sparsity

To evaluate the trade-off between closed-loop performance and sparsity, a performance metric is defined as:

$$\mathcal{J} = \frac{1}{T_{ss}} \sum_{i=1}^{m} \sum_{k=1}^{T_{ss}} |y_i(k) - 1|^2,$$

(31)

where $T_{ss}$ is the maximum settling time for all outputs, and $y_i$ is the rotor speed of the $i^{th}$ generator, with a steady-state per unit value of 1. Let the open-loop cost, closed-loop centralized MPC cost, and the dMPC closed-loop cost be denoted by $\mathcal{J}^{ol}$, $\mathcal{J}^{cent}$ and $\mathcal{J}^{dmpc}$, respectively. The dMPC performance loss index is then defined as:

$$\xi = \frac{\mathcal{J}^{dmpc} - \mathcal{J}^{cent}}{\mathcal{J}^{ol} - \mathcal{J}^{cent}}.$$

(32)

Note that (32) normalizes the dMPC cost between [0, 1] with 0 representing the optimal centralized cost, and 1 representing the open-loop cost. The closer $\xi$ is to 0, the better is the dMPC performance. A sparsity index is also defined as the ratio between the number of unidirectional communication links required by the dMPC controller, to that required by the centralized MPC. Since a centralized MPC will essentially require communication with all generators, the number of required links will be $2m$. Hence the sparsity index is given by:

$$\theta = \frac{1}{2m} \sum_{i=1}^{r} \text{card}(C_i^d) + \text{card}(C_i^u) + N_i,$$

(33)

where $\theta \in [0, 1]$, and $N_i$ is the number of controller-to-controller communication links needed due to possible overlapping modal areas. The closer $\theta$ to zero, more is the sparsity ($\theta$ to be exactly 0, however, has no physical meaning in this case as that would mean that there is no dMPC controller).

Table 1 provides a summary of main results for the three case studies listed in Section 7.2. Since the fault in Case
of dominant inter-area modes, resulting from a major disturbance in the network, are first identified. Generators which have highest contribution to these modes are then identified to form the communication topology for distributed control. Energy content of the dominant modes is extracted from the output open-loop frequency spectrum using the SDFT method. A dMPC problem is then solved for each distributed controller, under communication and actuation constraints. Simulations performed on the NPCC model show effectiveness of the control design by successfully eliminating sustained low-frequency oscillations, while also saving on both communication links and computation times when compared to a centralized control implementation.

Appendix A. Construction of $\chi^\kappa$, $\bar{\chi}^\kappa$, $z^\kappa$, $\bar{z}^\kappa$

Let $I^\kappa_g$ be the index set of all buses in area $\kappa$ that contains a PMU, and let $I^\kappa_g$ be the index set of all generator buses in area $\kappa$. $\chi^\kappa$ is constructed as the ordered vector $\chi^\kappa = \{V_i, \theta_i, I_i, \phi_i\}, \forall i \in I^\kappa_g$. Here, $I_i$ and $\phi_i$ denote the vector of magnitudes and phase angles of the currents leaving bus $i$, $z^\kappa$ is constructed as the ordered vector $z^\kappa = \{V_i, \theta_i, I_i, \phi_i\}, \forall j \in I^\kappa_g$. $\bar{\chi}^\kappa$ is obtained from polar to rectangular transformation of $\chi^\kappa$, i.e., $\bar{\chi}^\kappa = \{V_i \cos \theta_i + j V_i \sin \theta_i, \ldots\}, \forall i \in I^\kappa_g$. $\bar{z}^\kappa$ is obtained from polar to rectangular transformation of $z^\kappa$, i.e., $\bar{z}^\kappa = \{V_i \cos \theta_i + j V_i \sin \theta_i, \ldots\}, \forall i \in I^\kappa_g$.

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