supplement to handout #13

Problem 5.57
consider the following system

\[ x(w) \xrightarrow{H(w)} y(w) \]

Determine its frequency response:
(a) \( H(w) \) is lowpass with cutoff \( w_c \).

Let's look at \( H(w) \).

The new system is
\[ y(w) = -x(w) \cdot H(w) + x(w) = x(w) [1 - H(w)]. \]
Therefore,
\[ H_{new}(w) = 1 - H(w). \]
(b) $H(w)$ is highpass with cutoff $w_c$.

Now the new filter, $H_{\text{new}}(w)$, is lowpass.

**New example**

Let's construct a "reasonable" lowpass. It will have poles at frequencies $\pm \Theta$ (both are complex conjugates), and a double zero at $-1$ (on unit circle).

The radii of the poles will be $\frac{5}{3}$ and so

$$H(z) = \frac{(z+1)^2}{(z-re^{j\Theta})(z-re^{-j\Theta})}.$$

Reasonable constraints are,

$$H(0) = H(\Theta) = 1.$$
Let's start with constraint #1:

\[ w = 0 \implies z = e^{jw} = e^{j0} = 1. \]

\[
H(z=1) = \frac{(1+1)^2}{(1-re^{j\theta})(1-re^{-j\theta})} \cdot G = 1
\]

\[
4G = (1-re^{j\theta})(1-re^{-j\theta})
\]

\[
= 1 - r(e^{j\theta} + e^{-j\theta}) + r^2
\]

\[
= 1 + r^2 - 2r \cos\theta.
\]

Constraint #2:

\[ w = \theta \rightarrow z = e^{j\theta}. \]

\[
H(z = e^{j\theta}) = \frac{(e^{j\theta}+1)^2}{(e^{j\theta} - re^{j\theta})(e^{j\theta} - re^{-j\theta})} \cdot G = 1.
\]

\[
(e^{j\theta} + 1)^2 \cdot G = (e^{j\theta}(1-r)) \cdot (e^{j\theta} - re^{-j\theta})
\]

We have two parameters, G and r. These should (hopefully) be solvable from constraints *1* and *2* above.