Textbook Questions:

5.1

(a) The signals $x(n)$ and $y(n)$ do not have Fourier transforms. The reason for this is that both their summations don’t converge, i.e., $\sum_{n=-\infty}^{\infty} |x(n)|$ and $\sum_{n=-\infty}^{\infty} |y(n)|$ go to infinity, and the range of convergence does not include the unit circle. (Note that the sums of their squares, i.e., $\sum_{n=-\infty}^{\infty} |x(n)|^2$ and $\sum_{n=-\infty}^{\infty} |y(n)|^2$, also diverge. Therefore, there is no mean square type of convergence to any Fourier transform either.)

Because the Fourier transforms do not exist, computing $H$ by dividing $Y$ by $X$ is not defined. The system function $H$ is not defined.

(b) Here the transforms are defined, and we can divide $Y$ by $X$:

$$X(\omega) = \frac{1}{1 - \frac{1}{2} e^{-j\omega}},$$

$$Y(\omega) = \frac{1}{1 - \frac{1}{8} e^{-j\omega}},$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1 - \frac{1}{2} e^{-j\omega}}{1 - \frac{1}{8} e^{-j\omega}},$$

and the system could be LTI. (A system with this transform would yield the requisite $y(n)$ if the input was $x(n)$.)

(c) Clearly, an LTI system that simply multiplies by 3 would be fine. Therefore, this could be an LTI system.
5.4

(a)

\[ y(n) = \frac{x(n) + x(n-1)}{2} \]

\[ Y(w) = \frac{1}{2}(1 + e^{-jw})X(w) \]

\[ H(w) = \frac{1}{2}(1 + e^{-jw}) \]

\[ = (\cos \frac{w}{2})e^{-jw/2} \]

Refer to fig 5.4-1.

(b)

![Graphs of |H(w)| and theta(w)]

Figure 5.4-1:
\[ y(n) = \frac{x(n) - x(n-1)}{2} \]

\[ Y(w) = \frac{1}{2} (1 - e^{-jw}) X(w) \]

\[ H(w) = \frac{1}{2} (1 - e^{-jw}) \]

\[ = \left( \sin \frac{w}{2} \right) e^{-jw/2} e^{j\pi/2} \]

Refer to fig 5.4-2.

Figure 5.4-2:
5.7

(a)

\[ y(n) = x(n) + x(n - 4) \]

\[ Y(w) = (1 + e^{-j4w})X(w) \]

\[ H(w) = (2\cos 2w) e^{-j2w} \]

Refer to fig 5.7-1.

(b)

![Figure 5.7-1](image)

Figure 5.7-1:

\[ y(n) = \cos \frac{\pi}{2} n + \cos \frac{\pi}{4} n + \cos \frac{\pi}{2} (n - 4) + \cos \frac{\pi}{4} (n - 4) \]

But \( \cos \frac{\pi}{2} (n - 4) = \cos \frac{\pi}{2} \cos 2\pi + \sin \frac{\pi}{2} \sin 2\pi = \cos \frac{\pi}{2} n \)

and \( \cos \frac{\pi}{4} (n - 4) = \cos \frac{\pi}{4} \cos \pi - \sin \frac{\pi}{4} \sin \pi = -\cos \frac{\pi}{4} n \)

Therefore, \( y(n) = 2\cos \frac{\pi}{2} n \)
5.9

\[ x(n) = A \cos \frac{n\pi}{4} \]

(a) \[ y(n) = x(2n) = A \cos \frac{2n\pi}{4} \Rightarrow w = \frac{\pi}{2} \]

(c)

\[
\begin{align*}
y(n) & = x(n)\cos \pi n \\
& = A \cos \frac{n\pi}{4} \cos \pi n \\
& = A \left( \frac{\cos \frac{5\pi}{4} n}{2} + \frac{\cos \frac{3\pi}{4} n}{2} \right) \\
\text{Hence, } w & = \frac{3\pi}{4} \text{ and } w = \frac{5\pi}{4}
\end{align*}
\]
Matlab Questions:

\[
y = filter([1/2 -1/2], [1], [0 1 2 3 4 0 0])
\]

freqz([1/2 -1/2], [1])

Answer:

\[
y = \\
0 \quad 0.5000 \quad 0.5000 \quad 0.5000 \quad 0.5000 \quad -2.0000 \quad 0
\]