Abstract—The advent of Bluetooth Low Energy (BLE) enabled Beacons is poised to revolutionize the indoor contextual aware services to the users. Due to the lower energy consumption and higher throughput, BLE could therefore be an integral pillar of an Internet of Things (IoT) Location Based Service (LBS). Tracking a user with high accuracy is known as Micro-Location. This is a requirement of many IoT user-centric applications for indoor environments. Although several technologies have been used for tracking purposes, the accuracy has always been a serious issue. At the same time, each vendor would install different technologies. In this work, we propose to use the cutting-edge and commercially available Apple’s iBeacon protocol and iBeacon BLE sensors for micro-location. We propose to leverage a control theoretic approach, namely particle filtering, in order to increase the tracking accuracy in an indoor environment. We performed extensive experiments and our results show that the proposed beacon based micro-location system can be used to locate a user in an indoor environment with an error as low as 0.27 meters.

Index Terms—iBeacon, Internet of Things, Micro-Location, Particle Filtering.

I. INTRODUCTION

The idea of Internet of Things is gaining wide scale attention, however a concept of such transcending effect has certain issues associated with it. Ultra low power communication is one of essential needs of the IoT [1] which can be satisfied using the Bluetooth Low Energy (BLE). BLE is the latest radio technology of the Bluetooth Special Interest Group (SIG) and is also known as the Bluetooth smart. It has been particularly designed to keep the energy consumption low while providing higher throughput, lower latency, but with a comparatively shorter range (up to 50m). The low energy consumption would allow the sensor or other entities to communicate with other devices using a coin cell battery for a time span of up to two years.

Location Based Services (LBS) have been around for quite a while but the first generation of the LBS did not receive much attention due to its network centric approach. However, the second generation of the LBS is more user centric and is therefore finding its use on a large scale in the smart building domain. Realizing the significance of LBS, Apple introduced the iBeacons in Worldwide Developers Conference (WWDC) 2013 as part of its iOS 7.0. The beacon is the device that emits BLE signals whereas the iBeacon is the name of the proprietary protocol. This technology standard allows the mobile applications (apps) running on either iOS or Android platform to receive these BLE signals. The signals can be used for identifying proximity as well as provisioning of contextual aware services. Despite the relevant infancy of iBeacons, it is considered to be a revolutionary product that will penetrate the consumer market to a great extent. According to ibeaconinsider [2], the interaction of the user with the iBeacon advertised products increased by 19 times, while the in-store application usage was 16.5 times more for users receiving a beacon message. Similarly the device shipment is forecasted to be over 60 million by 2019 [3]. Realizing the potential of iBeacons, numerous large retail vendors such as Target, McDonalds, Macy’s etc. have started using them in stores to enhance the user experience. Beacons are intended for proximity based services, and not for micro-location, as they rely on the Received Signal Strength Indicator (RSSI), which may have significant fluctuations for indoor environments. Moreover, the iBeacon deployment in smaller scale or larger scale environments has not been standardized, and is purely based on experience. This may lead to suboptimal services offered to the customers. Hence, identifying the need to a more targeted study, we perform extensive experiments in a small space, i.e. 1m x 1m, and a large space i.e. 11m x 6m. We enhance the technology by proposing the use of a control theoretical approach that can take multiple RSSI values and determine the location of the user with a relatively high accuracy. We believe that this study is important for a wide audience, both engineers that deploy beacons, as well as corporations that want to offer iBeacon services based on the location of the user inside their business, i.e. educational spaces, museums or mega-stores. Moreover, we alter the parameters of the particle filtering algorithm and showcase the number of particles in order for a device to be captured accurately with the minimal consumption on the battery of the device. Hence the contribution of this work can be listed as follows:

- We leverage Apple’s iBeacons, a technology mainly intended for proximity, to provide micro-location services.
- We provide the mathematical formulation behind our study and create an actual prototype. The prototype is
tested in different environments with variable parameters to enhance accuracy.

- We highlight the experimental results to show the effectiveness of the proposed model towards accurate indoor positioning.
- We provide recommendations based on our experimental results for an actual iBeacon based deployment.

The paper is structured as follows: In section II, we present the related work while in section III, we describe the BLE technology as well the functioning of iBeacons. We provide an insight into the technology and its technical specifications. We describe particle filtering in section IV. The experimental setup and results are presented in section V, while we conclude the paper in section VI.

II. RELATED WORK

LBS are widely used in a number of outdoor facilities for navigation services. However providing highly accurate positioning services in indoor/GPS constrained environments is a challenging task due to the presence of obstacles and the inherent complexities in an indoor environment [4], [5]. There are a number of different techniques that can be used for indoor positioning such as Time Of Arrival (TOA), Return Time Of Flight (RTOF), Time Difference of Arrival (TDOA), RSS-based, Received Signal Phase Method, Angle of Arrival (AOA), Scene Analysis and Proximity algorithms [6].

RSSI based methods have been used extensively in [7]–[9]. More specifically, Zigbee, Wireless Local Area Networks (WLANs) and other similar wireless technologies have been used to track the users in an indoor environments. However, the problem with such protocols is that the access points require constant energy, and the protocols have been designed towards faster access rather than proximity based services. For reducing the RSS index drift, [9] uses both Heron-bilateration location estimation and Kalman-filter drift removal. This also reduces the positioning error as well as computation complexity. Such an approach further causes a decrease in the cost of deploying an RFID based indoor tracking system without jeopardizing the accuracy and granularity of localization. A hybrid indoor localization estimation technique that utilizes extended Kalman filter is presented in [10]. The presented technique integrates fingerprinting and trilateration resulting in a simple and robust approach for localization. Rather than converting the value of RSSI into distance, the euclidean distance formula is used.

In contrast with [9], [10], for improving the performance of the system, we utilize particle filters (PF) which tend to perform better than Kalman filters [11] and Unscented Kalman Filters (UKF) [7]. Furthermore our model is computationally efficient as it utilizes only particle filtering rather a combination of techniques as done in [12], [13] for improving the accuracy. Finally, we did not modify any of the core architecture of the BLE stack as was done with WLAN in [13]. PF is also used in [14] for localization using the Radio Frequency Identification (RFID) tags. The RFID system operates on Ultra High Frequency (UHF) and consists of a standard RFID reader, the passive tags that serve as the reference points. Proprietary designed semi-passive tag are attached to the object for tracking purposes. The semi-passive tag uses backscatter modulation for transferring the sensed information to the reader. While their setup was verified in a smaller space, we used comparatively extensive set of experiments to verify the accuracy of our model in a smaller and comparatively larger space. Our proposed system attains an accuracy as high as 0.27 meters which is higher than Magnetic Field Mapping (up to 2 meters), WLAN (3 meters), Zigbee (3 meters), GPS (for outdoor environments and 10 meters), and hybrid of RF and IR or RF and Ultrasonic technologies (1 meters).

III. BLUETOOTH LOW ENERGY AND IBEACONS

A. Bluetooth Low Energy

The low energy version of the Bluetooth that is specified in version 4 is known as Bluetooth Low Energy/Bluetooth smart [1]. In BLE, as opposed to the classic Bluetooth, the slave device advertises on either one or several of the three allocated advertisement channels for discovery purposes. The Master periodically scans the channels for discovering slaves. Once the slave device is discovered, the data transfer takes place in form of periodic connection events where both the slave and master wake up in synchronized manner for exchanging various frames. The devices save energy by sleeping for the rest of the time. In Bluetooth smart the core specifications have been enhanced to allow two different types of implementations, the single mode and the dual mode [15]. Overall, the BLE technology has a huge potential and will certainly be an important part of future IoT. Table I provides a summary of the specifications of BLE.

B. iBeacons

Apple’s iBeacon is a protocol that is meant to be used with the BLE enabled beacons. This technology assists the mobile applications in recognizing that any BLE enabled receiver is in the proximity of the BLE beacon. These beacons can communicate with any BLE enabled receiver and transmit data. The feature is included in iOS 7 and beyond. Similarly the Android 4.3+ systems also supports the iBeacons. It is a novel exciting technology that can enable the applications with better location awareness. The Beacon can be used for creating a region, known as a geofence, around an entity that will allow the iOS or the Android system to identify when it either enters or exits the location. Such geofences help in providing proximity based services such as when a user enters the geofence of any store, he will be provided with relevant information. The format of the Beacon advertisement is standardized by Apple. The advertising packet of the Beacon consists of three different components.

- Universally Unique Identifier (UUID): 16 Byte value used to differentiate an organization’s iBeacon from others. For a certain brand X, all of its beacons would have the same UUID hence assisting the mobile application to know about which network the beacons belong to.
Data packets size range from 8-27 octets in size and the throughput is 1Mbps

Frequency Hopping
Adaptive Frequency hopping is used that assists in minimizing the interference

Latency
The minimal latency for both connection setup and data transfer is 3ms.

Range
Range can be over 100m due to higher modulation index

Robustness
A 24-bit CRC is used for all the packets, hence robustness is enhanced

Topology
Optimized to work with one-to-one connections however can support one-to-many connections using a start topology

Security
BLE uses full AES-128 encryption using CCM for encrypting and authenticating the packets

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Rate</td>
<td>Data packets size range from 8-27 octets in size and the throughput is 1Mbps</td>
</tr>
<tr>
<td>Host Control</td>
<td>BLE has intelligent controllers that allows the host to sleep assisting in energy savings</td>
</tr>
<tr>
<td>Frequency Hopping</td>
<td>Adaptive Frequency hopping is used that assists in minimizing the interference</td>
</tr>
<tr>
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<td>The minimal latency for both connection setup and data transfer is 3ms.</td>
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<tr>
<td>Security</td>
<td>BLE uses full AES-128 encryption using CCM for encrypting and authenticating the packets</td>
</tr>
</tbody>
</table>

This mandatory field has to be included in every single advertisement.

- Major Value: 2 Byte value used for the specification of any iBeacon within a group. For the brand X in the example above, the beacons deployed in particular store in city Y will have the same major value. It is an optional field and is not necessarily advertised in every advertisement.

- Minor Value: 2 Byte value used for identification of particular beacons. For the brand X above, in store Y, the beacon in a particular section say shoes section will have its own unique minor value. It is also an optional field.

When the iOS or the Android system detects the signal from the beacon, Received Signal Strength Indication (RSSI) is used to not only determine the proximity to a specific beacon but also the accuracy of the proximity estimation. The higher the signal strength, the better will be the proximity measurement and higher will be the confidence of the iOS about the proximity to the beacon and vice versa.

Once the BLE enabled device receives a signal from an iBeacon with a UUID, the application running on the device contacts either the server or cloud to find out what type of functionality is associated with the specific UUID. It can be a coupon, an event notification or any other contextual aware entity. Figure 1 explains this concept.

**IV. PARTICLE FILTERING**

In the section below we describe the mathematical formulation that we used in our prototype. Micro-location is a non-linear bayesian tracking problem therefore we define our tracking problem as per [16]. The state sequence \( \{y_i, i \in N\} \) of the target is given by

\[
y_i = g_i(y_{i-1}, m_{i-1}) \quad (1)
\]

where \( g_i \) is a non-linear state function of \( y_{i-1} \). \( \{m_{i-1}, i \in N\} \) is an independent, identically distributed (i.i.d) process representing the noise sequence. The basic idea behind positioning is to estimate the state \( y_i \) recursively using the measurement \( z_i \) where \( z_i \) is given by

\[
z_i = h_i(y_i, n_i) \quad (2)
\]

where \( h_i \) is a non-linear function. \( \{n_i, i \in N\} \) is the i.i.d measurement noise sequence. We are particularly interested in estimating the filtered \( y_i \) on the basis of all measurements available. \( z_{1:i} = \{z_k, k = 1, \ldots, i\} \) up to time \( i \). On the basis of the Bayesian approach, the tracking problem recursively calculates the belief in state \( y_i \) at a time \( i \), on the basis of the measurements obtained \( z_{1:i} \) up to time \( i \). Therefore we must construct the pdf \( p(y_i | z_{1:i}) \). Let’s assume that the prior \( p(y_0 | z_0) = p(x_0) \) is known. So the pdf \( p(y_i | z_{1:i}) \) can be recursively obtained by a) predicting b) updating. Now for prediction state, suppose that the pdf \( p(y_{i-1} | z_{1:i-1}) \) is available at time \( i - 1 \). During the prediction stage, equation 1 is used for attaining the prior state pdf at time \( i \) using Chapman-Kolmogorov equation

\[
p(y_i | z_{1:i-1}) = \int p(y_i | y_{i-1}) p(y_{i-1} | z_{1:i-1}) dy_{i-1} \quad (3)
\]

During the update state, at time \( i \), the measurement \( z_k \) becomes available which can then be used to update the pdf using the bayes rule

\[
p(y_i | z_{1:i}) = \frac{p(z_i | y_i)p(y_i | z_{1:i-1})}{p(z_i | z_{1:i-1})} \quad (4)
\]

where

\[
p(z_i | z_{1:i-1}) = \int p(z_i | y_i)p(y_i | z_{1:i-1}) dy_i \quad (5)
\]

Equation 3 and 4 are the prediction and update state and lay the foundation for optimal beays solution. Determining it analytically is not possible as it is conceptual solution [16]. Therefore we use particle filtering to obtain the results that is a sequential Monte Carlo (MC) method that involves recursively calculating the relevant probability distributions [11].

It uses the concept of importance sampling and uses discrete random measures for approximating probability distributions. Particle Filtering is suitable for non-linear and non-Gaussian environments. There are different types of particle filtering methods of which Sequential Importance Sampling (SIS) is the most renowned one [7]. The basic idea in SIS type of
particle filtering is the representation of the required posterior probability density function (pdf) using a number of random samples that have specific weights [16]. The weights and samples are used to calculate the estimates.

Let \( \{y_{0,i}^k, w_{0,i}^k\} \) be the set of random measures that are used to characterize the posterior pdf \( p(y_{0,i}|z_{1:i}) \) where the set \( \{y_{0,i}^k, k = 0, \ldots, N_s\} \) is the support points set having weight given by \( \{w_{0,i}^k, k = 0, \ldots, N_s\} \) and \( y_{0,i} \{y_{j,i}, j = 0, \ldots, i\} \) is the set of the states up till time \( i \). After normalizing the weights, the posterior probability density at \( i \) will be approximately

\[
p(y_{0,i}|z_{1:i}) \approx \sum_{k=1}^{N_s} w_{0,i}^k \delta(y_{0,i} - y_{0,i}^k) \tag{6}
\]

Hence this results in a discrete weighted approximation of the true posterior probability. Importance sampling (see [17]) is used to choose the weights so the weights will be

\[
w_{0,i}^k \propto p(y_{0,i}|z_{1:i}) \tag{7}
\]

where \( q(y_{0,i}|z_{1:i}) \) is the importance density.

Since the process is sequential, then at every single iteration, the obtained samples could be an approximation to the \( p(y_{0:i−1}|z_{1:i−1}) \) and we need to approximate \( p(y_{0:i}|z_{1:i}) \) with a new sample set. In case the importance density is chosen for factorizing such that

\[
q(y_{0,i}|z_{1:i}) = q(y_{i}|y_{0:i−1}, z_{1:i})q(y_{0:i−1}|z_{1:i−1}) \tag{8}
\]

Then new samples \( y_{0,i}^k, q(y_{0,i}|z_{1:i}) \) can be obtained through the augmentation of the existing samples \( y_{0,i}^k \sim q(y_{0,i−1}|z_{1:i−1}) \) with the new state \( y_{i}^k \sim q(y_{i}|y_{0:i−1}, z_{1:i}) \). The weight updated equation can be found to be (see [16])

\[
y_{i}^k = \frac{w_{0,i−1}^k p(z_i|y_{i}^k)p(y_{i}^k|y_{0:i−1}^k)}{q(y_{i}^k|y_{0:i−1}^k, z_{1:i})} \tag{9}
\]

Similarly the posterior filtered probability density \( p(y_{i}|z_{1:i}) \) is approximated as

\[
p(y_{i}|z_{1:i}) \approx \sum_{k=1}^{N_s} w_{i}^k \delta(y_{i} - y_{i}^k) \tag{10}
\]

Hence the SIS algorithm recursively propagates the weights and particles as every measurement is obtained sequentially.

V. EXPERIMENTAL SET UP AND RESULTS

In order to evaluate the indoor accuracy of the proposed beacon implementation, we incorporated the particle filtering algorithm, described in the previous section, as part of an application in an iPhone. We forked the iOS application from [18] and modified it to fit in our experiments. Figure 2 shows two snapshots of the application we developed. Having used the particle filtering cocoapods, we also chose to stay out of any proprietary SDK as provided by the iBeacon vendors, and used the CoreLocation framework in the iPhone SDK. The CoreLocation and CoreBluetooth Frameworks are mandatory for the beacon to interact with an iOS device. CoreLocation Framework equips the application with the capability to listen to the beacons and notify the users about it. Furthermore, in order to reduce the fluctuation in the RSSI, the framework continuously averages RSSI values. We use the RSSI values to calculate the distances between the device and beacon, through the log-normal shadowing model [19], which has been shown to provide the most accurate results for indoor environments.

\[
RSSI = -10\log(d) + d_0 \tag{11}
\]

where \( d \) is the distance, and \( d_0 \) is the reference loss value for a distance of 1m [19]. Particle filtering along with trilateration was implemented on the mobile application [18] which is used to obtain the position of the user using various number of beacons that are placed in areas of different dimensions. We deployed our application on an Apple iPhone 4s running iOS 8.1. We used Series 10 beacons from Gimbal which have a transmission rate of 100 milliseconds. Table II presents application related information.

<table>
<thead>
<tr>
<th>Device</th>
<th>Apple iPhone 4s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wireless Interface</td>
<td>Bluetooth V4.0 / 2.4GHz</td>
</tr>
<tr>
<td>Operating System</td>
<td>iOS 8.1</td>
</tr>
<tr>
<td>Beacons</td>
<td>Gimbal Series 10</td>
</tr>
<tr>
<td>Gimbal range</td>
<td>50 meters</td>
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<tr>
<td>Transmission Frequency</td>
<td>100 ms</td>
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<tr>
<td>Major Value</td>
<td>Yes</td>
</tr>
<tr>
<td>Minor Value</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The variables in our experimental set up are:

- **Number of particles:** We varied the number of particles to obtain the optimal number of particles that results in the highest possible positioning accuracy. We varied the number of particles from 400-2000 using an increment
Fig. 3. Average Error Vs Number of Particles for Different Number of Beacons for scenario 1.

of 200 particles. Due to the energy and processing constraints of the smart phone, we did not move beyond 2000 particles.

- **Number of beacons:** We changed the number of beacons in the tracking area in order to obtain the optimum performing system with optimum accuracy. In our setup, we started with 3 beacons and continued increasing beacons until the addition of beacons did not improve the results significantly.

- **Tracking Area:** We also altered the tracking area and varied it between 1m x 1m, and 11m x 6m. While we chose 1m x 1m space for provision of an insight into how beacons can be used in smaller spaces, we chose the 11m x 6m as it covered our entire laboratory and replicated the commercial deployment scenario as real world scenarios have obstacles.

When the number of beacons were more than 3, then we used the n-point trilateration technique in combination with particle filtering to obtain the position of the user. In n-point trilateration, the position of the entity is assumed to be the intersection area of the range of all the reference points. In order to calculate the average error of the proposed model, we slightly modify the equation used in [20]. We compare the actual position of the user holding the iPhone with the estimated position that our application produces. The estimated position is the average of the $<\text{est}>$ coordinates of the particles.

$$<\text{Error}> = \sum_{i=1}^{n} \frac{\sqrt{(X_i - X_{<\text{est}>})^2 + (Y_i - Y_{<\text{est}>})^2}}{n}$$

(12)

where $(X_i, Y_i)$ is the actual point while the $(X_{<\text{est}>}, Y_{<\text{est}>})$ is the average estimated point. Using equation 12, we calculated the error incurred in locating an entity using the beacons and particle filtering. Furthermore standard deviation ($\sigma$) and Mean Square Error (MSE) were also calculated.

**A. Scenario 1**

The location of the mobile phone was tracked in a 1m x 1m area. The actual position of the mobile phone was compared with the estimated one using beacons and particle filtering at different points.

Table III provides the experimental results for the current scenario. The error (in meters), MSE, and standard deviation of the error is tabulated. Based on the results, it is evident that 4 beacons provided comparatively better accuracy and resulted in smaller error while tracking the object. While 5 and 6 beacons were comparable with 4 beacon based tracking, 3 beacons based tracking was the worst in terms of performance. Indeed the optimum performing system results in an error of 0.27 meters that is obtained with 1000 particles and 4 beacons. While intuitively, the increase in number of beacons should have caused an increase in accuracy, the higher number of beacons in a smaller space can cause interference among beacons that affects accuracy. The performance of the system varies for different number of particles because the effect of the number of particles on the filter accuracy relies on a) how accurately the true density is approximated by the proposal density b) the true density’s complexity [21]. Therefore for every specific set of experiment, the density varies that will require some particular number of particles to result in better accuracy. That is why there is fluctuation in the average error for every set of beacons with respect to the number of particles. Through the extensive set of experiments, we were able to obtain the optimum number of particles for different beacons to obtain the highest accuracy.

Since increasing beacons did not improve the performance significantly, it is viable to not add more beacons as the interference will increase even more in such a small area. Figure 3 shows the plot of average error vs different number of beacons. One can observe that the system with 4, 5 and 6 beacons outperform 3 beacons. The system performance improved when we added beacons; however after adding beacon 6, the performance degraded due to the interference among the beacons.

**B. Scenario 2**

We deployed the gimbal beacons in the High Performance Computing (HPC) lab in the Knoy Hall of Technology Building of Purdue University. We used the proposed model to track the mobile phone in 11m x 6m area. We started with 3 beacons and kept increasing until the addition of further beacons did not bring significant performance improvement (or performance degradation due to interference). Once we reached 8 beacons, the interference among the beacons started to affect the performance adversely so we stopped adding further beacons. Table IV shows the error (in meters) related information for the proposed tracking system. The system with 5 beacons provided the lowest error i.e. an error .97 meters is obtained with 5 beacons and 1000 particles. This accuracy
value was great for indoor environments as the user can be tracked within a single step. It also show the viability of using iBeacons in combination with particle filtering. In addition, 7 beacons system performs much better overall and due to larger space, the use of higher number of beacons facilitated the performance of the system until the space was saturated with beacons. Figure 4 shows the plot of average error vs number of beacons. It can be seen that increasing the number of beacons provided better results up to 7 beacons, however the addition of the 8th beacon deteriorated the performance so we stopped adding further beacons. A comparison of scenario 1 and scenario 2 shows that the error has increased with an increase in the tracking area. This can be attributed to the presence of obstacles, and various entities in the larger space that affect the 2.4 GHz BLE. Furthermore, increasing the number of beacons in the larger space can facilitate the tracking accuracy and reduce the error incurred. Also, the fluctuations in the tracking error have increased in scenario 2 that is because of the drastic fluctuation in the RSSI values in a bigger space due to fore-mentioned reason of presence of obstacles and different entities.

VI. CONCLUSION

In this paper, we proposed an accurate and efficient micro-location system that uses the BLE enabled beacons to locate an entity or user in an indoor/GPS constraint environment through particle filtering. We created a mathematical formulation, prototyped it, and created an exemplar application on an iOS device. We evaluated our prototype with extensive experiments by alternating the number of particles, number of beacons and the tracking area. Based on the experiments, we conclude that increasing the number of beacons in a specific space will only enhance accuracy until the area is saturated with beacons. For every set of beacons and tracking area, there is an optimal number of particles that will provide the lowest error. Hence, we propose that the beacons should be properly placed at higher altitudes within the space to avoid obstacles. Also the positioning of the beacons should be properly planned before the actual deployment. The tracking space should not be overloaded with beacons as the beacons can interfere among each other, affecting the accuracy adversely. The proposed model resulted in fairly accurate indoor tracking and we tuned our system to obtain tracking errors as low as 0.27 meters in smaller space while 0.97 meters in the larger space. In future, we aim to explore self adaptive particle filters for further enhancing the accuracy of indoor positioning.

REFERENCES

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<table>
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### Table IV

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### References


