ECE 421 Introduction to Signal Processing

Final – Spring 2018

May 4, 2018

Please remember to justify your answers carefully.

There are 9 pages in total on this test.

Last name: _______________  First name: _______________

Student ID: _______________
This question includes two unrelated parts. In both of them, we consider the sequence
\[ x(n) = 2\delta(n + 1) + 3\delta(n - 1). \]

a) Determine the autocorrelation sequence, \( r_{xx}(l) \), that corresponds to \( x(n) \).

b) Compute the Fourier transform, \( X(\omega) \), that corresponds to \( x(n) \).
Consider the finite duration discrete time signal, \( x(n) = [1, 3, 1, 3] \).

a) Compute the 4-point discrete Fourier transform (DFT), \( X(k) \), of the signal \( x(n) \).

b) The signal is periodic. In this part, you will compute the DFT of one of the periods. To do so, (i) specify the period, \( k \); (ii) specify one period of the signal, i.e., \( \tilde{x} = x(0), \ldots, x(k-1) \); and (iii) compute the \( k \)-point DFT of \( \tilde{x} \).
c) Write down MATLAB code that calculates the answers to parts a and b.
An analog (continuous time, continuous amplitude) filter is proposed with the following transfer function in the Laplace domain,

\[ H(s) = \frac{s}{s+3}. \]

Your goal in this question will be to map this filter to a discrete time filter.

a) Select an appropriate sampling parameter \( T \) to ensure that a bilinear transformation can map this analog filter to a digital one where the analog frequency \( \Omega = 3 \) in the denominator (i.e., \( s + 3 \)) corresponds to digital frequency \( \omega = \frac{\pi}{2} \).

b) Determine an expression for the transfer function, \( H(z) \), after the analog filter is mapped to the discrete time domain using the time sampling parameter \( T \) you found in part a. (If you are not sure about your answer for part a, then you may assume that \( T = 10 \).)
c) Identify any possible poles and zeros in the digital filter, and sketch a pole-zero plot. Make sure to clearly label your axes. Based on the pole-zero plot, what type of digital filter is this (e.g., low pass, band pass, high pass)? (If you are not sure about your answer for part b, then you may assume in parts c and d that \( H(z) = 1 + z^{-2} \).)

d) Convert the transfer function, \( H(z) \), to a difference equation. (The result of the conversion should look something like \( y(n) = 2y(n - 1) - 3x(n) + 4x(n - 1) \).)
Recall from Project 1 that the final demodulated signal maintained the frequency and phase of the original signal due to frequency folding, but the amplitude was scaled by some ratio. The MATLAB script below implements AM demodulation using frequency folding; it generates a modulated signal $S_1$ and downsamples it by a ratio of 20 to obtain the demodulated signal $S_2$. After plotting the (i) original signal (prior to modulation) $S_{\text{orig}}$; (ii) modulated signal $S_1$; and (iii) demodulated signal $S_2$ on the same graph, the amplitude ratio is determined by dividing $S_{\text{orig}}$ by $S_2$. There are multiple mistakes in the script, including coding errors (e.g., syntax) and implementation errors (e.g., wrong variables). Please identify at least 3 of these errors. For each error you identify, circle/underline the error, explain its nature, and how to correct it.

% signal parameters
fc=80*10^-3; % carrier frequency
forig=20*10^-3; % original signal frequency
Ts=1/(fc*10); % plot resolution: 10 samples/carrier frequency
T=0:T:5/fc; % investigate 5 periods
T2=0:T:5/(fc/20);% downsample by ratio of 20
fh=fc+forig; % high and low frequencies
fl=fc-forig;

% signal generation
S1=1/4*cos(2*pi*fh*T)+1/4*cos(2*pi*fl*T);
S2=1/4*cos(2*pi*Fh*T2)+1/4*cos(2*pi*fl*T2);
plot(T,cos(2*pi*forig*T),'-',T,S1,'--',T,S2,'*-');
title('Effect of Frequency Folding');
ylabel('Signals');
xlabels('t (sec)');
legend('original','reconstructed after folding',...
     'AM modulated after low pass');

% calculate ratio between original and modulated signals
Sorig=cos(2*pi*forig*T); % original Signal
Ratio=S2/Sorig; % amplitude ratio between original/modulated signals
Question 5 (LTI systems)

This question considers a linear time invariant (LTI) system that for an input
\[ x(n) = \delta(n) \] generates the output \[ y(n) = \delta(n - 2) + 2\delta(n - 3). \]

a) Sketch the impulse response. As before, make sure to clearly label your axes.

b) Provide a difference equation relating the output \( y(n) \) to the input \( x(n) \).
(As before, the result of the conversion should look something like \[ y(n) = 2y(n - 1) - 3x(n) + 4x(n - 1). \])

c) Suppose a constant signal \( x(n) = 2 \) is an input to the LTI system. What is \( y(n) \)? (Hint: the output will be identical for all \( n \).)
c) Instead of the constant input of part c, we now input a periodic version of the signal from question 2 to the LTI system. (That is, the input has the form \( x(n) = \ldots 1, 3, 1, 3, 1, 3, \ldots \).) Please explain carefully how you could compute the output, \( y(n) \). Full credit will be given for comprehensive explanations; computing the output will provide modest extra credit.