Direct Sums

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This supplement provides more details about direct sums.

**Direct sums:** Consider two vector spaces, $V_a$ and $V_b$, and two corresponding signals, $x_a \in V_a$ and $x_b \in V_b$. A direct sum, denoted by $V_c = V_a \oplus V_b$, is the union of all sums of the form $x_a + x_b$. That is,

$$V_a \oplus V_b = \{ x : \exists x_a \in V_a, x_b \in V_b, x = x_a + x_b \}.$$ 

As a simple example, consider $V_a$ to be the set of all signals band limited to $\omega_1$, and $V_b$ the set of all signals band limited between $\omega_1$ and $\omega_2 > \omega_1$. The direct sum is the set of all signals band limited to $\omega_2$. 