A SIMULATION MODEL FOR WELFARE POLICY ANALYSIS

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Abstract—This paper describes a new approach to large-scale simulation modeling for the analysis of public welfare policies. Whereas conventional microanalytic simulation models extrapolate the dynamic behavior of a representative sample of individuals to the corresponding target population, the new modeling technique disaggregates an exogenous demographic forecast into enough detail to evaluate the impact and cost of proposed social programs. This approach uses a multivariate extension of the Johnson translation system of probability distributions to estimate the percentage of individuals in a target population who satisfy given program eligibility criteria. The methodology is demonstrated in a prototype decision support system for analysis of the Institutional Care and Community Care programs of the Texas Department of Human Services.

INTRODUCTION

Throughout the world, the need for more rigorous public policy analysis has been intensified in recent years by budget reductions and by the redefinition of national, regional, and local roles in public decision making. In developing and analyzing policy options, decision makers have been forced to match resources with objectives and to identify those actions that most effectively contribute to solving a set of problems confronting the public. In recognition of this need, many government agencies have sought to increase their analytical capabilities. Some agencies, for the first time, have turned to computer simulation models to forecast the economic and social impact of policy alternatives.

This paper reports the results of an effort to develop a large-scale computer simulation model for the analysis of public welfare policies [1–3]. The project was a component of a major research effort directed at understanding and enhancing policy analysis in the Texas Department of Human Services (TDHS) [4]. It afforded the opportunity to work closely with agency staff in identifying managerial, technical, and political problems associated with the design and implementation of computer simulation techniques for public policy analysis.

As an alternative to traditional microanalytic simulation modeling [5, 6], we evolved a modeling technique based on disaggregation of a given demographic forecast into enough detail to evaluate the impact and cost of proposed social programs. In a study of the welfare services provided to the aged and disabled population of Texas, we implemented a prototype decision support system based on this alternative approach. The study required a method for estimating the percentage of individuals in each of 80 target populations (defined by 10 service regions and 8 age cohorts) who satisfy 3 specified program eligibility criteria. Consequently we have developed algorithms for fitting and graduating trivariate data sets based on an extension of the Johnson translation system of univariate probability distributions [7, 8].

This paper is organized as follows. The second section discusses the major decision variables and exogenous variables that are relevant to the analysis of public welfare policies. The third section surveys previous research on simulation models for welfare policy analysis. The fourth section describes our modeling approach, and the fifth section details the statistical methodology that was used to implement this approach. In the sixth section we summarize the main findings of this research. In the Appendix we establish the fundamental results that underlie the statistical and numerical methods used in the simulation model.

A FRAMEWORK FOR WELFARE POLICY ANALYSIS

Public agencies, particularly welfare agencies, must estimate the social impacts and costs of alternative methods of service delivery. The task of improving these forecasts is becoming more important as the use and distribution of public resources is scrutinized more closely. These forecasts are used in the political processes that determine which social programs merit public funding. To the extent that these forecasts are accurate and believable, then public decision makers are able to make informed decisions about the allocation of resources. The accuracy and believability of these estimates significantly increases the credibility of an agency and the relative influence that an agency will have in the political decision process.

Two major sets of policy (decision) variables must be considered by the decision maker in analyzing alternative methods of service delivery. The first set concerns client eligibility, which is usually determined by an income needs test together with a functional needs test. In order to be eligible for welfare assistance, a person must be poor and sick, or poor and
a single parent, or poor and out of work. Poverty is the common denominator, and it is measured by monthly income and countable resources (savings, investments, etc.). To evaluate a client’s functional need for service, we require a functional disability score that takes into account the client’s physical and mental health, his/her social resources (family, friends, social organizations, etc.), and his/her ability to perform the activities of daily living. Note that the income, resources, and functional disability score of an individual are not mutually independent variables; and an accurate statistical description of the dependencies among these variables in the service population is essential for valid analysis of welfare policies.

The second set of policy variables that must be considered concern the combination of program options available for addressing a given problem. Each option has variable costs per unit of service and variable participation rates. While each program option will serve some identifiable subset of the eligible population, a welfare agency generally has some latitude in determining how it will organize and deliver a given program of service. Therefore, policy choices must be made that affect both the cost per unit of service and the participation rate.

In addition to the two sets of policy variables, there are two sets of environmental (exogenous) variables that determine the impact of policy decisions. The environmental variables in the first set describe the change in magnitude and composition of the target population over time, while the variables in the second set describe the relevant inflation rates for the cost of services. Such exogenous variables cannot be controlled by the decision maker, yet they may be constrained by law, by technology, and sometimes by public opinion. Policy analysis involves estimating the impact and cost associated with a wide range of policy variables under various conditions and assumptions.

SIMULATION MODELS FOR WELFARE POLICY ANALYSIS

Computer simulation models for welfare policy analysis have been available for nearly two decades [6]. In the United States, the main catalyst for the development of these models was the surge in federal spending for welfare programs accompanying President Lyndon Johnson’s “War on Poverty.” In the mid-1960s, federal legislation involving billions of dollars was being introduced in Congress to address various social issues. Yet at that time, neither the analytical tools nor the data were available to government analysts for answering questions about the impact and costs of the proposed social programs. Instead, crude estimates and sometimes distorted facts may have been used as the basis for making policy recommendations [5].

Weak policy analysis was recognized as an impediment to passing federal legislation. Consequently, analysts at the U.S. Department of Health, Education, and Welfare (HEW) and the Office of Economic Opportunity (OEO) supported the development of a general purpose computer simulation model for analyzing proposed income transfer programs. In 1968 the President’s Commission on Income Maintenance Programs sponsored the development of a microanalytical simulation model known as RIM (Regional Income Model) [9]. Despite simplistic relations and crude data, RIM was used extensively at HEW and OEO. Subsequent modeling efforts undertaken by The Urban Institute resulted in two more sophisticated analysis programs—TRIM (Transfer Income Model) [10] and DYNASIM (Dynamic Simulation of Income Model) [6]. TRIM, in particular, has been widely adopted by many federal agencies such as the U.S. Department of Housing and Urban Development (HUD) and the Congressional Budget Office, as well as by many private organizations interested in demographic projections for the United States. However, there are no reports of state agency applications of TRIM.

Microanalytic simulation models focus on a microcosm made up of a representative sample of individuals or households. This initial sample generally is derived from census data, and it contains sufficient detail to reflect the policy changes being investigated. These models also require estimates of the rates of change in relevant demographic characteristics. These rates are used to “age” the sample to determine the impact and cost of proposed social programs. The impact on the sample is then projected to the entire population.

Microanalytic simulation models could be used to analyze proposed social programs at the regional level. However, the models have several drawbacks that impede their adoption. The most serious disadvantage is the amount and quality of data needed to drive these models. Often such data are not available at the regional level in any readily usable form. Another drawback is the complexity of the functional relationships used in the models. These relationships are not transparent either to analysts or to policy decision makers. The complexity of the models and the required large amounts of data make microanalytical simulation models intimidating to use, especially if analysis is required rapidly and repetitively under alternative program scenarios.

Despite the problems with microanalytic simulation models, high-level personnel at the TDHS believed that some type of simulation modeling in the form of a decision support system should be used for welfare policy analysis. A computer simulation committee made up of analysts, program directors, and outside university advisors was established to outline the specifications for a decision support/simulation model. The committee identified the following requirements for the model:

1. it must include options for altering the demographic forecast as well as the cost and eligibility characteristics of each program to be analyzed;
2. it must be interactive with a convenient “help” option so that minimal hard-copy documentation is required to run the model;
3. it must be statistically valid;
4. it must be based on data that will be available in the foreseeable future; and
5. its underlying methodology must be transparent to the decision makers.
ADSSIM: AGED AND DISABLED SERVICES SIMULATION MODEL

Modeling objectives

ADSSIM was developed as an impact evaluation and forecasting tool for the Institutional Care and Community Care Programs of the TDHS [1–4]. It enables policy analysts to anticipate the effects of changes in program eligibility criteria on client loads, costs, and other relevant performance measures. It also allows decision makers to forecast these measures for new and existing services over an extended planning horizon. ADSSIM was designed to make the best use of available data and to be as consistent as possible with accepted procedures for evaluating new and existing programs. ADSSIM is a prototype model that is intended to help frame future decision support systems for the TDHS.

In contrast to conventional microanalytic simulation models, ADSSIM takes an entirely different tack for deriving projections; and this new approach makes ADSSIM more robust against uncertainty and more easily adaptable to significantly altered demographic or economic circumstances. Whereas microanalytical models extrapolate an “aged” sample to the entire population, ADSSIM decomposes an externally generated population forecast into separate forecasts of the subpopulations that are eligible for specified social services. This top-down approach recognizes that many different population forecasts are available to various state agencies, and program analysis must be consistent with the forecasts adopted. Additionally, the top-down approach permits the use of multiple demographic forecasts as a strategy for sensitivity analysis. This is particularly important when policy makers are faced with an uncertain scenario such as the one that has unfolded in Texas over the last 6 yr. When using ADSSIM, the analyst is able to draw on the expertise of professional forecasters to drive the model rather than having to rely exclusively on the model itself for demographic forecasting algorithms. In most policy situations, there are several different sources of information about future trends. For example, demographic forecasts used by the Governor’s Office are likely to be different from those used by the Legislature. Almost any demographic forecast can be selected to drive ADSSIM.

Model structure

ADSSIM operates interactively, leading the analyst through the structure of a particular policy option so that all policy variables and assumptions are revealed as he responds to a sequence of prompts. In most cases, the analyst can choose to use default values for various inputs, or he can enter new values as deemed appropriate.

The following notation shall be used to describe the functional relations employed in ADSSIM:

- $X_2 =$ dollar value of countable resources for the same randomly chosen person;
- $X_3 =$ functional disability score for the same randomly chosen person;
- $x_{ij} =$ upper limit on the person’s monthly income in order to be eligible for service $j$;
- $x_{ij} =$ upper limit on the person’s countable resources in order to be eligible for service $j$;
- $x_{ij} =$ lower limit on the person’s functional disability score in order to be eligible for service $j$;
- $G_d(x_{ij}, x_{ij}^2, x_{ij}^3) =$ percentage of persons in the target population defined by cohort $i$ and region $j$ who are eligible for service $j$;
- $\psi_{ij} =$ participation rate for service $j$ that is expected to occur within the subpopulation of eligible clients belonging to cohort $i$ in region $j$ during year $t$;
- $N_j =$ average number of units of service $j$ required annually for each participating client;
- $C_j =$ average annual cost per unit of service $j$ in the base year of the study;
- $r =$ annual rate of inflation in the cost of service $j$;
- $t^0 =$ base year of the study.

The projected number of persons in region $i$ participating in service $j$ during year $t$ is given by

$$Y_{it} = \sum_j \psi_{ij} G_d(x_{ij}, x_{ij}^2, x_{ij}^3) K_{it}.$$  (1)

The projected costs of this service by region and for the entire state are

$$K_{jt} = Y_{it} N_j C_j (1 + r)^{t - t^0}.$$  (2)

and

$$K_{jt} = \sum_{i=1}^v K_{jt},$$  (3)

where $v$ is the number of regions in the state.

ADSSIM can project client loads and costs by region and by age cohort over a designated forecast horizon. These projections can be aggregated across regions and/or across age cohorts as needed. While the projections in equations (2) and (3) are based on the total assumption of service costs by the TDHS, copayment programs have been proposed. ADSSIM allows for the analysis of copayment programs by specifying a monthly income level above which clients will contribute to the cost of services. Copayment is factored into the total cost projections.

Equation (1) assumes that the eligibility criteria are fixed for the forecast horizon. However, ADSSIM allows for the income and resource criteria to vary; for example, they may be indexed to the cost of living.

Figure 1 displays the ADSSIM process flow. This was found to be intuitively appealing to analysts and
decision makers at the TDHS, and it satisfies the basic requirements specified by the simulation committee. However, the model structure poses a major analytical problem for implementation—namely, the estimation of the trivariate distribution functions $G_{x}(x_{1}, x_{2}, x_{3})$ for all service regions and age cohorts as well as for arbitrary values of the eligibility limits $x_{1}, x_{2}, x_{3}$. The TDHS participates in a biennial survey that can record among other things the monthly income, countable resources, and functional disability score for each respondent. The results of such a survey constitute the appropriate basis for estimating these distributions.

MODELING THE TRIVARIATE TARGET POPULATIONS

Johnson’s univariate translation systems

In this study as in many other practical applications, marked departures from normality were observed in many of the univariate data sets that were encountered. Since statistical theory for normally distributed populations is much simpler and more extensively developed than for any other underlying distribution, there is a strong motivation to seek a one-to-one transformation of the original data that will yield normally distributed observations. At least in principle, this can always be done for continuous data [see display (8) below]; and then relevant percentages for each univariate target population can be obtained by computing the corresponding normal probabilities.

Suppose that the continuous random variable $X$ has the (unknown) distribution function $F_{x}(x) = \Pr\{X \leq x\}, -\infty < x < +\infty$. To translate $X$ into a new variate $Z$ having the standard normal distribution

$$\phi_{z}(z) = \Pr\{Z \leq z\} = \left(2\pi\right)^{-1/2} \int_{-\infty}^{z} \exp\left(-\frac{1}{2}t^{2}\right) dt.$$  

Johnson [7] proposed several transformations having the general form

$$Z = \gamma + \delta f[(X - \xi)/\lambda] \quad (\delta > 0, \lambda > 0).$$

If on the other hand we start with a standard normal variate $Z$, then the selection of a particular function $f(\cdot)$ implicitly defines a family (or system) of distributions for $X$, and we seek to approximate the target distribution $F_{x}(\cdot)$ as closely as possible by taking appropriate values for the parameters $\gamma, \delta, \lambda$, and $\xi$. Johnson defined 4 such systems that are capable of describing a wide variety of continuous populations:

(1) the lognormal system $S_{1}$:

$$Z = \gamma + \delta \ln[(X - \xi)/\lambda] \quad (X > \xi; \lambda \equiv 1); \quad (4)$$

(2) the unbounded system $S_{2}$:

$$Z = \gamma + \delta \ln((X - \xi)/\lambda) + \left(\frac{[(X - \xi)/\lambda^{2} + 1]}{1}\right)^{1/2} \quad (-\infty < X < \infty); \quad (5)$$

(3) the bounded system $S_{3}$:

$$Z = \gamma + \delta \ln\left[\frac{(X - \xi)/\lambda}{1 - (X - \xi)/\lambda}\right] \quad (\xi < X < \xi + \lambda); \quad (6)$$

(4) the normal system $S_{4}$:

$$Z = (X - \xi)/\lambda \quad (-\infty < X < \infty; \gamma \equiv 0, \delta \equiv 1); \quad (7)$$

In general applications of Johnson’s translation method to continuous populations, the identification of an appropriate function $f(\cdot)$ and the estimation of the required parameters $\gamma, \delta, \lambda, \xi$ can at best yield a transformed variate $\gamma + \delta f[(X - \xi)/\lambda]$ having approximately the standard normal distribution. However, observe that the random variable $F_{r}(X)$ is uniformly distributed on the unit interval $(0, 1)$ (see p. 216 of [11]); thus the inverse transform method [12] ensures that the relation

$$Z = F_{x}^{-1}\left(F_{r}(X)\right) \sim F_{x}(X) \quad (8)$$

holds exactly. Moreover, the translation $F_{x}^{-1}F_{r}(\cdot)$ is strictly increasing (and hence one-to-one) on the space of $X$ (that is, on the range of values that $X$ can assume). Thus for a given cutoff value $x^{*}$, the probability of the basic event $\{X \leq x^{*}\}$ can be readily obtained as $F_{x}(x^{*})$, where $x^{*}$ is the translated cutoff value. The widespread success of Johnson’s translation method stems from the flexibility of the functional forms (4) through (7) in approximating the exact translation $F_{x}^{-1}F_{r}(\cdot)$.

If $F_{r}(\cdot)$ has known moments $\mu_{k} = E[X^{k}]$ and $\mu_{k} = E[(X - \mu)^{k}]$ for $2 \leq k \leq 4$, then in principle we can identify the appropriate function $f(\cdot)$ and evalu-
ate all necessary parameters exactly (that is, to the limits of machine accuracy). In terms of the skewness \( \beta_1 \equiv \mu_3/\mu_2 \) and kurtosis \( \beta_2 \equiv \mu_4/\mu_2^2 \), Fig. 2 shows that Johnson's translation systems can accommodate all possible points in the \((\beta_1, \beta_2)\) plane. Generally \( \beta_1 \) and \( \beta_2 \) determine the functional form \( f(\cdot) \) as well as the shape parameters \( \gamma \) and \( \delta \); then in terms of the auxiliary variate \( Y = f^{-1}(Z - \gamma)/\delta \), we obtain the scale parameter \( \lambda = [\text{Var}(X)/\text{Var}(Y)]^{1/2} \) and the location parameter \( \xi = E[X] - \lambda E[Y] \).

When the only available information about \( F_\alpha(\cdot) \) is contained in a random sample \( \{x_j, 1 \leq j \leq n\} \) taken from this distribution, the usual procedure is to fit a Johnson curve to the sample data by either the method of moments or the method of percentiles. The method of moments conforms to the fitting scheme outlined above, but it uses sample moments instead of the theoretical moments:

\[
m_1 = n^{-1} \sum_{j=1}^{n} x_j; \quad m_k = n^{-1} \sum_{j=1}^{n} (x_j - m_1)^k, \quad 2 \leq k \leq 4; \quad \beta_1 = m_3/m_2; \quad \beta_2 = m_4/m_2^2.
\]

To implement this method within each system, various computational schemes have been proposed [7, 13-17]. Unfortunately this method has the following serious deficiencies [18]: (a) the statistics \( \beta_1 \) and \( \beta_2 \) have large variances; (b) in small samples \( \beta_1 \) and \( \beta_2 \) are highly biased; and (c) both \( \beta_1 \) and \( \beta_2 \) are extremely sensitive to outliers. Slifker and Shapiro [18] developed an alternative technique for system identification and parameter estimation that is based on the method of percentiles described below.

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The method of moments estimates the four required parameters by matching four selected quantiles of the standard normal distribution with the corresponding estimated quantiles of the target distribution. If the selected percentages are \( \{z(i): 1 \leq i \leq 4\} \), then the corresponding standard normal quantiles \( \{z(\alpha_i)\} \) are defined by

\[
z(\alpha_i) \equiv \Phi^{-1}[\alpha(i)], \quad \alpha(i) \equiv \min\{z: \Phi(z) \geq \alpha(i)\}, \quad 1 \leq i \leq 4.
\]

Similarly, the quantile of order \( \alpha(i) \) for \( F_\alpha(\cdot) \) is \( x(\alpha_i) \equiv F_\alpha^{-1}[\alpha(i)], \quad 1 \leq i \leq 4 \). In terms of the order statistics \( x_1^* \leq \cdots \leq x_4^* \) computed from the sample \( \{x_j, 1 \leq j \leq n\} \), the estimator of \( x(\alpha_i) \) is given by

\[
x(\alpha_i) = x(\alpha_i) = \hat{x}(\alpha_i) = x(\alpha_i) - \hat{\xi},
\]

where \( [r] \equiv \text{greatest integer} \leq r \) (see p. 215 of [19]). Once the function \( f(\cdot) \) has been identified by the location of the point \( (\hat{\beta}_1, \hat{\beta}_2) \) in Fig. 2 or by some other means, the method of percentiles attempts to solve the four nonlinear equations

\[
z(\alpha_i) = \gamma + \delta f(\hat{\xi} - \xi)/\lambda, \quad 1 \leq i \leq 4,
\]

in the four unknowns \( \gamma, \delta, \lambda, \xi \). To implement this method within each system, various percentile-matching schemes have been proposed [7, 18, 20-22]. All of these schemes can yield parameter estimates that are \textit{infeasible} in the following sense:

\[
\xi > \min\{x_i\} - x_1^* \quad \text{or} \quad \xi + \hat{\lambda} > \max\{x_i\} - x_4^*. \quad (10)
\]

(Actually the moment-matching schemes described above also have this deficiency.) As a basic principle of our research, we sought to develop univariate curve-fitting procedures that are guaranteed to avoid such anomalies. In fact, condition (10) prevents the use of the multivariate curve-fitting procedure discussed in the subsection below; see the remark following step 2 of that procedure.

\section*{Curvature-fitting algorithms for Johnson's systems}

A univariate percentile-matching procedure. To avoid infeasible parameter estimates when fitting Johnson curves and to take advantage of the extra information that is available when an endpoint is known, we developed a general percentile-matching procedure based on a modified Newton-Raphson method in which the step length is adjusted if necessary to keep each trial solution within a prespecified feasible region.

We discuss in some detail fitting an \( S_4 \) curve when neither endpoint is known. Corresponding to the user-modifiable default percentages \( \{\alpha(1) = 0.07, \alpha(2) = 0.3118, \alpha(3) = 0.6882, \alpha(4) = 0.931, \} \), the quantiles \( \{z(\alpha_0)\} \) and \( \{x(\alpha_0)\} \) are computed first. With the notation \( u = [u_1, u_2, u_3, u_4]' = [\gamma, \delta, \lambda, \xi]' \), the percentile-matching condition (9) can be compactly expressed as \( h(u) = 0 \).

\[
h(u) = \begin{bmatrix}
    h_1(u) = u_1 + u_2 \ln \frac{x(\alpha_0) - u_4}{u_4 + u_4 - \hat{x}(\alpha_0)} - z(\alpha_0), \quad 1 \leq i \leq 4, \\
    h(u) = \begin{bmatrix} h_1(u), h_2(u), h_3(u), h_4(u) \end{bmatrix}, \quad \text{the percentile-matching condition (9) can be compactly expressed as } h(u) = 0.
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can be obtained with one of Johnson's univariate
a standardized trivariate normal distribution with
Zi, and Zi, 1 <
correlation φ between the transformed coordinates
(Note that if condition (10) were to occur for the ith
coordinate, then we could not compute the trans-
variate fitting procedure would fail at this step.)
translations [8]:
approximation to the ith coordinate function r,(X)
5
i < 3,
1
In the Appendix we
prove the existence of a transformation z that is
one-to-one on the space of X and that yields
Z = τ(X) ∼ N3,
\left[\begin{array}{ccc}
1 & & \\
& 1 & \\
& & 1
\end{array}\right],
(11)
a standardized trivariate normal distribution with
The development of the distribution-fitting algo-
rithms and the associated software required for this
study has clearly revealed the need for a
recently been redesigned to provide the trivariate
modeling technique.
modeling coupled with a general method for aggre-
gating and disaggregating multiattribute service pop-
ulations. As a prototype decision support system for
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An important conclusion of this research is that
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to reducing the basic data collection effort to a
manageable size, the top-down approach gives the
analyst greater control over the scenarios that he
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SUMMARY AND CONCLUSIONS
The main finding of this study is that a feasible and
effective tool for welfare policy analysis can be based
on a top-down approach to large-scale simulation
modeling coupled with a general method for aggre-
gating and disaggregating multiattribute service pop-
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Evaluating the fitted trivariate distribution functions
Given the cutoff values x,j, x,j, and x,j for the 3
numerical eligibility criteria, we seek to compute the
probability of randomly selecting an individual
from the target population whose corresponding
attributes X = [X1, X2, X3] satisfy the conditions
X1 < x,j, X2 < x,j, and X3 > x,j. The algorithm for
evaluating the fitted trivariate distribution function uses the transformed cutoff values
z,j = y,j + δ,j(1 - 1/ξ,j)/ξ,j, 1 < j < 3,
in the fundamental relation
Pr[X1 < x,j, X2 < x,j, X3 > x,j]
= Pr[Z1 < z,j, Z2 < z,j, Z3 > z,j]
= Pr[Z1 < z,j, Z2 < z,j]
− Pr[Z1 < z,j, Z2 < z,j, Z3 < z,j]
≈ Φ1(z,j, z,j, 0, ρ12) − Φ3(z,j, z,j, ρ12, ρ23).
Although we were able to use an IMSL routine [23]
to compute the standardized bivariate normal distribu-
tion Φ2,(.), we had to develop a new computational
procedure to evaluate the trivariate normal distribu-
tion Φ3 with acceptable speed and accuracy. This
development is summarized in the Appendix.
methods for fitting Johnson distributions to univariate and multivariate data sets. References [24-26] detail a univariate fitting procedure based on non-linear weighted least-squares estimation of the empirical cumulative distribution function. The software to implement this new procedure [27] has evolved directly from the original curve-fitting package [2] that was developed to support ADSSIM. Work is currently underway on a multivariate extension of the least-squares fitting procedure [28]. These innovations provide further evidence of the viability of the ADSSIM approach to the design of large-scale simulation models for welfare policy analysis.

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REFERENCES


APPENDIX

A Normalizing Translation for a Continuous Random Vector

Let $X = [X_1, \ldots, X_p]$ denote a continuous random vector whose space $S_p$ is a subset of $p$-dimensional Euclidean space $R^p$ and whose $i$th marginal distribution is $F_{i}(x_i) = \Pr(X_i \leq x_i)$ for all $x_i \in R$, $1 \leq i \leq p$. Let $U(0,1)$ denote the uniform distribution on the unit interval $I = (0,1)$, and let $N_p(0, \Sigma)$ denote a $p$-dimensional standard normal distribution with a null mean vector and a correlation-type dispersion matrix $\Sigma = \{\|n_i\|^2\}$. In view of (8), it is natural to seek a normalizing translation that maps $X$ into a new random vector $Z \sim N_p(0, \Sigma)$ where

$$
\sigma_n = E[\Phi^{-1}(F(X_i))] = \{\|n_i\|^2\}
$$

$1 \leq h, l \leq p$. (A.1)

Proposition

There exists a normalizing transformation $\tau: R^p \to R^p$ such that

$$
\tau(X) \sim N_p(0, \Sigma).
$$

(A.2)

where the entries of $\Sigma$ are given by (A.1). Moreover, the restriction $\tau: S_p \to R^p$ is one-to-one.

Proof

Both $R^p$ and $R^q$ are complete separable metric spaces (see [29], Chapter VIII, §7, Example 1 and Chapter XIV, §2.
Example 4). Moreover, \(R^2\) and \(R^1\) are equinumerable sets (Theorem 2.12 of [30]) asserts the existence of a mapping \(\Psi : R^2 \rightarrow R^1\) such that: (a) \(\Psi\) is one-to-one, (b) \(\Psi\) maps \(R^2\) unto \(R^1\), and (c) both \(\Psi\) and \(\Psi^{-1}\) are Borel measurable functions.

Let \(Z \sim N_{R}(\theta, \Sigma)\). Define the random variables \(W = \Psi(Y)\), \(X = \Psi(Z)\) with the corresponding distribution functions \(F_{W}(r) = \Pr\{W \leq r\}\), \(F_{Y}(r) = \Pr\{Y \leq r\}\) for all \(r \in R^1\), and the inverse distribution functions \(F_{W}^{-1}(u) = \inf\{r \in R^1 : F_{W}(r) \geq u\}\), \(F_{Y}^{-1}(u) = \inf\{r \in R^1 : F_{Y}(r) \geq u\}\) for all \(u \in I\). Note that we define the spaces of the variables \(W, X, Y,\) and \(Z\) as \(S_{W} = F_{W}^{-1}(I)\), \(S_{Y} = \Psi^{-1}(S_{W})\), \(S_{X} = F_{Y}^{-1}(I)\), and \(S_{Z} = \Psi^{-1}(S_{W})\) respectively.

For any fixed \(r \in R^1\), we have \(\Pr\{W = r\} = \Pr\{Y = \Psi^{-1}(r)\} = 0\) since \(\Psi\) is one-to-one and \(X\) is a continuous random vector. Therefore \(W\) is continuous, and the same argument shows that \(Y\) is continuous. It follows that \(F_{W}^{-1} \sim F_{X}^{-1} \sim F_{Y}^{-1}\) for all \(u \in I\). In view of the definition of \(Y\), we have

\[
\tau(X) = \Psi^{-1} \circ F_{Y}^{-1} \circ F_{W}^{-1} \circ \Psi^{-1}(Z) \sim N_{R}(\theta, \Sigma).
\]

Finally if \(x', x'' \in S_{Y}\) are distinct points, then it is easy to check that \(\tau(x')\) and \(\tau(x'')\) are also distinct; thus the restriction of \(\tau\) to \(S_{Y}\) is one-to-one.

**Computing Trivariate Normal Probabilities**

**The S-function formulation**

To complete the implementation of ADSSIM, we needed a computational procedure for evaluating the trivariate distribution function \(\Phi(z_1, z_2, z_3; \rho_{12}, \rho_{13}, \rho_{23})\) at arbitrary cutoff values \(z_1, z_2, z_3\). For this purpose Steck [31] defined the \(S\)-function

\[
S(z, a, b) = \frac{1}{2\pi} \int_{0}^{\infty} \frac{1}{1 + u^2} \exp\left[-\frac{1}{2}(1 + u)^2\right] \frac{1}{2\pi} \int_{0}^{\infty} \frac{1}{1 + v^2} \exp\left[-\frac{1}{2}(1 + v)^2\right] \, du, \quad -\infty < z, a, b \leq \infty; \tag{A.3}
\]

in terms of the auxiliary functions

\[
T(z, a) = \frac{1}{2\pi} \int_{0}^{\infty} \frac{1}{1 + u^2} \exp\left[-\frac{1}{2}(1 + u)^2\right] \, du, \quad -\infty < z, a \leq \infty; \tag{A.4}
\]

\[
\text{sign}(z) = \begin{cases} \text{1}, & z \geq 0; \\ \text{-1}, & z < 0; \tag{A.5} \end{cases}
\]

and

\[
\mathcal{A}(z_1, z_2) = \begin{cases} \text{0}, & \text{sign}(z_1) = \text{sign}(z_2); \\ \text{1}, & \text{otherwise}. \tag{A.6} \end{cases}
\]

Steck's formulation of \(\Phi\) can be represented in the first orthant \((z_1, z_2, z_3)\) (nonnegative) and in the last orthant \((z_1, z_2, z_3)\) (nonpositive) as

\[
\Phi(z_1, z_2, z_3; \rho_{12}, \rho_{13}, \rho_{23}) = \begin{cases} \frac{1}{2} \sum_{i=1}^{3} \left[ 1 - \Delta(a_i, c_i) \right] \Phi(z_i) - \frac{3}{2} \sum_{i=1}^{3} \left[ T(z_i, a_i) + T(z_i, c_i) \right] - \frac{3}{2} \sum_{i=1}^{3} \left[ S(z_i, a_i, b_i) + S(z_i, c_i, d_i) \right], \tag{A.7} \end{cases}
\]

where the conditional cutoff factors \(\{a_i, c_i, d_i\}, i \leq 3\) are defined below:

\[
u_i = \frac{z_i^2}{1 - \rho_{ii}}, \quad \mu_i = \frac{z_i^2}{1 + \rho_{ii}}; \tag{A.8}
\]

\[
u_1 = \frac{z_1^2}{1 - \rho_{11}}, \quad \mu_1 = \frac{z_1^2}{1 + \rho_{11}}; \tag{A.9}
\]

\[
u_2 = \frac{z_2^2}{1 - \rho_{22}}, \quad \mu_2 = \frac{z_2^2}{1 + \rho_{22}}; \tag{A.10}
\]

\[
u_3 = \frac{z_3^2}{1 - \rho_{33}}, \quad \mu_3 = \frac{z_3^2}{1 + \rho_{33}}. \tag{A.11}
\]

The conditional cutoff factors \(\{b_i, d_i\}, i \leq 3\) are computed from (A.5) through (A.7) as follows:

\[
u_1' = \left(1 - \mu_1 \right)^{2}/2, \quad \mu_1' = \left(1 + \mu_1 \right)^{2}/2; \tag{A.12}
\]

\[
u_2' = \left(1 - \mu_2 \right)^{2}/2, \quad \mu_2' = \left(1 + \mu_2 \right)^{2}/2; \tag{A.13}
\]

\[
u_3' = \left(1 - \mu_3 \right)^{2}/2, \quad \mu_3' = \left(1 + \mu_3 \right)^{2}/2. \tag{A.14}
\]

The conditional cutoff factors \(\{b_i, d_i\}, i \leq 3\) are computed from (A.5) through (A.7) as follows:

\[
u_1' = \left(1 - \mu_1 \right)^{2}/2, \quad \mu_1' = \left(1 + \mu_1 \right)^{2}/2; \tag{A.15}
\]

\[
u_2' = \left(1 - \mu_2 \right)^{2}/2, \quad \mu_2' = \left(1 + \mu_2 \right)^{2}/2; \tag{A.16}
\]

\[
u_3' = \left(1 - \mu_3 \right)^{2}/2, \quad \mu_3' = \left(1 + \mu_3 \right)^{2}/2. \tag{A.17}
\]

We had to recast Steck's original formulas for the conditional cutoff factors into the equivalent forms (A.8) through (A.9) to facilitate machine computation of these quantities. In particular, these new forms clarify the analysis for the special cases in which some of the variables \(z_i, a_i, b_i, c_i, d_i\) assume one of the values \(0, -\infty, +\infty\).

If the point \((z_1, z_2, z_3)\) does not fall in the first or last orthant and, say, \(z_1\) has a different sign from the other coordinates, then we have

\[
\text{sign}(z_1) = \text{sign}(z) = -\text{sign}(z_1) \Rightarrow \Phi(z_1, z_2, z_3; \rho_{12}, \rho_{13}, \rho_{23}) = \left[ \Phi(z_1) + \Phi(z_2) - \Delta(z_1, z_2) \right] - T(z_1, a_1) - T(z_2, c_2) - \left[ \Phi(z_1, z_2, -z_3; \rho_{12}, \rho_{13}, -\rho_{23}) \right]. \tag{A.18}
\]

To implement the computing formulas (A.4) through (A.10), we used IMSL routines [23] for evaluation of \(\Phi(z, a)\) and \(T(z, a)\). Unfortunately, we were unable to find a routine for evaluating the \(S\)-function in any numerical-analysis library; thus we were forced to develop a portable computational procedure for \(S(z, a, b)\) that could satisfy the speed and accuracy requirements of ADSSIM.

**Computing the S-function**

For each year and for each service to be analyzed during the interactive execution of ADSSIM, the \(S\)-function must be computed 480 times. Initially we evaluated (A.3) using an IMSL numerical quadrature algorithm. Because the resulting interactive response times seemed to be too large, we sought a faster evaluation procedure based on a series expansion of the \(S\)-function. Taking a third-order Taylor expansion of \(\Phi(z, a)\) about the point \(u = z(1 + a^2 + a^2 + a^2 + a^2)\) and integrating the result term-by-term as in (A.3), we have

\[
S(z, a, b) = \frac{1}{2\pi} \int_{0}^{\infty} \frac{w(z, a, b)}{k+1} \, dw, \quad \Phi(z, a). \tag{A.19}
\]
\[ u_1 = u_1(z, a, b) \equiv \begin{cases} \sqrt{3}, & \text{if } u_1 \leq \sqrt{3} \\ \frac{|z - u_1|^3}{2 |a b|} |ab| \leq 1 \\ \frac{z}{(a, b, 1)} & \text{otherwise} \end{cases} \]

Thus the absolute value of the expression
\[ R(z, a, b) = \frac{D(z, a, b)}{12\pi} \left[ (1 + a^2 + a^2 b^2 + t^2) r(a, b, t) \right] \]

\[ \times \frac{d^2}{dt^2} \Phi \left[ z r(a, b, \tau(t)) \right] dt, \]

(A.11)

where
\[ r(a, b, t) = (1 + a^2 + a^2 b^2 + t^2)^{1/2}, \]
\[ w_0(a, b) = \tan^{-1}\left[ b \left/ r(a, b, 1) \right. \right], \]
\[ w_1(a, b) = \tan^{-1}(b) - r(a, b, 1) w_0(a, b), \]
\[ w_2(a, b) = \left[ 1 - r(a, b, 1) \right] w_0(a, b), \]
\[ -2 r(a, b, 1/2) w_1(a, b), \]
\[ + a \cdot \ln\left( \frac{a b + r(a, b, 1)}{r(a, b, 0)} \right) \] (A.12)

and where \( \tau(t) \) is a point in the interior of the interval joining \( \frac{1}{2} \) and \( t \) that corresponds to Lagrange's form of the remainder. A bound for the last term on the right-hand side of (A.11) is based on the following result [2]:

\[ 0 < \tau, a, b < \infty \Rightarrow \]
\[ D(z, a, b) = \sup_{0 < \tau < 1} \left\{ \left| \frac{d^2}{dt^2} \Phi \left[ z r(a, b, \tau(t)) \right] \right| \right\}, \]

\[ = \max_{\tau \in [1, 2]} \left\{ \left| (2\pi)^{1/8} \exp\left(-\frac{1}{2} u_1^2 \right) u_1^4 - 1 \right| \right\}, \]

where
\[ u_1 = u_1(z, a, b) \equiv z r(a, b, 0), \]

and

\[ (A.11) \]

Thus the absolute value of the expression
\[ R(z, a, b) = \frac{D(z, a, b)}{12\pi} \left[ (1 + a^2 + a^2 b^2 + t^2) r(a, b, t) \right] \]

\[ \times \frac{d^2}{dt^2} \Phi \left[ z r(a, b, \tau(t)) \right] dt, \]

(A.11)

An ADSSIM support routine controls the evaluation of \( S(z, a, b) \) using formulas derived by Stock [31] to convert all such computations for arbitrary arguments \(-\infty < z, a, b < \infty\) to the regular case in which \( 0 < z, a, b < \infty\). In the regular case, the first three terms of equation (A.11) can be used to approximate the S-function, provided that the maximum absolute truncation error (A.12) does not exceed a user-specified tolerance. If (A.12) is too large, then the S-function is estimated by numerical quadrature. In our applications of ADSSIM, we required \(|R(z, a, b)| < 10^{-k}\); and since this error bound was never exceeded, we found that the first three terms of (A.11) always provided an adequate approximation to the S-function.

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