ECE 421 Introduction to Signal Processing

Midterm 1 – Spring 2015

February 23, 2015

Please remember to justify your answers carefully.

There are 9 pages in total on this test.

Name: ______________________  Student ID: ______________________
Consider a system $H$ that processes an input $x(n)$ as follows, $y(n) = 3^n x(n + 2)$. We will evaluate different properties of this system.

(a) Is this a causal system? Why?

(b) Is the system bounded input bounded output (BIBO) stable or not? Why?
Consider a sampling frequency of \( F_s = 10 \text{kHz} \).

(a) What is the highest range of analog frequencies that can be sampled and then perfectly reconstructed with \( F_s = 10 \text{kHz} \)?

(b) Find two different continuous-time signals that will produce the sequence

\[
x(n) = \cos(0.25\pi n)
\]

when sampled with \( F_s = 10 \text{kHz} \).
(c) Let $x_a(t) = \cos(3500\pi t) + 2\sin(12000\pi t)$. What are the discrete time frequencies of the sinusoids in the sampled sequence when $F_s = 10kHz$?
Question 3

A digital filter obeys the following difference equation,
\[ y(n) = \frac{3}{4} y(n - 1) - \frac{1}{8} y(n - 2) + \frac{7}{8} x(n). \]

The input of the filter is an impulse, i.e., \( x(n) = \delta(n) \), and the initial conditions are
\[ y(-1) = -1 \text{ and } y(-2) = 1. \]

We will compute the output \( y(n) \) in steps using the one sided z transform.

(a) Express the one-sided z transforms of \( y(n - 1) \) and \( y(n - 2) \) using \( Y^+(z) \), which is the one-sided z transform of \( y(n) \), and the initial conditions.

(b) What is the one-sided transform of \( x(n) \)?
(c) Using your results from parts (a) and (b), express the difference equation (from the beginning of Question 3) in terms of one-sided z transforms. (Your answer should look something like $Y(z) + c_1 z^{-k_1} Y(z) + c_2 z^{-k_2} + c_3 z^{k_3}$; it will be an expression in the one-sided z domain.)

If you did not solve parts (a) and (b), you can assume that $Z \{ y(n - 1) \} = z^2 Y(z)$, $Z \{ y(n - 2) \} = 2z^{-2} Y(z)$, and $X(z) = 3z$.

(d) Express $Y(z)$ using partial fractions. (Your answer should look something like $Y(z) = \frac{13}{1 + z^{-1}} - \frac{2}{1 - 4z^{-1}}$.)

If you did not solve part (c), you can assume $Y(z)[1 - 1.5z^{-1} + 0.5z^{-2}] = 7z^{-1}$.
(e) Compute the output $y(n)$ using the inverse one-sided z transform. If you did not solve part (d), you can assume that $Y^+(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{\frac{1}{2}}{1-\frac{1}{4}z^{-1}}$. 
Question 4

A linear time invariant system is described by the following difference equation,

\[ y(n) = 0.5y(n - 1) + bx(n), \]

where \( b \) is a constant that will be determined soon.

(a) Find the transfer function in the Fourier domain, \( H(\omega) = \frac{Y(\omega)}{X(\omega)}. \)

(b) What is the squared magnitude of the transfer function, \( |H(\omega)|^2? \)
(c) Find the value of $b$ such that $|H(\omega)| = 1$ at frequency $\omega = 0$.
If you did not solve part (b), you can assume that $|H(\omega)|^2 = \sin(\omega) + b \cos(\omega)$. 