Homework 03 – Problem 3

Dror Baron

Let me describe Problem 3 in Homework 5 based on the parameters given to me by the software.

**Problem:** I was given a difference equation

\[ y(n) - 0.25y(n - 1) = 2x(n) \]

with an input

\[ x(n) = 0.15^n u(n) \]

and initial conditions

\[ y(-1) = 2. \]

We need to compute \( y(n) \).

**Solution:** We compute the one sided \( z \) transform of the difference equation,

\[ Y^+(z) - 0.25[z^{-1}Y^+(z) + y(-1)] = 2X^+(z). \]  \hspace{1cm} (1)

We know that \( y(-1) = 2 \). Moreover, the one sided \( z \) transform of \( x(n) \) is

\[
Z^+\{x(n)\} = \sum_{n=0}^{+\infty} x(n)z^{-n} \\
= \sum_{n=0}^{+\infty} 0.15^n u(n)z^{-n} \\
= \sum_{n=0}^{+\infty} 0.15^n z^{-n} \\
= \frac{1}{1 - 0.15z^{-1}}.
\]

Substituting into the one transform of the entire difference equation (1),

\[ Y^+(z)[1 - 0.25z^{-1}] = \frac{2}{1 - 0.15z^{-1}} + 0.5, \]

where \( 2X^+(z) = \frac{2}{1 - 0.15z^{-1}} \) and the 0.5 comes from the initial condition \( y(-1) = 2 \) being multiplied by 0.25. Dividing by \( [1 - 0.25z^{-1}] \),

\[ Y^+(z) = \left[ \frac{1}{1 - 0.25z^{-1}} \right] \left[ \frac{2}{1 - 0.15z^{-1}} + 0.5 \right] \]
\[
\begin{align*}
&= \frac{2}{(1 - 0.25z^{-1})(1 - 0.15z^{-1})} + \frac{0.5}{1 - 0.25z^{-1}} \\
&= \frac{2 + 0.5(1 - 0.15z^{-1})}{(1 - 0.25z^{-1})(1 - 0.15z^{-1})}.
\end{align*}
\]

A partial fraction expansion could be
\[
Y^+(z) = \frac{a}{1 - 0.15z^{-1}} + \frac{b}{1 - 0.25z^{-1}}.
\]

We see that
\[
a(1 - 0.25z^{-1}) + b(1 - 0.15z^{-1}) = (a+b) + z^{-1}(-0.25a - 0.15b) = 2 + 0.5(1 - 0.15z^{-1}) = 2.5 - 0.075z^{-1}.
\]

The constants \(a = -3\) and \(b = 5.5\) can be determined, for example using Matlab. Plugging them in,
\[
Y^+(z) = \frac{-3}{1 - 0.15z^{-1}} + \frac{5.5}{1 - 0.25z^{-1}}.
\]

Taking the inverse one sided transform yields
\[
y(n) = -3(0.15)^nu(n) + 5.5(0.25)^nu(n).
\]