Compressive Imaging via Approximate Message Passing with Wavelet-Based Image Denoising

Jin Tan, Yanting Ma, and Dror Baron

Atlanta, GA
December 4th, 2014

Supported by NSF and ARO
Compressive Imaging
Compressive imaging

Less radiation

Less power consumption
Compressive imaging

Dimension $N$

$N \gg M$

Dimension $M$

Linear measurements

Dimension $M$

Dimension $N$
Compressive imaging

- Length-N input $x$
- Matrix $A$, dimension $M \times N$, $M < N$
- Additive white Gaussian noise
- Well known: modest # measurements $M$ suffices for robust signal reconstruction

$$y = A x + z$$
A

\[ z \sim N(0, \sigma_z^2) \]

Reconstruction

\[ y \rightarrow \text{Reconstruction} \rightarrow x \]
Approximate Message Passing
Approximate message passing (AMP)  
[Donoho, Maleki, & Montanari 2009]

- Fast iterative algorithm
- Based on belief propagation

\[
y = h + z = Ax + z
\]

\[
v = x + \text{noise}
\]
Approximate message passing (AMP)

Initialize \( x^0 = 0 \)

At iteration \( t \), do

Residual: \[ r^t = y - Ax^t + \frac{r^{t-1}}{M/N} \langle \eta_{t-1}'(x^{t-1} + A^T r^{t-1}) \rangle \]

Noisy image: \( \nu^t = x^t + A^T r^t \)

Denoising: \( x^{t+1} = \eta_t(\nu^t) \)
Approximate message passing (AMP)

Initialize $x^0 = 0$

At iteration $t$, do

Residual:  
$$ r^t = y - Ax^t + \frac{r^{t-1}}{M/N} \langle \eta'_{t-1}(x^{t-1} + A^T r^{t-1}) \rangle $$

Noisy image:  
$$ v^t = x^t + A^T r^t $$

Denoising:  
$$ x^{t+1} = \eta_t(v^t) $$
Approximate message passing (AMP)

Initialize $x^0 = 0$

At iteration $t$, do

Residual: \[ r^t = y - Ax^t + \frac{r^{t-1}}{M/N} \langle \eta'_{t-1}(x^{t-1} + A^Tr^{t-1}) \rangle \]

Noisy image: \[ v^t = x^t + A^Tr^t \]

Denoising: \[ x^{t+1} = \eta_t(v^t) \]

Standard AMP: $\eta_t(v^t)$ is scalar
Wavelet-Based Image Denoising
Wavelet-based image denoising

Wavelet-based, convenient for Onsager term computation

Original image

Wavelet transform
Wavelet-based image denoising

Adaptive Wiener filter [Mihcak et al. 1999]

Values of neighboring coefficients
Wavelet-based image denoising

Adaptive Wiener filter [Mihcak et al. 1999]

Compute variances of wavelet coefficients based on neighborhood, $\sigma_i^2$

Adaptive Wiener filtering: $\frac{\sigma_i^2}{\sigma_i^2 + \sigma_z^2} \cdot i$-th noisy wavelet coefficient
Wavelet-based image denoising

Adaptive Wiener filter [Mihcak et al. 1999]

Non-scalar denoiser

Compute variances of wavelet coefficients based on neighborhood, $\sigma_i^2$

Adaptive Wiener filtering: $\frac{\sigma_i^2}{\sigma_i^2 + \sigma_z^2} \cdot i$-th noisy wavelet coefficient
AMP with Adaptive Wiener Filter
(AMP-Wiener)
AMP-Wiener

Initialize $x^0 = 0$

At iteration $t$, do

Residual: $r^t = y - Ax^t + \frac{r^{t-1}}{M/N} \langle \eta'_{t-1}(x^{t-1} + A^T r^{t-1}) \rangle$

Noisy image: $v^t = x^t + A^T r^t$

Denoising: $x^{t+1} = \eta_t(v^t)$
AMP-Wiener

Denoising: \( x^{t+1} = \eta_t(v^t) \)

✓ Noise variance is approx. \( \|r^t\|_2^2 / M \) [Montanari 2012]

✓ Divergence problem
AMP-Wiener

Denoising: $x^{t+1} = \eta_t(v^t)$

- Noise variance is approx. $||r^t||_2^2/M$ [Montanari 2012]

- Divergence problem: damping [Rangan et al. 2014]
  $\alpha \cdot \text{current estimate} + (1 - \alpha) \cdot \text{previous estimate}$
  $(0 < \alpha \leq 1)$
Numerical Results
Numerical results

Original
Numerical results

Iteration 1
Numerical results

Iteration 3
Numerical results

Iteration 7
Numerical results

Iteration 30
Numerical results

A: i.i.d. zero-mean Gaussian, Measurement rate 0.3
Average over 591 images

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>NMSE(dB)</th>
<th>Runtime(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbo-BG</td>
<td>-20.37</td>
<td>12.39</td>
</tr>
<tr>
<td>Turbo-GM</td>
<td>-20.72</td>
<td>12.47</td>
</tr>
<tr>
<td>MCMC</td>
<td>-20.31</td>
<td>&gt;400</td>
</tr>
<tr>
<td>AMP-Wiener</td>
<td>-21.00</td>
<td>3.34</td>
</tr>
</tbody>
</table>

[Turbo-BG/GM: Som & Schniter 2012]
[MCMC: He & Carin 2009]
Numerical results
**Numerical results**

**AMP-Wiener** [Tan et al., May 2014]

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>NMSE(dB)</th>
<th>Runtime(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbo-BG</td>
<td>-20.37</td>
<td>12.39</td>
</tr>
<tr>
<td>Turbo-GM</td>
<td>-20.72</td>
<td>12.47</td>
</tr>
<tr>
<td>MCMC</td>
<td>-20.31</td>
<td>&gt;400</td>
</tr>
<tr>
<td>AMP-Wiener</td>
<td>-21.00</td>
<td>3.34</td>
</tr>
</tbody>
</table>

\[ r^t = y - Ax^t + \frac{r^{t-1}}{M/N} \langle \eta'_{t-1} (x^{t-1} + A^Tr^{t-1}) \rangle = \eta_t \]
Numerical results

AMP-Wiener [Tan et al., May 2014]

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>NMSE(dB)</th>
<th>Runtime(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbo-BG</td>
<td>-20.37</td>
<td>12.39</td>
</tr>
<tr>
<td>Turbo-GM</td>
<td>-20.72</td>
<td>12.47</td>
</tr>
<tr>
<td>MCMC</td>
<td>-20.31</td>
<td>&gt;400</td>
</tr>
<tr>
<td>AMP-Wiener</td>
<td>-21.00</td>
<td>3.34</td>
</tr>
</tbody>
</table>

\[ r_t = y - Ax_t + \frac{r_{t-1}}{M/N} \langle \eta'_{t-1} (x^{t-1} + A^T r^{t-1}) \rangle \]

AMP-BM3D with Monte Carlo
[Metzler et al., June 2014]
Numerical results

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>NMSE(dB)</th>
<th>Runtime(sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbo-BG</td>
<td>-20.37</td>
<td>12.39</td>
</tr>
<tr>
<td>Turbo-GM</td>
<td>-20.72</td>
<td>12.47</td>
</tr>
<tr>
<td>MCMC</td>
<td>-20.31</td>
<td>&gt;400</td>
</tr>
<tr>
<td>AMP-Wiener</td>
<td>-21.00</td>
<td>3.34</td>
</tr>
<tr>
<td>AMP-BM3D</td>
<td>-25.27</td>
<td>16.06</td>
</tr>
</tbody>
</table>

A: i.i.d. zero mean Gaussian
Radio astronomy imaging system

\[ y = \text{blurring kernel} \ast x + z \]
Conclusion
Conclusion

Approximate message passing:
Convert matrix channel problem to scalar denoising problem
Conclusion

Denoising via adaptive Wiener filter:

\[ x^{t+1} = \eta_t(v^t) \]
Conclusion

Adaptive Wiener filter:
A robust denoiser with simple derivative

\[ r^t = y - Ax^t + \frac{r^{t-1}}{M/N} \langle \eta'_{t-1}(x^{t-1} + A^T r^{t-1}) \rangle \]

Onsager term
Questions?