Problem 1:

(a) We are asked to compute the 5-point DFT of several sequences. One approach to this question involves using Matlab. For example, for a sequence \(x_1(n) = [2, 3, 2, 4, 2]\), the DFT can be computed in Matlab using the following commands.

\[
x1=[2 3 2 4 2];
x1f=fft(x1)
\]

Another approach would use the definition of the DFT,

\[
X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}.
\]

(b) We are asked to define \(y_1(n)\) such that \(Y_1(k) = X_1(k)X_2(k)\). One approach to do so uses Matlab, where we (i) compute \(X_1(k)\) and \(X_2(k)\) using the fft command, (ii) multiply them to form \(Y_1(k)\), and (iii) compute \(y_1(n)\) using the ifft command.

A second approach to compute \(y_1(n)\) directly from \(x_1(n)\) and \(x_2(n)\) involves the circular convolution of \(x_1\) and \(x_2\).

A third approach relies on \(x_2(n)\) having the simple form \([0, 0, 0, 1, 0]\), which implies that \(x_1(n)\) is shifted in a cyclical way to the right by 3 units, or to the left by 2 units. Because \(x_1(n) = [2, 3, 2, 4, 2]\), shifting it to the right by 3 units yields \(y_1(n) = [2, 4, 2, 2, 3]\).

(c) We need to compute \(x_3(n)\) such that \(Y_2(k) = X_1(k)X_3(k)\). To do so, we compute \(Y_2(k)\) and \(X_1(k)\), for example using Matlab’s fft command. Next, we derive \(X_3(k)\) by dividing \(Y_2(k)\) by \(X_1(k)\). Finally, we compute \(x_3(n)\) from \(X_3(k)\), for example by applying the ifft command.