Part I: Questions from the textbook

4.4 Consider the following periodic signal:

\[ x(n) = \{ \ldots, 1, 0, 1, 2, 3, 2, 1, 0, 1, \ldots \} \]

where the underlined 3 corresponds to time \( n = 0 \).

(a) Sketch the signal \( x(n) \) and its magnitude and phase spectra.
(b) Using the results in part (a), verify Parseval’s relation by computing the power in the time and frequency domains.

4.6 Determine and sketch the magnitude and phase spectra of the following periodic signals.

(a) \( x(n) = 4 \sin(\pi(n-2)/3) \).
(c) \( x(n) = \cos(2\pi n/3) \sin(2\pi n/5) \).
(h) \( x(n) = (-1)^n \) for all \( n \).

4.9 Compute the Fourier transform of the following signals.

(b) \( x(n) = 2^n u(-n) \).
(g) \( x(n) = \{ -2, -1, 0, 1, 2 \} \), where the underlined 0 corresponds to time \( n = 0 \).

4.17 Let \( x(n) \) be an arbitrary signal, not necessarily real valued, with Fourier transform \( X(\omega) \). Express the Fourier transforms of the following signals in terms of \( X(\omega) \).

(c) \( y(n) = x(n) - x(n-1) \).
(e) \( y(n) = x(2n) \).

4.19 Let \( x(n) \) be a signal with Fourier transform as follows: (i) \( X(\omega) = 0 \) for \( |\omega| > \pi/2 \); (ii) \( X(\omega) \) increases linearly from 0 to 1 as \( \omega \) goes from \(-\pi/2\) to 0 \( \left( X(\omega) = \frac{2}{\pi} \left( x - \frac{\pi}{2} \right) \right) \); and (iii) decreases
linearly from 1 to 0 as \( \omega \) goes from 0 to \(+\frac{\pi}{2}\) to 0 \( X(\omega) = 1 - x^2 \pi \). Determine and sketch the Fourier transforms of the following signals:

(a) \( x_1(n) = x(n) \cos(\pi n/4) \).

(b) \( x_2(n) = x(n) \sin(\pi n/2) \).

**Matlab:**

In class we discussed how signals with discontinuities contain high frequency components. To see this, consider the following continuous signals:

(a) \( x_a(t) = 1 \) when \(|t| < 1\), else 0.

(b) \( x_b(t) = 2 + t \) for \(-2 < t < 0\) and \( x_b(t) = 2 - t \) for \(0 < t < 2\); else the signal is 0.

The first signal looks like a box centered around \( t=0 \), and the second signal is a triangle centered around \( t=0 \). It can be shown that \( x_b(t) = x_a(t) * x_a(t) \). That is, the second signal is the first signal convolved with itself. Therefore, it should be easy for you to calculate the Fourier transform of the second signal once you’ve calculated the transform of the first.

Your role in this question is to compute the Fourier transforms both analytically and numerically for \(|F| < 100\). To clarify, the integral \( X(F) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi Ft} \, dt \) is the standard Fourier transform for aperiodic continuous time signals. To compute \( X(F) \) analytically, take the integral. To compute it numerically, the integration is replaced by a summation over small steps (because the signals are zero for \(|t| > 2\), you can examine a well-sampled range of \( t \), for example \( t=-2:dt:2 \) where \( dt \) will be some small number such as 0.001); but don’t forget to multiply the expression inside the summation by \( dt \), else the summation will not be normalized properly. That is, \( X(F) \approx \sum_t x(t)e^{-j2\pi Ft} \, dt \) approximates the integral better as \( dt \) becomes small. (Note that this summation needs to be calculated for each frequency \( F \).)

If you calculate the transform numerically at 1000 frequencies using a stepsize \( dt=0.001 \) (approximately 1000 values for \( t \)), you will be performing several million calculations. If you do so using loops in Matlab, your code will run quite slowly, and you are encouraged to see how defining operations with vectors can significantly accelerate the calculation.

**What to submit?** Please plot for both signals how the summations become better approximations to the analytical answers as \( dt \) in the summations becomes smaller. Please attach the 2 plots, your calculations for the integrals, and the Matlab code used to perform the numerical integration and to plot the results.

Finally, take a look (no need to submit anything) how the second signal’s spectra decays to zero more rapidly at high frequencies than the first signal’s spectra.