2.12.2013

This module:
Overviews some properties of the different Fourier transforms that we have seen, in particular analogies between time and frequency.

Reading material:
section 4.3

Motivation
When should we discuss things in the time domain? And when in the frequency domain?

We will see that the different Fourier transform tools that we have developed contain some analogs to one another. Therefore, the more familiar you will be with these concepts, the easier it will become to think about signals and systems in BOTH time and frequency domains.
Let's start discussing the various properties:

1) Continuous time signals have aperiodic spectra, whereas discrete time signals have periodic spectra.
   Why? Because in the continuous case, there is no periodicity, whereas in discrete time everything is periodic modulo $2\pi$.

2) Periodic signals have discrete spectra, whereas aperiodic signals have continuous spectra.
   Why? Periodic signals can be written as a linear combination of sinusoids that share the same period.
   In contrast, for aperiodic signals we must break up this symmetry and allow sinusoids that are not periodic into the mix.
There are also similarities between the transforms and inverse transforms:

1) \[ X_a(\mathcal{F}) = \mathcal{F} \{ x_a(t) \} = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi Ft} \, dt \] 
   \[ x_a(t) = \int_{-\infty}^{\infty} X_a(\mathcal{F}) e^{j2\pi Ft} \, dF \]
   \text{continuous time aperiodic}

2) \[ C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j(2\pi/N)k n} \] 
   \[ x(n) = \sum_{k=0}^{N-1} C_k e^{j(2\pi/N)k n} \] 
   \text{discrete periodic}

3) \[ C_k = \frac{1}{T} \int_{-T/2}^{T/2} x_a(t) e^{-j2\pi k F_0 t} \, dt \] 
   \[ x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(a) e^{j\omega n} \, d\omega \] 
   \text{discrete aperiodic}

4) \[ X_a(\mathcal{F}) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k F_0 t} \] 
   \[ x(n) = \sum_{n=-\infty}^{\infty} X(\mathcal{F}) e^{-j\omega n} \] 
   \text{discrete aperiodic}