SOME QUICK POINTS FOR SOLVING PROBLEMS ON 3-PH TRANSFORMERS WITH UNBALANCED LOADS

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These types of problems will always consist of 3 elements:

1. The transformer primary (connected in either $Y$ or $\Delta$)
2. The transformer secondary (connected in either $Y$ and $\Delta$)
3. The load impedances (connected in either $Y$ or $\Delta$)

First of all, ALWAYS remember the following four points:

1. Even if the load is unbalanced, the **Line to Line** voltage for all three elements will remain ‘perfectly’ balanced. For example, if the transformer rating is given as 4 KV $Y/400$ V $\Delta$, then that means that no matter what the load is, the line voltage phasors on the primary will always have a magnitude of 4 KV and will 120 degrees apart from each other, and the line voltage phasors on the secondary will always have magnitude 400 V and will be 120 degrees apart from each other.

2. Naturally from the above point, it also means that the phase voltages of the primary and secondary will also be perfectly balanced, respectively.

3. Caution! - However, the above point does not necessarily mean that the phase voltage of the load will be balanced as well. If the load is connected in $\Delta$ then that is true (since for $\Delta$ line and phase voltage mean the same), but if the load is connected in $Y$ then you cannot claim that the phase voltage across this $Y$ is balanced any more because, as we saw in lecture 7, due to the unbalanced impedances the neutral point of the $Y$ will be ‘floating’, and hence there is no way to determine what the potential of the neutral is. More on this later!

4. Since the load is connected across the secondary, the line-to-line voltage of the secondary and that across the load will always be the same.

Next, consider the following possible combinations for the connections of the three elements. The objective is to determine the line currents drawn from the primary side when the load is unbalanced.
1. **Primary in \( \Delta \), Secondary in \( Y \), Unbalanced load in \( \Delta \)**

This is the most convenient case. Say the transformer rating is given as 4 KV \( \Delta \)/ 400 V \( Y \). This means that the line-to-line voltage across the secondary, and hence across the load is 400 V.

Before you start, compute the quantity \( a \), which we defined as the turns ratio of each transformer in the 3-phase bank. Remember that

\[
a = \frac{\text{phase voltage on secondary}}{\text{phase voltage on primary}}
\]

If the transformer rating is 4 KV \( \Delta \)/ 400 V \( Y \), then

\[
a = \frac{400}{\sqrt{3}} \frac{1}{4000} = \frac{1}{10\sqrt{3}}.
\]

Next, follow the steps listed below:

1. Denote the secondary line voltages as \( 400\angle 0 \), \( 400\angle -120 \), and \( 400\angle 120 \). As mentioned in (4) on page 1, these are also the line voltages across the load. Since the load is in \( \Delta \) it means that these are also the phase voltages across the load. The load impedances will be typically given to you, so find the current phasors flowing through each impedance using Ohm’s law. These currents will be the phase currents in the load.

2. Apply KCL at the terminals of the loads, and calculate the line currents flowing into the load by proper phasor subtraction. These currents will also be the line currents flowing out of the secondary.

3. Since the secondary is in \( Y \) this means that the calculated line currents are the **phase currents** flowing through the corresponding phases of the secondary.

4. From transformer law, we know that the current in each phase of the secondary gets reflected in the corresponding phase of the primary linearly. Hence, compute the primary phase currents by simply multiplying the secondary phase currents by \( a \).

5. Finally, apply KCL at the primary terminals to calculate the line currents by proper phasor subtraction. This will give you the line currents drawn from the primary. You can verify that the three current phasors add up to zero - i.e., they are balanced. But they will not be perfectly balanced.

2. **Primary in \( \Delta \), Secondary in \( Y \), Unbalanced load in \( Y \)**

This case is bit trickier than Case 1. Since the load is unbalanced and in \( Y \), hence it is no longer possible to compute the phase voltage of the load because of the floating neutral problem. Hence, the only thing that you may do in this case is to convert the \( Y \) load to an equivalent \( \Delta \)-load by star-delta conversion formulae. Once that is done, then the case
becomes exactly similar to Case 1. Therefore, follow the steps listed for Case 1 to find the primary line currents.

3. Primary in $\Delta$, Secondary in $\Delta$, Unbalanced load in $\Delta$

Say, the transformer rating is 4 KV $\Delta$/ 400 V $\Delta$. Then $a = \frac{400}{4000} = \frac{1}{10}$. Next, follow the steps listed below:

1. Denote the secondary line voltages as $400 \angle 0$, $400 \angle -120$, and $400 \angle 120$. As mentioned in (4) on page 1, these are also the line voltages across the load. Since the load is in $\Delta$ it means that these are also the phase voltages across the load. The load impedances will be typically given to you, so find the current phasors flowing through each impedance using Ohm’s law. These currents will be the phase currents in the load.

2. Apply KCL at the terminals of the loads, and calculate the line currents flowing into the load by proper phasor subtraction. These currents will also be the line currents flowing out of the secondary.

3. Recall how the $\Delta - \Delta$ situation for balanced load was handled in lecture 6. There we saw that for balanced loads, the secondary line currents and primary line currents are linearly related to each other. It turns out that this is also true for unbalanced load. Hence, simply multiply the calculated secondary line currents by $a$ to obtain the primary line currents. You can, again, verify that these currents add up to zero, i.e., they are balanced but not perfectly balanced.

4. Primary in $\Delta$, Secondary in $\Delta$, Unbalanced load in $Y$

Here, again, the floating neutral problem will arise. Hence, simply convert the $Y$ load to an equivalent $\Delta$ load by star-delta conversion. Once that is done, then this case becomes exactly the same as the previous case. Therefore, follow the steps listed above to obtain the primary line currents.

All of these 4 cases can be repeated with the primary being in $Y$. In practice, however, the primary is not preferred to be in $Y$ so that the line currents can be maintained to be balanced. However, for the sake of completeness, let us discuss these cases also.
5. **Primary in Y, Secondary in Y, Unbalanced load in Δ**

Say, the transformer rating is 4 KV $Y/400$ V $Y$. Then $a = \frac{400}{\sqrt{3}} = \frac{1}{10}$. Using the same argument as Case 1, find the load currents, and then the line currents on the secondary, which are also the phase currents on the secondary as the secondary is now in $Y$. The phase currents on the primary will be these currents multiplied by $a$. Since primary is in $Y$, the calculated phase currents will also be the line currents.

6. **Primary in Y, Secondary in Y, Unbalanced load in Y**

Convert the load to Δ-load by star-delta conversion, and repeat the case above.

7. **Primary in Y, Secondary in Δ, Unbalanced load in Δ**

This is a degenerate case, and no unique solution can be found for computing the primary line currents from the secondary line currents (Try to find out WHY).

8. **Primary in Y, Secondary in Δ, Unbalanced load in Y**

Same as above. This is also a degenerate case, and no unique solution can be found for computing the primary line currents from the secondary line currents (Try to find out WHY).