Textbook Questions:

2.1

(a) 
\[ x(n) = \left\{ \ldots, 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 0, \ldots \right\} \]

Refer to fig 2.1-1.

(b) After folding \( s(n) \) we have

\[ x(-n) = \left\{ \ldots, 0, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots \right\} . \]

After delaying the folded signal by 4 samples, we have

\[ x(-n+4) = \left\{ \ldots, 0, 0, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots \right\} . \]

On the other hand, if we delay \( x(n) \) by 4 samples we have

\[ x(n-4) = \left\{ \ldots, 0, 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 0, \ldots \right\} . \]

Now, if we fold \( x(n-4) \) we have

\[ x(-n-4) = \left\{ \ldots, 0, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, 0, \ldots \right\} . \]
(c) 
\[ x(-n + 4) = \left\{ \ldots, 0, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots \right\} \]

(d) To obtain \( x(-n + k) \), first we fold \( x(n) \). This yields \( x(-n) \). Then, we shift \( x(-n) \) by \( k \) samples to the right if \( k > 0 \), or \( k \) samples to the left if \( k < 0 \).

(e) Yes.
\[ x(n) = \frac{1}{3} \delta(n - 2) + \frac{2}{3} \delta(n + 1) + u(n) - u(n - 4) \]

2.8

(a) True. If 
\[ v_1(n) = T_1[x_1(n)] \] and \[ v_2(n) = T_1[x_2(n)], \]
then 
\[ \alpha_1 x_1(n) + \alpha_2 x_2(n) \]
yields 
\[ \alpha_1 v_1(n) + \alpha_2 v_2(n) \]
by the linearity property of \( T_1 \). Similarly, if 
\[ y_1(n) = T_2[v_1(n)] \] and \[ y_2(n) = T_2[v_2(n)], \]
then 
\[ \beta_1 v_1(n) + \beta_2 v_2(n) \rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n) \]
by the linearity property of \( T_2 \). Since 
\[ v_1(n) = T_1[x_1(n)] \] and \[ v_2(n) = T_2[x_2(n)], \]
it follows that 
\[ A_1 x_1(n) + A_2 x_2(n) \]
yields the output 
\[ A_1 T_1[x_1(n)] + A_2 T_2[x_2(n)], \]
where \( T = T_1 T_2 \). Hence \( T \) is linear.

(b) True. For \( T_1 \), if 
\[ x(n) \rightarrow v(n) \] and \[ x(n - k) \rightarrow v(n - k) \],
For \( T_2 \), if 
\[ v(n) \rightarrow y(n) \]
and \( v(n - k) \rightarrow y(n - k) \).
Hence, For \( T_1 T_2 \), if 
\[ x(n) \rightarrow y(n) \] and \[ x(n - k) \rightarrow y(n - k) \]
Therefore, \( T = T_1 T_2 \) is time invariant.
(c) True. $T_1$ is causal $\Rightarrow$ $v(n)$ depends only on $x(k)$ for $k \leq n$. $T_2$ is causal $\Rightarrow$ $y(n)$ depends only on $v(k)$ for $k \leq n$. Therefore, $y(n)$ depends only on $x(k)$ for $k \leq n$. Hence, $T$ is causal.

(d) True. Combine (a) and (b).

(e) True. This follows from $h_1(n) * h_2(n) = h_2(n) * h_1(n)$

(f) False. For example, consider

$T_1 : y(n) = nx(n)$ and 
$T_2 : y(n) = nx(n + 1)$.

Then,

$$T_2[T_1[\delta(n)]] = T_2(0) = 0.$$ 
$$T_1[T_2[\delta(n)]] = T_1[\delta(n + 1)]$$ 
$$= -\delta(n + 1)$$ 
$$\neq 0.$$ 

(g) False. For example, consider

$T_1 : y(n) = x(n) + b$ and 
$T_2 : y(n) = x(n) - b$, where $b \neq 0$.

Then,

$$T[x(n)] = T_2[T_1[x(n)]] = T_2[x(n) + b] = x(n).$$

Hence $T$ is linear.

(h) True.

$T_1$ is stable $\Rightarrow$ $v(n)$ is bounded if $x(n)$ is bounded.

$T_2$ is stable $\Rightarrow$ $y(n)$ is bounded if $v(n)$ is bounded.

Hence, $y(n)$ is bounded if $x(n)$ is bounded $\Rightarrow T = T_1T_2$ is stable.

(i) Inverse of (c). $T_1$ and for $T_2$ are noncausal $\Rightarrow T$ is noncausal. Example:

$T_1 : y(n) = x(n + 1)$ and 
$T_2 : y(n) = x(n - 2)$

$\Rightarrow T : y(n) = x(n - 1),$ which is causal. Hence, the inverse of (c) is false.

Inverse of (h): $T_1$ and/or $T_2$ is unstable, implies $T$ is unstable. Example:

$T_1 : y(n) = e^{x(n)}$, stable and $T_2 : y(n) = ln[x(n)]$, which is unstable.

But $T : y(n) = x(n)$, which is stable. Hence, the inverse of (h) is false.
2.49

(a)

\[ y(n) = 0.8y(n-1) + 2x(n) + 3x(n-1) \]
\[ y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1) \]

The characteristic equation is

\[ \lambda - 0.8 = 0 \]
\[ \lambda = 0.8 \]
\[ y_h(n) = c(0.8)^n \]

Let us first consider the response of the system

\[ y(n) - 0.8y(n-1) = x(n) \]

to \( x(n) = \delta(n) \). Since \( y(0) = 1 \), it follows that \( c = 1 \). Then, the impulse response of the original system is

\[ h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1) \]
\[ = 2\delta(n) + 4.6(0.8)^{n-1} u(n-1) \]

(b) The inverse system is characterized by the difference equation

\[ x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1) \]

Refer to fig 2.49-1

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**Figure 2.49-1:**
2.62

(a)

\[ \gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l) \]

\[ \gamma_{xx}(-3) = x(0)x(3) = 1 \]

\[ \gamma_{xx}(-2) = x(0)x(2) + x(1)x(3) = 3 \]

\[ \gamma_{xx}(-1) = x(0)x(1) + x(1)x(2) + x(2)x(3) = 5 \]

\[ \gamma_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7 \]

Also \( \gamma_{xx}(-l) = \gamma_{xx}(l) \)

Therefore \( \gamma_{xx}(l) = \{1, 3, 5, 7, 5, 3, 1\} \)

(b)

\[ \gamma_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)y(n-l) \]

We obtain

\[ \gamma_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\} \]

we observe that \( y(n) = x(-n + 3) \), which is equivalent to reversing the sequence \( x(n) \). This has not changed the autocorrelation sequence.
Matlab Question:

cclc
clear
dis_sig = randint(1,5,10); %random signal case
%dis_sig = [1 2 1 1 0];    %2.62 (a)
%dis_sig = [1 1 2 1 0];    %2.62 (b)

autocorr_pos = zeros(1,5); %Consider correlation with positive l here
for temp_l_plus1 = 1:1:5   %index of r. Matlab cannot have index 0 so the
   actual l should be 1 smaller in use.
   for temp_n = temp_l_plus1:1:5  %infact l ranges from 0 to 4 here
      autocorr_pos(temp_l_plus1) = autocorr_pos(temp_l_plus1) +
      dis_sig(temp_n) * dis_sig(temp_n - (temp_l_plus1 - 1) ); %function (2.6.9) in
      the book
   end
end

autocorr_neg = fliplr(autocorr_pos); %autocorrelation vector is symmetric
over l = 0
autocorr = [ autocorr_neg(1:4)  autocorr_pos ]; %Combine these two vector.
Notice r(0) is on both vectors, keep just one.