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Mini-course at MTNS 2016, Minnesota, USA

Composite Control of Networks via Singular Perturbation Theory

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Motivation & Objectives

Systems that communicate through networks are ubiquitous in this day and age. The complexity of such systems is continuously increasing as a result of the decreasing cost of communication (with the prevalence of the internet for example) and the growing number of *smart* devices used for monitoring and actuation. There is also, on the other hand, an increasing need to take the different privacy and security concerns of the interconnected systems into consideration. However, many conventional analysis and design methods, such as the ones that make use of the Information Technology (IT) concepts, may not be adequate in dealing with such systems, especially if they are dependent on central monitoring and control strategies. This necessitates the need for adapting alternative control-theoretic methods that help in re-framing the complex and potentially large-scale problems into relatively convenient ones for which efficient solutions can be found.

Taking deeper look at many natural and engineered networks, such as social networks, wireless sensor networks and power systems networks, it can be seen that they tend to be organized into a number of interconnected *clusters*. The notion of clustering here is loosely defined as the ratio between the strength of the internal connections to the external connections. This special structure is shown to exhibit two time-scales behavior, where the average of the states within each cluster evolve in a slower time-scale compared to the relative motion of the internal states. This motivates considering *singular perturbation theory* as a main tool in their analysis and control. Singular perturbation theory has been shown to be effective in dealing with systems that exhibit multi-time-scale properties. It can effectively allow us to consider each time-scale component of the system separately and, hence, can lead to considerable simplification of the entire system dynamics. By decomposing the system into its slow and fast components, one can also have a deeper understanding of the system dynamics and can design targeted control strategies independently for each time-scale dynamics. Traditionally, however, singular perturbation has been used to primarily study model reduction of standalone centralized systems without consideration to any spatial structure, and to study how these reduced-order models can be used for control designs. Our focus, in contrast, is to explore how network-theoretic concepts can be merged with these model reduction and control techniques. More specifically, we will combine graph theory and singular perturbation theory to show how network structure can be an additional design component that can be exploited for conventional singular perturbation-based designs.

From a different prospective, the majority of the so called control problems of the cyber-physical systems, where networks constitute a vital part of their structure, so far have either been addressed very incrementally, or have been defined in a broad and intuitive way without rigorous mathematical formulation. As a step towards bridging this gap, we propose to organize this mini-course in MTNS 2016 to inform and educate our control audience, especially young students and scientists, about the tremendous potential of exploiting two-time-scale properties of networks for the purpose of their analysis and control. In particular, our goal in this mini-course is to educate control engineers about modeling and control of multi-time-scale networked systems, to formulate problems on making the control systems *distributed* and *scalable*, and to point out how singular perturbation theory can play an instrumental role in solving these problems. The total duration of the tutorial will be 125 minutes. It will consist of 3 talks of respective lengths 50 minutes, 50 minutes and 25 minutes. Details of these talks are given as follows.

Time schedule

Talk	Duration	Title	Speaker
Talk 1	50 minutes	Singular Perturbation Theory and its Use in State-Feedback Control	Almuatazbellah Boker
Talk 2	50 minutes	Two-Time-Scale Modeling and Control of Clustered Networks	Almuatazbellah Boker, Aranya Chakrabortty
Talk 3	25 minutes	Role of Singular Perturbation for Attack Localization in Graphs	Thomas Nudell

Description of Talks

- **Talk 1: Singular Perturbation Theory and its use in State-Feedback Control**

  - This 50-min talk will cover the fundamental aspects of singular perturbation theory. In particular, it will cover the following topics:

  1. Modeling techniques of two-time scale systems: a discussion on the standard form of singularly perturbed systems.
  2. Linear time-invariant systems: brief review of block-triangular and block-diagonal forms and their eigenspace properties; observability and controlability properties.
  3. Composite control of linear systems: eigenvalues assignment; near optimal regulators; high-gain feedback; robust output-feedback design.
  4. Composite control of nonlinear systems: stability analysis.

- **Talk 2: Two-Time-Scale Modeling and Control of Clustered Networks**

  - This 50-min talk will cover the following topics:

  1. Two-time scale modeling of clustered networks: representing clustered networks in the standard singular perturbation models; review of multi-cluster modeling technique, where each cluster can have an approximate and unique model that represents the whole network.
  2. Cluster-level composite control of networks: overview of the cyber-physical structure of the closed-loop system, control aggregation concept.
  3. Control problems/applications: distributed consensus control; design of robust controllers with respect to inter-cluster feedback communication delays.

- **Talk 3: Role of Singular Perturbation for Attack Localization in Graphs**

  - This 25-min talk will cover the following topics:

  1. The graph Laplacian of clustered networks: eigenvalues and eigenvectors.
  2. Discrete nodal domains of Laplacian eigenvectors: an introduction to basic concepts and theory.
  3. Discrete nodal domains and transfer function residues: a brief theoretical foundation.
  4. Tractable attack localization algorithms in clustered graphs, i.e., putting it all together: singular perturbation, transfer function residues, and discrete nodal domains.

All the talks will be made interactive to promote participation from the audience. Matlab demos, visualization charts and graphics will be shown as much as possible to illustrate theoretical concepts. A detailed handout listing references to textbooks and online study materials for further reading will be provided to the attendees at the end of the course.

Our target audience for this course are faculty members, postdoctoral researchers, graduate students, and industry practitioners who want to learn about the recent advancements in the field of control of clustered networks and the constructive role that singular perturbation theory can play in this field. The session will provide an insightful thrust to network system education for control engineers, especially for new graduate students who are looking for fresh research problems. We will encourage interested audience to interact with the speakers after their talks, thereby promoting the chances of collaborative research.