Question 1

**Part a)** In order for the signal to be periodic, we would need both \( \cos(\frac{4}{5}n) \) and \( \sin(\frac{5}{3}n + \frac{1}{3}) \) to be periodic. The cosine is not periodic, because \( \frac{4}{5} \) is not of the form \( 2\pi q \) where \( q \) is a rational number. For the sine, first we can ignore the phase shift of \( +\frac{1}{3} \), because it has no impact on periodicity. Having ignored the phase, we would need \( \frac{5}{3} \) to have the form \( 2\pi q' \) for some other rational number \( q' \). We can see that the sine is not periodic either.

**Part b)** The only frequency is \( F = 500 \), which yields the form \( \sin(2\pi Ft) = \sin(1000\pi t) \). The Nyquist rate is \( 2F \), which is 1000 samples per unit time. An easy way to verify this result is that when we advance \( \frac{1}{2F} = 0.001 \) time units, the phase advances by \( 1000\pi \times 0.001 = \pi \) radians, which is half a cycle. If \( x(t) \) is sampled at 800 Hz, the result is

\[
x(n) = x\left(t = \frac{n}{800}\right) = 0.7 \sin\left(1000\pi \frac{n}{800}\right) = 0.7 \sin\left(\frac{5}{4} \pi n\right).
\]

Note that \( \sin\left(\frac{5}{4} \pi n\right) = \sin\left(-\frac{3}{4} \pi n\right) \), and so \( x(n) = 0.7 \sin\left(-\frac{3}{4} \pi n\right) \).

**Part c)** It’s easy to see the result with a table.

<table>
<thead>
<tr>
<th>( n )</th>
<th>(-4)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-n + 2)</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>(x(-n + 2)))</td>
<td>0</td>
<td>-2</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>(2x(-n + 2)))</td>
<td>0</td>
<td>-4</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

In the sketch, I expected to see:

- Horizontal axis with an arrow pointing right.
- Vertical axis with an arrow pointing up.
- The horizontal axis is labeled with \( n \) (toward the right side), and near the top of the vertical axis \( 2x(-(n + 2)) \).
- To show the scale, need some numerical values along each axis.
Question 2  
**Part a)** Not including a sketch – quite time consuming on a computer. I can make a handwritten sketch and scan it in, if this would help. The idea is to break the difference equation into two parts. The input $x(n)$ enters on the left side, and there’s a block on the left side that computes $v(n) = 3x(n) + 2x(n - 1)$. The output of that block enters a second block that turns $v(n)$ into $y$ as follows: $y(n) = 2y(n - 1) + v(n)$. The output $y(n)$ will leave from the right side.  
**Part b)** One way to solve this question is to first partition the block diagram into a right block and left block. Second, swap the order of the blocks (order of convolution doesn’t matter). At this time, the blocks will have a form I style, which can be converted into a difference equation along the lines of Part a.

Question 3  
**Part a)** Let us write the $z$ transform of the difference equation, 
\[ Y(z) + 5z^{-1}Y(z) + 6z^{-2}Y(z) = X(z). \]  
Next, can write  
\[ Y(z)[1 + 5z^{-1} + 6z^{-2}] = X(z). \]  
the transfer function divides the output $Y(Z)$ by the input $X(z)$, 
\[ H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 + 5z^{-1} + 6z^{-2}}. \]  
**Part b)** The key task is to identify the zeros and poles. 
\[ H(z) = \frac{1}{1 + 5z^{-1} + 6z^{-2}} = \frac{1}{(1 + 2z^{-1})(1 + 3z^{-1})} = \frac{z^2}{(z + 2)(z + 3)}. \]  
There are poles are at $p_1 = -2$ and $p_2 = -3$. Both poles are outside the unit circle. Note that there is a double zero. The system is stable, meaning that the unit circle must be inside the ROC. But with poles outside the unit circle, the responses must be anti-causal. The ROC for $p_1 = -2$ is $\text{ROC}_1 = \{z : |z| < 2\}$, the ROC for $p_2 = -3$ is $\text{ROC}_2 = \{z : |z| < 3\}$, and the entire ROC is the intersection, $\text{ROC}_1 \cap \text{ROC}_2 = \{z : |z| < 2\}$. (Note that “$\cap$” means intersection of sets.)  
The sketch of the zeros and poles should contain:
• Horizontal axis with arrow at its right labeled with “Re(z).”

• Vertical axis with arrow at its top labeled with “Im(z).”

• Poles at $p_1 = -2$ and $p_2 = -3$. These are labeled by $-2$ and $-3$, respectively.

• A double zero at the origin, $z = 0$. The double nature of the zero is highlighted by the number 2 near the zero.

• The ROC is given by shading the circle of radius 2 centered around the origin. This circle should intersect the pole $p_1 = -2$.

As explained above, the system is anti-causal, because both poles are outside the unit circle yet it is stable.

Question 4

Part a) We have only covered the period so far. The Fourier transform will be studied in class soon.

To compute the period $N$, we must have $\frac{\pi N}{4} = 2\pi k$ for some integer $k$. It can be seen that $N = 8$, because each sample advances the phase by $\pi/4$ radians, and a cycle is $2\pi$ radians; 8 samples complete a cycle.