Homework 8

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Problem 2: LTI systems cannot produce frequencies that differ from those in the input. That is, if \(X(\omega_1) = 0\) and \(Y(\omega_1) \neq 0\), then \(H\) cannot be LTI (because if it was, then \(Y(\omega_1) = 0 \cdot H(\omega_1) = 0\)).

In light of this observation, consider the signal \(x(n) = A \cos(2\pi 0.23 n)\).

(a) For \(y(n) = x(2n)\), the frequency doubles, meaning that \(y(n) = A \cos(2 \cdot 2\pi 0.23 n)\). The frequency in \(y(n)\) is \(f = 2 \cdot 0.23 = 0.46\) cycles per sample. This frequency did not exist in \(x(n)\), and so \(H\) is not LTI.

(b) For \(y(n) = x^2(n)\), we have \(y(n) = A^2 \cos^2(2\pi 0.23 n)\), which can be shown to have frequency components at DC \((f_1 = 0)\) and the second harmonic \((f_2 = 2 \cdot 0.23 = 0.46)\). Again, this is not LTI, because the DC component did not exist in \(x(n)\).

(c) For \(y(n) = \cos(\pi n) \cdot x(n) = A \cos((\pi - 0.46\pi) n) = \cos(0.54\pi n)\), and so \(f = 0.54/2 = 0.27\) cycles per sample. This frequency was not present in \(x(n)\), and \(H\) is not LTI.

Problem 3: Our band pass filter allows frequencies \(\frac{6\pi}{16} \leq \omega \leq \frac{10\pi}{16}\) to pass through, and so its bandwidth is \(\frac{10\pi}{16} - \frac{6\pi}{16} = \frac{4\pi}{16} = \frac{\pi}{4}\).

Next, because the equivalent low pass is symmetric, its cutoff frequency is half the bandpass, \(\omega_c = \frac{1}{2} \cdot \frac{\pi}{4} = \frac{\pi}{8}\).

The center frequency of the band pass is
\[
\omega_{bc} = \frac{1}{2} \left[ \frac{6\pi}{16} + \frac{10\pi}{16} \right] = \frac{8\pi}{16} = \frac{\pi}{2}.
\]

The time domain response of the low pass is
\[
h_{lp}(n) = \frac{\sin(\omega_c n)}{\pi n},
\]
the impulse response corresponding to the modulation sequence is
\[
d(n) = 2 \cos(\omega_c n),
\]
and the band pass is their product,
\[
h(n) = h_{lp}(n)d(n) = 2 \frac{\sin(\omega_c n)}{\pi n} \cos(\omega_c n).
\]