Let me describe Problem 1 in Homework 7 based on the parameters given to me by the software.

**Problem:** We are given a periodic signal,

\[ x(n) = 7 \cos(2\pi \frac{13}{28}(n - 12) - \frac{\pi}{4}) . \]

1) We need to compute the period \( N \). The period provides an integer number of \( 2\pi \) radians. Seeing 28 in the denominator, let’s try that. If \( N = 28 \), then we have \( 2\pi \frac{13}{28} \) radians, which is 13 cycles. Moreover, because 13 is a prime number, we cannot reduce the period \( N \) further by dividing it by some other integer.

2) We are asked to express \( x(n) \) as a sum of exponentials. Recall that \( \cos(\theta) = \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta} \), and so we see that the phase is \( \theta = 2\pi \frac{13}{28}(n - 12) - \frac{\pi}{4} \), and the sum of exponentials becomes

\[ x(n) = \frac{7}{2}e^{j2\pi \frac{13}{28}(n-12)} + \frac{7}{2}e^{-j2\pi \frac{13}{28}(n-12)} , \]

where \( \frac{7}{2} \) is due to the 7 being in \( x(n) \) being multiplied by \( \frac{1}{2} \) in each exponent.

3) Recall that the indices have the form

\[ c_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}. \]

Therefore,

\[ c_k = \frac{1}{N} \sum_{n=0}^{N-1} \left[ \frac{7}{2}e^{j2\pi \frac{13}{28}(n-12)} + \frac{7}{2}e^{-j2\pi \frac{13}{28}(n-12)} \right] e^{-j2\pi kn/N} \]

\[ = \frac{7}{2} \frac{1}{28} \sum_{n=0}^{27} e^{j2\pi \frac{13(n-12)-kn}{28}} - j\frac{\pi}{4} + e^{j2\pi \frac{-13(n-12)-kn}{28}} + j\frac{\pi}{4}. \]

The first exponent is non-zero only when \( k = 13 \). The second exponent is non-zero only when \( k = -13 \). Note, however, that \( k \in \{0, 1, \ldots, N - 1 \} \). The \( k = -13 \) corresponds to \( k = 15 \) after adding 28 (things are modulo \( N \)), and so \( k_1 = 13 \) and \( k_2 = 15 \).

4) The actual coefficients are computed as follows,

\[ c_{13} = \frac{7}{2}e^{-j2\pi \frac{13}{28} - j\frac{\pi}{4}} . \]
\[ c_{15} = \frac{7}{2} e^{j2\pi \frac{15}{28} + j\frac{\pi}{4}}. \]

Note that the \( \frac{1}{28} \) term is canceled out by all 28 terms in each summation having the same value.