Please recall the course academic integrity policy for tests:
No cooperation or “collaboration” between students is allowed. Especially during an online course experience, it could be tempting to text or email a friend. This is not allowed. You will be allowed to use your notes, books, a browser, and software such as Matlab.\footnote{You can use the browser to access Moodle, the course webpage, and look up technical topics. Similar to a normal test, you must not communicate with other people.} However, while working on the test you should not text, email, or communicate with other people (certainly not other students) in any way, unless you are consulting with the course staff. By submitting the test, you will be acknowledging that you completed the work on your own without the help of others in any capacity. Any such aid would be unauthorized and a violation of the academic integrity policy.
Question 1 (Linear convolution via DFT; 6 points.)

Let \( x(n) = \{6, 2, 0, 3\} \) be the input sequence to a finite impulse response (FIR) filter \( h(n) = \{1, 2\} \) (the underline corresponds to time zero). In this question, we will look at linear convolution using zero padding.

(a) Compute the linear convolution, \( y(n) \), using the time domain formula, i.e., \( y(n) = \sum_m x(m) h(n - m) \).

(b) How many zeros need to be padded to \( x(n) \) and \( h(n) \), respectively, in order to avoid aliasing in the output, \( y = x \ast h \), if we want to use the discrete Fourier transform (DFT) and inverse DFT (IDFT) to calculate \( y(n) \)? (Make sure to provide two numbers, one for \( x \) the other for \( h \).)
(c) What are $\tilde{h}$ and $\tilde{x}$ after zero padding $h$ and $x$?

(d) Suppose that $x$ and $h$ have been defined in Matlab as row vectors. The following Matlab code is supposed to calculate $y(n)$. There are missing parts below. Please write down the missing parts in Lines 3, 5 and 8.

```
1. Lx=length(x);
2. Lh=length(h);
3. Ly= __;
4. xpad=[x zeros(1,Ly-Lx)];
5. hpad= __;
6. xf=fft(xpad);
7. hf=fft(hpad);
8. yf= __;
9. y=ifft(xf);
```
Question 2 (DFT properties; 8 points.)
Consider the system below:
A real-valued discrete time signal, \( x(n) \), is of length \( N = 6 \). We are given several values of its discrete Fourier transform (DFT):

\[
X(0) = 5, \quad X(1) = 2 + j, \quad X(2) = 1 + 2j, \quad X(3) = 4.
\]

(a) Determine the missing values of \( X(k) \) for \( k \in \{4, 5\} \).

(b) We are given another length-6 discrete time signal, \( y(n) \), whose DFT, \( Y(k) \), is related to \( X(k) \) as follows,

\[
Y(k) = X(k)[0.5 + 0.5e^{j\pi k}].
\]

Compute \( Y(k) \) for \( k \in \{0, 1, 2, 3, 4, 5\} \). (If you are not sure about your answers in part (a), you may assume that \( X(4) = 35j \) and \( X(5) = 5 + 2j \).)
(c) Consider the DFT time shift property, 
\[ x(n - m)_N \leftrightarrow X(k)e^{-j\frac{2\pi km}{N}}, \]
where \( x(n - m)_N \) denotes time indices that are cyclical with respect to modulo \( N \). Use the time shift property to express \( y(n) \) in terms of \( x(n) \). (We expect a response such as \( y(n) = x(n) - 10 \) or \( y(n) = x(n + 1)_N - x(n - 2)_N \).)

(d) Find \( \sum_{n=0}^{5} x(n)x(n)^* \).
Consider an analog filter with system function

\[ H_a(s) = \frac{s + 2}{s^2 + 4s + 13}. \]

(a) Convert the analog filter into a digital infinite impulse response (IIR) filter \( H(z) \) using a bilinear transformation, where the digital filter is supposed to have a resonant frequency of \( \omega_r = \frac{\pi}{3} \).
(b) In order to simplify the calculation, in the rest of question 3 please use a digital IIR filter with transfer function

\[ H(z) = \frac{(z + 1)(z - 1)}{z^2 - 1.5z - 1} \]

Where are the zero(s) and pole(s) of \( H(z) \)?

(c) Suppose that this system is stable. What is the region of convergence (ROC) of this filter? Is \( H(z) \) a causal system?