ECE 421 Introduction to Signal Processing

Midterm 1 – Spring 2018

February 21, 2018

Please remember to justify your answers carefully.

There are 9 pages in total on this test.

Name: _____________________ Student ID: __________________
Consider the continuous time function given by the input signal,

\[ x_a(t) = 3 \cos(70\pi t + 0.6\pi) + 2 \sin(240\pi t). \]

a) What is the Nyquist rate for this signal?

b) The signal \( x_a(t) \) is sampled at \( F_s = 200 \) samples per unit time. Write an equation for the discrete time sampled signal, \( x(n) \).
c) Write MATLAB code to plot the discrete time sampled signal, $x(n)$. You can assume that the $n$ vector comprised of time indices has already been defined. Based on $n$, you need to compute $x(n)$ and then plot it.

d) Was $x_a(t)$ aliased when sampling at $F_s = 200$ samples per unit time? Make sure to justify your answer.

e) The discrete time signal $x(n)$ is passed through an ideal digital to analog (D/A) converter at the same sampling rate of $F_s = 200$ samples per unit time. Determine the continuous time output, $\hat{x}_a(t)$. 
Question 2 (properties of systems)
Consider the discrete time system, \( y(n) = T[x(n)] = x(n^2) \).

a) Is the system time invariant? If yes, please sketch the system block diagram using a block diagram (multipliers, adders, \( z^{-1} \) delay blocks, and so on). If not, please justify your statement.

b) Consider a linear time invariant (LTI) system whose transfer function in the Fourier domain is given by \( H(\omega) = Y(\omega)/X(\omega) = 1 - e^{-j\omega} \). Is this system bounded input bounded output (BIBO) stable? Justify your answer. (Hint: recall that the transfer function in the Fourier domain is closely related to that in the \( z \) domain.)
Question 3 (z-transform and its properties)
The z-transform of a causal discrete time signal, $x(n)$, is given by

$$X(z) = \frac{3 - z^{-1}}{(1 - 0.5z^{-1})(1 - 0.25z^{-1})}.$$ 

a) Provide an expression for $x(n)$. (Your answer should be symbolic, e.g. $x(n) = 2^n u(n)$.)
b) Compute $x(n)$ for $n \in \{-1, 0, 1\}$. (Provide numerical answers such as $x(1) = 13$; symbolic answers will not count.)

c) We are now given another discrete time signal, $\hat{x}(n)$, whose $z$-transform is given by

$$X(z) = \frac{z^{-1}(3 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.25z^{-1})}.$$ 

Please compute $\hat{x}(n)$. (Hint: you could do so using partial fractions, but that would probably take quite a bit of effort. Instead, compare $\hat{X}(z)$ to $X(z)$ in part a and utilize properties of the $z$-transform.)
Question 4 (LTI systems and difference equations)

An LTI system $H$ has poles at $p_1 = -3$ and $p_2 = 2$ and a single zero at $z_1 = 1$. That is, its transfer function is given by

$$H(z) = \frac{z - z_1}{(z - p_1)(z - p_2)}. \quad (1)$$

a) How many possible systems can be associated with this pole-zero pattern? Specify the region of convergence (ROC) for each of them.
b) What is the ROC if the system is known to be stable? Make sure to justify your answer.

c) Please sketch a pole-zero plot. Make sure to sketch the ROC of the stable system you found in part b.
d) Derive the difference equation for the system $H$ whose transfer function is given in (1).

e) Sketch a block diagram that can implement the difference equation from part d.