Allocating Manpower to Minimize $L_{\text{max}}$ in a Job Shop
Finding the Allocation that Minimizes The Lower Bound on $L_{\text{max}}$

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Job Shop Scheduling

- Job shop scheduling: determine an optimal schedule given a performance objective
- Has been an active area of research for more than 60 years [Potts & Strusevich 2009]
- Most job shop scheduling problems are NP-Hard [Lenstra et al. 1977]
- Basic assumption of most job shops: machines form the only constraint
- Typically a job shop does not have as many workers as machines that need operation — workers form a constraint!
Dual Resource Constrained Systems

- Dual Resource Constrained (DRC) system — a job shop in which both machines and workers are limited resources
- Partition the job shop into machine groups, workers will be assigned to these machine groups
- The way in which workers are allocated to the machine groups affects performance
- Research objective: methodology to determine the allocation of workers in the job shop that allows $LB(L_{max})$ to be minimized
A DRC job shop is “one in which shop capacity may be constrained by machine and labor capacity or both” [Treleven & Elvers 1985].

Hottenstein and Bowman [1998] found two main questions were researched:

1. **When** should workers transfer from one machine or machine group to another, and
2. To which machine or machine group (**where**) should they move

Current DRC system research focuses largely on the effects on job shop performance of:
- Cross-training of workers
- Incorporation of worker forgetfulness
Hodgson et al. [1998] considered the classic $N/M/L_{\text{max}}|r_j$ scheduling problem: Find a schedule that minimizes $L_{\text{max}}$ in a job shop that has $N$ jobs, $M$ machines, and the release times of the jobs ($r_j$) are not all the same.

This is an NP-Hard problem [Baker and Su 1974].

The approach was to:

1. Calculate a lower bound on $L_{\text{max}}$ for the job shop
2. Generate a schedule heuristically (using the Virtual Factory)
3. Compare the actual $L_{\text{max}}$ value to the lower bound value to evaluate the schedule
A relaxation involving preemption (suggested by Baker and Su [1974]) was used to determine a lower bound on $L_{\text{max}}$ on each machine.

The lower bound on $L_{\text{max}}$ for the job shop was the maximum over all machine lower bounds.

Powerful lower bound — $M$ chances for it to be tight.
The Virtual Factory

- Transient, deterministic simulation
- Schedules jobs at machines using the ‘Revised Slack’ rule
- ‘Revised slack’ is “a function of a job’s due-date, processing time on the current machine and all downstream operations, and estimated queuing times for all downstream operations” [Schultz et al. 2004]
- Termination occurs if the schedule $L_{\text{max}}$ value equals the lower bound $L_{\text{max}}$ value, or a certain number of iterations has occurred
- Equaled or bettered the series of benchmark problems established by Demirkol et al. [1998]
Model Assumptions

Together with the standard job shop assumptions, these assumptions transform the job shop into the DRC system considered in this research:

- Fewer workers than machines in the job shop
- A machine group must have at least one worker assigned to it, otherwise it cannot process any jobs
- Each machine requires one worker to operate it, and a worker cannot operate more than one machine at a time
- The allocation of workers, once determined, is static
- Every worker is fully cross-trained
Machine Group Network Flow Formulation

Want to determine $\text{LB}_\vartheta(L_{\text{max}})$ — a lower bound on $L_{\text{max}}$ for the job shop given an allocation $\vartheta$.

First find $\text{LB}_{mg}(L_{\text{max}})$ — a lower bound on $L_{\text{max}}$ for a machine group using a network flow formulation.

**Types of Nodes:**

- $s_i$: source node for all jobs in machine group $i$
- $j_{u,h}$: job node for job $u$ on machine $h$
- $t_{v,h}$: time period node for time period $v$ on machine $h$
- $\omega_v$: worker availability node for time period $v$
- $k_i$: sink node for machine group $i$
Dashed arcs represent arcs to and from other machines in the $i^{th}$ machine group.
### Arc Capacities and Flow

<table>
<thead>
<tr>
<th>Arc</th>
<th>Capacity</th>
<th>Flow on Arc</th>
<th>Relevant Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>((s_i, j_u, h))</td>
<td>([p_{u,h}, p_{u,h}])</td>
<td>(p_{u,h})</td>
<td>(u = 1, \ldots, n_h, \ h = 1, \ldots, M_{mg_i})</td>
</tr>
<tr>
<td>((j_u, h, t_v, h))</td>
<td>([0, 1])</td>
<td>({0, 1})</td>
<td>(u = 1, \ldots, n_h, \ h = 1, \ldots, M_{mg_i};) (v = ES_{u,h}, \ldots, LF_{u,h} + L_{\text{max}} - 1)</td>
</tr>
<tr>
<td>((t_v, h, \omega_v))</td>
<td>([0, 1])</td>
<td>({0, 1})</td>
<td>(v = \delta_h, \ldots, \tau_h, \ h = 1, \ldots, M_{mg_i})</td>
</tr>
<tr>
<td>((\omega_g, k_i))</td>
<td>([0, w_i])</td>
<td>({0, 1, \ldots, w_i})</td>
<td>(g = \psi, \ldots, T)</td>
</tr>
</tbody>
</table>

The capacities on each arc ensure that:

1. The entire job is processed
2. At most one unit of an operation is processed in a given time period
Machine Group Network Flow Formulation, cont.

- Formulation allows for preemption
- Network feasibility is dependent on the value of $L_{max}$ used
- We want to find the smallest $L_{max}$ value that allows a feasible network
- A series of networks must be solved to obtain the optimal $L_{max}$ value for a machine group
- How do we find a good starting $L_{max}$ value?
Constructive Algorithm to Find a Feasible $L_{\text{max}}$ Value

- Horn’s [1974] procedure allows us to find an optimal preemptive schedule for the $N/1/L_{\text{max}}|r_j$ problem.

- To construct a preemptive machine group schedule, at time $t$ operate the $w_i$ machines with the $w_i$ largest $L_{\text{max}}$ values in their remaining schedules.

- A schedule constructed this way aims to minimize $L_{\text{max}}$

- Provides an initial feasible $L_{\text{max}}$ value for the network flow approach.
Characterization of $\text{LB}_{mg}(L_{\text{max}})$

**Proposition 1**

The lower bound on $L_{\text{max}}$ for a machine group is a discrete, monotonic, non-increasing function of the number of workers assigned to the machine group.

- The lower bound on $L_{\text{max}}$ for a machine group is not convex

- A relatively simple search procedure to find the best lower bound on $L_{\text{max}}$ allocation is still possible
Search Procedure

- The search procedure uses Proposition 1 to find an allocation $\vartheta^*$ of workers to machine groups that minimizes $LB(L_{max})$ for the job shop.

- The allocation search algorithm is optimal:
  - If a unique allocation minimizes $LB(L_{max})$, the algorithm will find it,
  - If the allocation that minimizes $LB(L_{max})$ is not unique, one such allocation will be found.
Experimental Job Shop

- 1050 jobs, 10 machine groups each with 7 machines (total of 70 machines)
- Each job has Uniform(6,10) operations, and visits the same machine group no more than 3 times
- Operation time is Uniform(1,40)
- On average, each machine group sees 10% of the job shop workload
- Overall job shop staffing levels of 60%, 70%, 80%, and 90% (e.g. 60% staffing = 42 workers for allocation)
- 20 replications at each due-date range and staffing level combination
For each allocation $\vartheta$, you have a $\text{LB}(L_{\text{max}})$ value, and a $L_{\text{max}}$ value from the Virtual Factory

Allocation $\vartheta^*$: Allocation that minimizes $\text{LB}(L_{\text{max}})$

Allocation $\vartheta^{\text{Best}}$: Allocation that has the best Virtual Factory schedule (smallest $L_{\text{max}}$ schedule)

Performance to the Lower Bound (PLB): Difference between $L_{\text{max}}$ for a given allocation $\vartheta$ and $\text{LB}_{\vartheta^*}(L_{\text{max}})$ for a given allocation.

PLB is a measure of how good the allocation actually is.
In the following graphs:

- **Blue line**: average PLB of allocation $\vartheta^{\text{Best}}$ over 20 problems
- **Red line**: average PLB allocation $\vartheta^*$ over 20 problems
- **Green line**: worst case PLB amongst the 20 different $\vartheta^*$ allocations

Gap between the **Red** line and the **Blue** line: the average maximum improvement that can be achieved from enumerating the allocation search space
Min. LB Alloc. vs Best Alloc. Performance to the LB,
Symmetric Job Shop, 70 Machines, 60% Staffing

Best Alloc. Perf.
Min. LB Alloc. Perf.
Worst Min. LB Alloc. Perf.
Min. LB Alloc. vs Best Alloc. Performance to the LB,
Symmetric Job Shop, 70 Machines, 70% Staffing

Best Alloc. Perf.
Min. LB Alloc. Perf.
Worst Min. LB Alloc. Perf.
Min. LB Alloc. vs Best Alloc. Performance to the LB, Symmetric Job Shop, 70 Machines, 80% Staffing

Time Units

Due-date Range

Best Alloc. Perf.
Min. LB Alloc. Perf.
Worst Min. LB Alloc. Perf.

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Min. LB Alloc. vs Best Alloc. Performance to the LB, Symmetric Job Shop, 70 Machines, 90% Staffing

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Interpretation of Experimental Results

- On average, allocation $\vartheta^*$ yields a schedule with a $L_{\text{max}}$ value close to allocation $\vartheta^\text{Best}$

- Although the average performance of allocation $\vartheta^*$ is good, the worst case performance can be 5 or 6 average processing times more than the average

- Similar results have been found for an asymmetric job shop
Main Findings

- Allocation $\vartheta^*$ isn’t necessarily the allocation that allows the best schedule to be generated.

- On average, the PLB of allocation $\vartheta^*$ is good, but it can be improved upon.

- The PLB of allocation $\vartheta^*$ in the worst case highlights the need for investigation into how to find allocation $\vartheta^{\text{Best}}$ quickly.
Further Research

- Develop an intelligent search procedure to improve on allocation $\vartheta^*$
- Optimality criteria for an allocation
- Total (and even partial) enumeration of the allocation search space is computationally impractical: develop probabilistic methods so that we can state we have found allocation $\vartheta^{\text{Best}}$ with some level of confidence
Questions?

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