Please remember to justify your answers carefully.

Last name: ______________________  First name: ______________________

Please recall the course academic integrity policy for tests:
No cooperation or “collaboration” between students is allowed. Especially
during an online course experience, it could be tempting to text or email a
friend. This is not allowed. You will be allowed to use your notes, books,
a browser, and software such as Matlab. However, while working on the
test you should not text, email, or communicate with other people (certainly
not other students) in any way, unless you are consulting with the course
staff. By submitting the test, you will be acknowledging that you
completed the work on your own without the help of others in
any capacity. Any such aid would be unauthorized and a violation of the
academic integrity policy.

\[1\] You can use the browser to access Moodle, the course webpage, and look up technical
topics. Similar to a normal test, you must not communicate with other people.
Question 1 (sampling)

Consider an analog signal, \( x_a(t) = 0.4 \cos(120\pi t + 0.6\pi) + 0.7 \cos(480\pi t) \).

(a) What is the Nyquist rate required to sample \( x_a(t) \)?

(b) What is the maximum analog frequency that can be reconstructed by sampling an analog signal at \( F_s = 100 \) samples per unit time?
(c) Suppose that $x_a(t)$ is sampled at $F_s = 200$ samples per unit time. What is the resulting discrete time sampled signal $x(n)$?

(d) Was there aliasing in Part (c)? Make sure to justify your answer.

(e) The discrete time signal $x(n)$ is passed through an ideal digital to analog (D/A) converter using $F_s = 200$ samples per unit time. Determine the reconstructed signal in continuous time, $\hat{x}_a(t)$. 
Consider the following difference equation,

\[ y(n) + 1.5y(n-1) = x(n) + \frac{1}{2}x(n-1). \]

(a) Compute the transfer function \( H(z) = \frac{Y(z)}{X(z)} \).

(b) Determine the zeros and poles of the system \( H(z) \).
(c) Suppose that the system is causal.

- Determine the region of convergence (ROC).
- Draw the poles, zeros, and ROC.
- Is the system stable?
This question will guide you through a problem where a discrete time communication signal, \( x(n) \), is reflected off of some object in the environment. The receiver measures
\[
y(n) = x(n) + A \cdot x(n - k),
\]
where \( x(n) \) is the transmitted signal, \( A \cdot x(n - k) \) is the reflected signal, \(|A| < 1\) is the reflection’s amplitude, \( k > 0 \) is its delay, \( A \) is real valued, and \( k \) is integer. Our goal will be to estimate the amplitude, \( A \), and delay, \( k \), of the reflected signal.

(a) Recall that the autocorrelation for the transmitted signal, \( x(n) \), obeys
\[
R_{xx}(l) = \sum_n x(n)x(n + l).
\]
In this part, you will help us derive \( R_{yy}(l) \), the autocorrelation of the version measured at the receiver. In the derivation below, \( \alpha, \beta, \) and \( \gamma \) are missing; please specify these 3 expressions. (For your convenience, these missing parts are underlined and in red font.)

\[
R_{yy}(l) = \sum_n y(n)y(n + l)
= \sum_n [x(n) + A \cdot x(n - k)][x(n + l) + \alpha]
= \sum_n [(\beta + A \cdot x(n - k)x(n + l) + A \cdot x(n)x(n + l - k) + A^2 \cdot x(n - k)x(n + l - k)]
= \left[ \sum_n x(n)x(n + l) \right] + A \left[ \sum_n x(n - k)x(n + l) \right]
+ A \left[ \sum_n x(n)x(n + l - k) \right] + A^2 \cdot [\gamma]
= R_{xx}(l) + A \cdot R_{xx}(l + k) + A \cdot R_{xx}(l - k) + A^2 \cdot R_{xx}(l)
= R_{xx}(l)[1 + A^2] + A[R_{xx}(l + k) + R_{xx}(l - k)].
\]
(b) To keep things simple, in the remainder of the question we assume that the transmitted signal, \( x(n) \), has an impulse autocorrelation, 

\[
R_{xx}(l) = \delta(l).
\]

In part (b), suppose that the delay satisfies \( k = 3 \). Specify \( R_{yy}(l) \) as a function of \( A \). (Hint: \( R_{yy}(l) \) is nonzero for \( l \in \{-k, 0, k\} \). Compute \( R_{yy}(l) \) for those values of \( l \).)

(c) Based on part (b), how would you estimate \( A \) from \( R_{yy}(l) \)? Keep in mind that \( y(n) \) is being measured, and you must somehow estimate \( R_{yy}(l) \). (If you are unsure about \( R_{yy}(l) \), you may assume that \( R_{yy}(-k) = A, R_{yy}(0) = 2A, R_{yy}(+k) = A^2 \).)