Delay Optimal Scheduling for Energy Harvesting Based Communications

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Abstract—We consider the efficient scheduling problem for a bursty wireless link that is powered by a capacity-limited battery storing the harvested energy with the power grid as a backup. Specifically, we propose a scheduling scheme, which allows the source node to rely on the harvested energy supply to transmit packets whenever possible, and draw the grid power when necessary, but with an average power constraint. We formulate a two-dimensional Markov chain, and give an analysis on the average queuing delay and the average power consumption from the grid. Then, a linear programming problem is formulated to minimize the average delay under the constraint of a maximum allowable power consumption from the grid. By analyzing the corresponding optimization problem, we obtain the optimal scheduling policy and the optimal transmission parameters.

I. INTRODUCTION

Energy harvesting based communication has attracted much research attention in recent years. Equipped with energy harvesting devices, wireless nodes are able to gather energy from surrounding environments. Thus, the life of wireless networks, in particular sensor networks powered by capacity limited batteries, can be substantially extended. Energy harvesting can also help reduce carbon emission and environmental pollution, as well as reliance on traditional energy resources, and thus have been regarded as a key technology leading to wireless green communications.

Recently, some works have focused on developing efficient transmission and resource allocation algorithms for wireless communications subject to different objectives and energy harvesting profiles. For example, the optimal transmission problem has been investigated for an energy harvesting wireless link power by batteries of either finite or infinite capacity in [1], [2]. A save-then-transmit protocol was proposed in [3] to minimize the delay constrained outage probability by using two batteries that are alternatively used and recharged. Another line of work also focused on the queueing performance analysis for optimal energy management policies [4] [5].

Due to the bursty energy harvesting profile, a node should accumulate a sufficient amount of energy before each packet transmission if it relies only on harvested energy supply. In this case, the waiting time could be undesirably long. Intuitively, this situation can be greatly relieved if the grid power can be used to transmit backlogged packets when needed. Hence, there exists a tradeoff between the waiting time and the consumed energy from the grid.

In this work, we investigate the delay optimal scheduling problem for a wireless link powered by a capacity limited battery storing harvested energy and the backup power grid. In our system, the source will first seek energy supply from the battery whenever available, and resort to the power grid when necessary, but with an average power constraint. The source schedules packet transmissions with one of energy supplies based on the data packet queue status and the energy storage status at the battery. By two-dimensional Markov chain modeling and Linear Programming (LP) formulation methods, we succeed to find the optimal scheduling policy and the optimal transmission parameters. It is found that the source relies on the harvest energy supply if the data queue length is below a critical threshold, and resorts to the power grid otherwise in the face of a depleted battery.

The rest of this paper is organized as follows. Section II briefly presents the system model and the stochastic scheduling scheme. In Section III, we formulate a two-dimensional Markov chain. In Section IV, a two-step procedure is applied to obtain the optimal scheduling parameters. Section V demonstrates simulation results and Section VI concludes this paper.

II. SYSTEM MODEL

We consider a wireless link powered mainly by a battery storing the harvested energy and further by the power grid when necessary, as shown in Fig.1. Suppose that the data packets arrive at the source buffer according to a Bernoulli arrival process [6] with probability η1. The system is assumed to be time-slotted, and at the beginning instant of each slot, \( k_1 \in N \) data packets arrive at the data queue with arbitrarily large and indeterministic capacity \( Q_1 \). Let \( q_1[t] \in Q_1 = \{0, 1, 2, \cdots, Q_1\} \) be the length of the data queue at the end of time slot \( t \), updated as \( q_1[t] = \min\{q_1[t-1]+a_1[t]-v_1[t], Q_1\} \), where \( a_1[t] \in \{k_1, 0\} \) and \( v_1[t] \in \{1, 0\} \) denote the number of data packets arriving and served in each time slot \( t \), respectively. Without loss of generality, it is assumed that at most one packet is transmitted in each slot due to the capacity limitation of the wireless link.

We also adopt a probabilistic energy harvesting model. Assume that a unit of harvested energy arrives at the beginning of a time slot with probability \( \eta_2 \), which can be used to

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transmit $k_2$ packets. The harvested energy is stored in the battery with the maximum capacity $E$ Joule, and discarded when the battery is full. Let $\bar{e}_s$ (Joule) denote the amount of energy needed for transmission of one data packet. The battery storage is modeled as a virtual energy queue with a finite capacity $Q_2 = \lfloor E/\bar{e}_s \rfloor$. Similarly, the length of the energy queue $q_2[t] \in Q_2 = \{0, 1, 2, \ldots, Q_2\}$ is updated as $q_2[t] = \min\{q_2[t-1] + a_2[t] - v_2[t], Q_2\}$, where $a_2[t]$ and $v_2[t]$ are the corresponding arrival and service processes. It is assumed that the packet and energy arrival processes are independent, and the newly harvested energy can be used for data transmission in the same slot. For notational convenience, we set $q[t] = (q_1[t], q_2[t])$, $a[t] = (a_1[t], a_2[t])$ and $v[t] = (v_1[t], v_2[t])$.

The source should always transmit using the energy stored in the battery or newly arriving energy packet when possible, which corresponds to the case $q_2[t-1] > 0$ or $a_2[t] > 0$. When the harvested energy is not available, i.e., $q_2[t-1] = a_2[t] = 0$, the source schedules the transmission of data packets with the grid energy probabilistically according to the data queue status $q_1[t-1]$ and the data packet arrival status $a_1[t]$. To this end, we define two sets of parameters: $\{g_i\}$ and $\{f_i\}$. With $q_1[t-1] = i$, the source transmits one data packet with probability $g_i$ ($i \geq 0$) with the grid energy if $a_1[t] > 0$, and with probability $f_i$ ($i > 0$) if $a_1[t] = 0$, respectively. Accordingly, the service process $\nu[t]$ is expressed as

$$\nu[t] = \begin{cases} (1, 1) & w.p.1, \ a[t] \cup q[t-1] \subset \Psi_1, \\ (1, 0) & w.p.g_1, \ a[t] = (k_1, 0), q[t-1] = (i, 0), \\ (1, 0) & w.p.f_1, \ a[t] = (0, 0), q[t-1] = (i, 0), \\ (0, 0) & \text{otherwise,} \end{cases}$$

where $\Psi_1 = \{a[t] \cup q[t-1] | a_1[t] + q_1[t-1] > 0, a_2[t] + q_2[t-1] > 0\}$.

In this work, the queueing system is modeled as a discrete-time Markov chain, where each state represents the buffer status $q[t]$. The performance metrics of interest are the average queueing delay given by

$$\tilde{D} = \frac{1}{\eta_1} \sum_{i=1}^{Q_1} i \sum_{j=0}^{Q_2} \pi(i, j),$$

and the normalized average power consumption from the grid given by

$$\tilde{P} = \sum_{i=0}^{Q_1} \pi(i, 0) \cdot \omega(i, 0)(p),$$

where $\pi(i, j)$ is the steady-state probability of state $(i, j)$, and $\omega(i, j)(p)$ is the probability that the source draws one unit power $p$ from the grid for one packet transmission at state $(i, j)$. In the sequel, we focus on investigating the delay optimal scheduling problem under the average power constraint $\tilde{P} \leq P_{max}$ for 1) Case I with $k_1 = k_2 = 1$; 2) Case II with $k_1 = 1, k_2 > 1$; and 3) Case III with $k_1 > 1, k_2 = 1$, respectively. These three cases capture the essence of the match and mismatch between the data arrival model and the energy harvesting capacity.

### III. Two-Dimensional Markov Chain Modeling

To analyze the proposed scheduling scheme, we formulate two-dimensional discrete-time Markov chains for Cases I, II and III, respectively, as shown in Fig. 2. In the figure, the solid lines represent the fixed state transitions while the dotted lines indicate state transitions that vary with the values of $k_2$ and $k_1$, respectively. Let $\Pr\{q[t+1]|q[t]\}$ be the one-step transition probability of the homogeneous Markov chain. For ease of expression, we define four constants as

$$\mu_0 = (1 - \eta_1)\eta_2, \ \mu_1 = (1 - \eta_1)(1 - \eta_2), \ \mu_2 = \eta_1(1 - \eta_2), \ \mu_3 = \eta_1\eta_2.$$  

We further define two subsets of $Q_i$ as: $Q_i^L = \{0, \ldots, Q_i-1\}$, $Q_i^R = \{1, \ldots, Q_i\}$, and set $\eta_i = 1 - \eta_i$, for $i = 1, 2, \ldots$.

In Case I, the Markov chain turns out to be the one-dimensional one, as plotted in Fig. 2(a), which consists of the queue states $(i, 0)$ and $(0, j)$ for all $i \in Q_1$, and $j \in Q_2$. The transition between the states $(i, 0)$ corresponds to the case that there is no storage of harvested energy in the battery. When one data packet arrives while no energy is harvested, $\lambda_{1,i}$ denotes the transition probabilities from state $(i, 0)$ to $(i+1, 0)$ if no data packet is delivered with the grid power in this slot (with probability $1 - g_i$). When neither data packets arrive, $\mu_{1,i}$ denotes the transition probabilities from state $(i, 0)$ to $(i-1, 0)$, if one backlogged data packet is transmitted using one newly arriving energy packet or using the grid power in this slot (with probability $f_i$) when no energy is harvested. Thus, the transition probabilities $\{\lambda_{1,i} \ (i \in Q_i^L)\}$ and $\{\mu_{1,i} \ (i \in Q_i^R)\}$ can be expressed as

$$\lambda_{1,i} = \Pr\{(i+1, 0)|(i, 0)\} = \mu_2(1 - g_i),$$

$$\mu_{1,i} = \Pr\{(i-1, 0)|(i, 0)\} = \mu_0 + \mu_1 f_i.$$  

And the transition between the states $(0, j)$ corresponds to the case that the data packet queue is empty. In this case, when no data but one energy packet newly arrives, the state $(0, j)$ will transfer to $(0, \min\{j+1, Q_2\})$ with the probability $\mu_0$.
for $j \in Q_2^f$. When one data packet arrives while no energy is harvested, the state $(0, j)$ will transfer to $(0, j - 1)$ with the probability $\mu_2$ for $Q_2 > j > 0$. The state $(0, Q_2)$ stay at itself with the probability $\eta_1$ due to the capacity limitation of the battery. It will transfer to $(0, Q_2 - 1)$ if one data packet newly arrives with probability $\eta_2$.

We then focus on the Markov chain of Case II shown in Fig.2(b) (where $k_2 = 2$ may be assumed when checking the following expressions). Similar to Case I, the transition in the first column $(j = 0)$ of Fig.2(b) corresponds to the case that there is no storage of harvested energy in the battery. The corresponding transition probabilities are expressed as $\lambda_{1,i} = \mu_2(1 - g_i)$, and $\mu_{1,i} = \Pr\{i - 1, 0\} = \mu_1 f_i$ for all $i \in Q_1^f$. When $k_2$ energy packets arrive at this slot, the state $(i, 0)$ $(i \in Q_1^f)$ will transfer to $(i, \min\{k_2, Q_2\} - 1)$ with probability $\mu_3$ and $(i - 1, \min\{k_2, Q_2\} - 1)$ with probability $\mu_0$, respectively, depending on whether there is a new data packet arrival or not.

We then pay attention to the transitions in the first row $(i = 0)$. When $k_2$ energy packets newly arrive, the state $(0, j)$ will transfer to $(0, \min\{j + k_2, Q_2\})$ with probability $\mu_0$, if no data packets arrive, and transfer to $(0, \min\{j + k_2, Q_2\} - 1)$ with probability $\mu_3$, if one data packet newly arrives and is delivered immediately. If no energy packets arrive while there is a new data packet arrival, the state $(0, j)$ for $0 < j < Q_2$ transfers to $(0, j - 1)$ with probability $\mu_2$, and remains the same with probability $\mu_1$, when neither data nor energy packets arrive. The case $j = Q_2$ requires special treatment, as the battery is full and the newly harvested energy has to be discarded anyway. Then let us consider the four transitions starting from the state $(i, j)$ for $i > 0$ and $0 < j < Q_2$. Corresponding to four combinations of data and energy arrival states, i.e., $a_1[t] \in \{0, 1\}$ and $a_2[t] \in \{0, 1\}$, the state $(i, j)$ will transfer to state $(i - 1, j - 1)$ with probability $\mu_1$, to state $(i - 1, \min\{k_2, Q_2\} - 1)$ with probability $\mu_0$, to state $(i, j - 1)$ with probability $\mu_2$, and $(i, \min\{k_2, Q_2\} - 1)$ with probability $\mu_3$, respectively.

Next we give an illustration of the transition probabilities of the Markov chain in Case III, as shown in Fig.2(c). The transition probabilities in the first column of Fig.2(c) are expressed as $\lambda_{1,i} = \mu_2(1 - g_i)$ $(i \in Q_1^f)$, $\mu_{1,i} = \mu_0 + \mu_1 f_i$ $(i \in Q_1^f)$.

$$\begin{align*}
\lambda_{1,i} &= \Pr\{(i + k_1 - 1, 0)\} = \mu_3 + \mu_2 g_i (i \in Q_1^f), \\
\mu'_{1,i} &= \Pr\{(i, 0)\} = \mu_1 (1 - f_i) (i \in Q_1^f).
\end{align*}$$

(6)

When $k_1$ data packets newly arrive, $\lambda_{1,i}$ and $\lambda_{1,i}$ denote the transition probabilities from state $(i, 0)$ to $(i + k_1 - 1, 0)$ and $(i + k_1, 0)$, respectively, depending on whether one data packet is delivered or not in this slot. And $\mu_{1,i}$ and $\mu'_{1,i}$ denote the transition probabilities from state $(i, 0)$ to $(i - 1, 0)$ and $(i, 0)$, respectively, for the case when there is no new data packet arrival. Similar to case II, there are four transitions starting from the state $(i, j)$ for $i \geq 0$ (including the first row) and $0 < j < Q_2$ with the transition probabilities $\mu_1$, $\mu_0$, $\mu_2$ and $\mu_3$, respectively, corresponding to four combinations of $a_1[t] \in \{0, 1\}$ and $a_2[t] \in \{0, 1\}$. When the battery is full ($j = Q_2$), the state $(i, Q_2)$ $(i > 0)$ transfers to $(i + k_1 - 1, Q_2 - 1)$ and $(i - 1, Q_2 - 1)$ with probabilities $\eta_1$ and $\eta_1$, respectively.

We order the $N = (1 + Q_1)(1 + Q_2)$ states as $(0, 0), \ldots, (0, Q_2), (1, 0), \ldots, (1, Q_2), \ldots, (Q_1, 0), \ldots, (Q_1, Q_2)$, and denote $P$ as the $N \times N$ transition probability matrix. We denote by $\pi$ the $N \times 1$ row vector containing steady-state probabilities, and by $e$ the $1 \times N$ column vector with all the elements equal to one. For notational convenience, we also define two sub-vectors of $\pi$ as: $\pi_i = [\pi(i, 0), \ldots, \pi(i, Q_2)]$ and $\pi_i = [\pi_0, \ldots, \pi_i]$. Given a set of parameters $\{g_i\}$ and
{f_i}, the steady-state probabilities \( \pi_{i,j} \) can be obtained by solving the linear equations \( \pi P = \pi \), \( \pi e = 1 \). Note that the transmission parameters \{g_i\} and \{f_i\} only influence the transition probabilities from the states \((i, 0)\), \(i \in Q_1\). We thus consider \( P_{0} \), a submatrix of \( P = P - I \), which excludes the state transitions starting from states \((i, 0)\). In this way, \( \pi P_{0} = 0 \) present the local balance equations at the states \((i, j)\) \((i \geq 0, j > 0)\).

**IV. DELAY OPTIMAL SCHEDULING UNDER POWER CONSTRAINT**

In this section, we adopt the two-step procedure [7] to find the optimal scheduling policy for the three cases in a unified way. Due to space limitation, most of the proofs for the results presented in this section are omitted, and the reader is referred to [8] for details.

**A. LP Formulation**

As the first step, we formulate the LP problem which aims at minimizing the average queue delay subject to the maximum average power constraint from the grid as:

\[
\begin{align*}
\min \quad & D = \frac{1}{\eta_1} \sum_{i=0}^{Q_1} \sum_{j=0}^{Q_2} \pi_{i,j} \\
\text{s.t.} \quad & \bar{P}(\pi) \leq P_{\max}, \tag{a} \\
& \sum_{j=0}^{Q_2} \pi_{i,j} \leq \Theta_u(i, \bar{\pi}_1) \quad (i > 0), \tag{b} \\
& \sum_{j=0}^{Q_2} \pi_{i,j} \geq \Theta_1(i, \bar{\pi}_{i-1}) \quad (i > 0), \tag{c} \\
& \pi_{i,j} \geq 0, \quad (\forall i,j), \tag{d} \\
& \sum_{i=0}^{Q_1} \sum_{j=0}^{Q_2} \pi_{i,j} = 1, \tag{e} \\
& \pi P_{0} = 0. \tag{f}
\end{align*}
\]

From the property of a Markov chain, the last three constraints \(d)-(f)\) are straightforward. The constraint \(a\) indicates the power constraint with a new expression of \( \bar{P} \) presented in Lemma 1 below, which depends only on the steady-state probabilities \(\pi_{i,0}\) and \(\pi_{i,1}\); \(i \in Q_1\). And the constraints \(b\) and \(c\) represent the relationship between the steady-state probabilities themselves due to the varying transmission parameters, as discussed later in Lemma 2. The optimal solution to (7) is denoted by \(\pi^* = [\pi^*_{i,j}]_{1 \times N}\) and the minimum average delay by \(D^*\).

**B. The Optimal Solution**

According to Loynes’s theorem, the queueing system is stable only when the average arrival rate is less than the average service rate. Hence, we will discuss the optimal solution to the LP problem (7) under the assumption that the queueing system is stable, i.e., \(P_{\max} > k_1 \eta_1 - k_2 \eta_2\).

From the theorem of linear programming, there must exist at least one optimal solution to the LP problem (7), since the average queueing delay \(\bar{D} = \frac{1}{k_1 \eta_1} \sum_{i=0}^{Q_1} i \pi_i\) is lower bounded by zero. And the optimal solution of an LP problem can be searched over the vertices (equivalently, extreme points) of the polyhedron for the LP feasible region. We also notice that \(\bar{D} = \frac{1}{k_1 \eta_1} \sum_{i=0}^{Q_1} i \pi_i\) is a weighted summation of the steady-state probabilities \(\pi_i = \sum_{j=0}^{Q_2} \pi_{i,j}\). Thus, based on this intuition, we obtain the structure of the optimal solution \(\pi^*\) in the following theorem.

**Lemma 1.** In Cases I, II, and III, the normalized average power consumption from the grid can be expressed as \(\bar{P} = \sum_{i=0}^{Q_1} \xi_i \cdot \pi_{i,0} - \sum_{i=0}^{Q_1} \eta_i \cdot \pi_{i,1}\), where the coefficients \(\xi_i\) and \(\zeta_i\) are presented in Table 1.

For ease of illustration, we define several constants as \(\tau = \frac{\eta_1}{\eta_1 - \eta_2}, \phi = \frac{k_2 \eta_2}{k_2 \eta_2 - \eta_1}\), and \(\phi_1 = \frac{\eta_1}{\eta_1 - \eta_2}\), and define \([x]^+ = \max\{0, x\}\).

**Lemma 2.** In Cases I, II and III, the steady-state probabilities \(\pi_{i,j}\) satisfy \(\Theta_u(i, \bar{\pi}_1) \leq \pi_i = \sum_{j=0}^{Q_2} \pi_{i,j} \leq \Theta_u(i, \bar{\pi}_1)\) for all \(i > 0\), where \(\Theta_u(i, \bar{\pi}_1)\) and \(\Theta_1(i, \bar{\pi}_{i-1})\) are linear functions of \(\pi_i\) and \(\pi_{i-1}\), respectively, as listed in Table 2. And \(\pi_i = \Theta_u(i, \bar{\pi}_1)\) and \(\pi_i = \Theta_1(i, \bar{\pi}_{i-1})\) hold when \(g_{i-k_1} = f_i = 0\) and \(g_{i-k_1} = f_i = 1\), respectively.

From the above two lemmas, \(\bar{P}, \Theta_u(i, \bar{\pi}_1)\) and \(\Theta_1(i, \bar{\pi}_{i-1})\) are all linear functions of the steady-state probabilities \(\{\pi_{i,j}\}\). Hence, we can represent them as \(\bar{P} = \pi a_0, \sum_{j=0}^{Q_2} \pi_{i,j} - \Theta_u(i, \bar{\pi}_1) = \pi a_1^u\) and \(\Theta_1(i, \bar{\pi}_{i-1}) - \sum_{j=0}^{Q_2} \pi_{i,j} = \pi a_1^1\), where \(a_0, a_1^u\) and \(a_1^1\) are \(N \times 1\) column vectors collecting corresponding coefficients, and will be used to compute the optimal solution later.

**Table I**

<table>
<thead>
<tr>
<th>Case I with (k_1 = k_2 = 1)</th>
<th>Case II with (k_1 = 1) and (k_2 &gt; 1)</th>
<th>Case III with (k_1 &gt; 1) and (k_2 = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\xi_i = 0 \quad (i \in Q_1))</td>
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<td>(\xi_i = 0 \quad (i \in Q_1))</td>
</tr>
<tr>
<td>(\eta_0 = \mu_2 + \mu_2)</td>
<td>(\eta_0 = \mu_2 + \eta_2 (Q_1 - i) \quad (i \in Q_1))</td>
<td>(\eta_0 = \mu_2 (Q_1 - i) + \mu_2 (i \in Q_1))</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>Case I with (k_1 = k_2 = 1)</th>
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</tr>
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<tbody>
<tr>
<td>(\Theta_u(i, \bar{\pi}_1))</td>
<td>(\Theta_1(i, \bar{\pi}_{i-1}))</td>
<td>(\Theta_u(i, \bar{\pi}_1))</td>
</tr>
<tr>
<td>(\phi \pi_{(i-1,0)})</td>
<td>(\tau \pi_{(i,0)} + \pi_{(i,0)} \phi_2)</td>
<td>(\pi_{(i-1,0)} + \pi_{(1,0)} \phi_2)</td>
</tr>
<tr>
<td>(\tau \pi_{(i-1,0)} \phi_2)</td>
<td>(\pi_{(i,0)} \phi_2 + \sum_{m=0}^{i-1} \pi m_{m(i+k_1+1)} \leq k_1)</td>
<td>(\tau \pi_{(i-1,0)} \phi_2 + \sum_{m=0}^{i-1} \pi m_{m(i+k_1+1)} \leq k_1)</td>
</tr>
</tbody>
</table>
Theorem 3. The optimal solution $\pi^*$ satisfies

$$
\pi^* a_i \leq p_{\text{max}},
$$

$$
\pi^* a_i^u = 0 (i = 1, \cdots, i^*-1),
$$

$$
\pi^* a_i^l = 0 (i = i^* + 1, \cdots, Q_1),
$$

where $i^*$ is the optimal threshold on the data queue length determined by $p_{\text{max}}$.

From Theorem 3 and Lemma 2, $\sum_{j=0}^{Q_2} \pi^*_{i,j} = \Theta_a(i, \pi^*_i)$ when $g_i-k_i = f_i = 0$ for $i < i^*$, and $\sum_{j=0}^{Q_2} \pi^*_{i,j} = \Theta_l(i, \pi^*_{i-1})$ when $g_i-k_i = f_i = 1$ for $i > i^*$, respectively. This implies that the source should schedule the packet transmission based on a threshold $i^*$. The optimal threshold $i^*$ is determined by the maximum allowable power constraint $p_{\text{max}}$, and can be obtained by comparing $p_{\text{max}}$ to the power thresholds $\{\tilde{p}_m\}$ ($m \geq 0$), i.e.,

$$
i^* = \arg \min_{\tilde{p}_m \leq p_{\text{max}}} m, \quad (9)
$$

where $\tilde{p}_m$ is used to measure the amount of grid power consumption from the grid when the scheduling policy strictly based on the threshold $m$ is applied: the source waits for the harvested energy if the number of backlogged data packets is less than or equal to $m$. Note that $\tilde{p}_m$ is a decreasing function of data queue length $m$, and $p_{\text{max}} \geq \tilde{p}_0$ indicates that the allowable grid energy supply is sufficient so that the source can use the grid power whenever it needs. Please refer to [8] for more details about the power thresholds $\{\tilde{p}_m\}$.

In Case I, the Markov chain is one-dimensional and transitions takes place only between adjacent states, as shown in Fig. 2(a). In this case, we can obtain elegant closed-form expressions for optimal transmission parameters as follows.

**Corollary 4. In Case I, when $p_{\text{max}} \geq \tilde{p}_0 = \mu_2 \alpha^{-1}$, we have $\pi^*_{i,0} = \alpha^{-1}$ and $\pi^*_{i,1} = 0$ for all $i > 0$. When $\eta_1 - \eta_2 < p_{\text{max}} < \tilde{p}_0$, $\pi^*_{i,0} = \frac{p_{\text{max}} - (\mu_2 - \mu_0)}{\mu_2 - \alpha (\mu_2 - \mu_0)}$ and

$$
\pi^*_{i,0} = \begin{cases} 
\pi^*_{i,0} \phi^i, & i \leq i^*-1, \\
1 - \alpha \pi^*_{i,0} - \pi^*_{i,0} \sum_{i=1}^{i^*-1} \phi^i, & i = i^*, \\
0, & i > i^*,
\end{cases}
$$

for $i > 0$, respectively, where the optimal threshold is obtained as $i^* = \Omega_{\phi}(\pi^*_{i,0}, 1 - \alpha \pi^*_{i,0})$ with the function $\Omega_{\phi}(a,b)$.**

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<td>$g^*_i$</td>
<td>$i = i^* - k_1$</td>
<td>$i &gt; i^* - k_1$</td>
</tr>
<tr>
<td>$f^*_i$</td>
<td>$i = i^*$</td>
<td>$i &gt; i^*$</td>
</tr>
</tbody>
</table>

Table III

**The optimal transmission parameters $g^*_i$ and $f^*_i$ for Cases I, II, and III.**
an optimal delay-power tradeoff for each $k_2$. The average queueing delay monotonically decreases with the increase of the maximum allowable power consumption $p_{\text{max}}$ from the grid due to the enhanced service rate. For the same reason, a larger $k_2$ means a higher amount of energy harvested each time, and leads to a much better delay-power tradeoff. It is also observed the delay-power curves of $k_2 = 5$ and $k_2 = 6$ are almost identical to each other. This owes to the fact that in the case of $k_2 = 6$, a part of harvest energy is wasted when recharging the battery with its capacity $Q_2 = 5$.

Similarly, we plot the optimal delay-power curves of the proposed scheme for Case III with different $k_1$ in Fig. 4. We set $\eta_1 = 0.1$, $\eta_2 = 0.3$, and $Q_2 = 5$. Similar to Case II shown in Fig. 3, a higher $p_{\text{max}}$ induces the reduced average queueing delay thanks to the enhanced service rate. The only difference between them is the behavior of the minimum average delay $D^*$. In Cases I and II, the average queueing delay is equal to zero if there exists sufficient energy whether from the battery or the grid, since one newly arriving data packet can always be delivered immediately. In Case III, however, at most one of $k_1$ data packets that newly arrive at this slot can be delivered, and the other packets shall wait for the next transmission opportunity. And more packets are queued when the data arrival rate is increased due to the growth of $k_1$ or $\eta_1$. As shown in Fig. 4, $D^*$ increases with the increase of $k_1$.

VI. CONCLUSIONS

In this paper, we studied the delay optimal scheduling problem over a communication link powered mainly by harvested energy together with the backup power grid. It is interesting to see that the threshold based transmission policy is the optimal. In particular, the source should wait for the harvested energy when the data queue length is below an optimal threshold, and resort to the grid if no harvested energy can be exploited while the data queue length exceeds the threshold ($i^*$ if there is no new data packet arrival, and $i^* - k_1$ if there is new data packet arrival). The optimal threshold $i^*$ is determined by the maximum allowable power from the grid $p_{\text{max}}$. In Cases I and II, only one data packet newly arrives each time and it can be delivered immediately. In these two cases, the source is allowed to exploit the grid power only when the threshold is exceeded and there is new data packet arrival ($g_{i^*}^1 = 1, f_{i^*}^1 = 0$). In Case III, multiple data packets arrive each time, and the data queue length could grow to infinity. To reduce the average queueing delay, the source should exploit the grid power to transmit as long as the threshold is reached, whether there is new data packet arrival or not ($g_{i^*}^2 = 1, f_{i^*}^2 = 1$). Simulation results confirmed our theoretical analysis. Notice that the method applied in this work can be used to deal with the optimal scheduling problem in continuous-time systems, since a similar linear programming can be formulated and solved. We will extend this work to the scenario where rate-flexible physical-layer transmissions are scheduled based on the randomly available amount of harvested energy and time-varying wireless channel conditions.

REFERENCES