Introduction to Financial Risk Analysis Using Monte Carlo Simulation

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Overview

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I. Introduction

*Risk* encompasses not only the probabilities of various outcomes but also the adverse or beneficial consequences resulting from those outcomes.

Figure 1: Basic Risk Paradigm
Applications of Risk Analysis and Assessment

- Involve complex problems—for example,
  - Legal actions;
  - Environmental health and safety issues;
  - Logistics systems engineering;
  - International economic development; or
  - Capital project ranking and portfolio management.

- Require comparing two or more options, where each features probabilistic outcomes and the associated economic consequences.
II. Elements of Financial Risk Assessment

A. Measures of Financial Performance

1. **Net Present Value** (NPV) is the present value of future net cash flows discounted at some interest rate per period, minus the initial investment,

\[
NPV = \sum_{t=0}^{n} \frac{CF_t}{(1 + R)^t},
\]

where \( CF_0 < 0 \) is the initial investment (cash outflow); \( CF_t \) is the cash flow at the end of period \( t \), and \( R \) is the discount (interest) rate per period.
2. **Internal Rate of Return** (IRR) is the discount rate that equates the present value of a project’s cash inflows to the present value of the project’s cash outflows:

\[
NPV = \sum_{t=0}^{n} \frac{CF_t}{(1 + IRR)^t} = 0.
\]

When using IRR as a financial performance measure, we usually assume the initial outlay is the only negative net cash flow and all subsequent cash flows are positive.
NPV and IRR are used to rank investment options.

- If we rank projects A and B based on NPV computed at the same discount rate $R$, then project A will outrank (is preferred to) project B if $\text{NPV}_A > \text{NPV}_B$.

- Similarly, if we rank projects A and B based on IRR, then project A will outrank (is preferred to) project B if $\text{IRR}_A > \text{IRR}_B$. 
### Table 1: Comparison of NPV and IRR for Projects A and B

<table>
<thead>
<tr>
<th></th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Period</strong></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td><strong>Net cash flow</strong></td>
<td>–100</td>
<td>50</td>
</tr>
<tr>
<td><strong>Cumulative NCF</strong></td>
<td>–100</td>
<td>–50</td>
</tr>
<tr>
<td><strong>Discounted NCF (@ 10%)</strong></td>
<td>–100</td>
<td>45.5</td>
</tr>
<tr>
<td><strong>Cumulative discounted NCF</strong></td>
<td>–100</td>
<td>–54.5</td>
</tr>
</tbody>
</table>
• When ranking alternative investment options or projects, we often construct graphs of the NPVs of the competing projects (*NPV profiles*) as a function of the discount rate, $R$.

• For selecting projects, NPV is a better comparison metric than IRR in some respects; however, IRR is widely used in industry.
Figure 2: Net Present Value Profiles of Projects A & B
The NPV profile in Figure 2 demonstrates why NPV may be a superior in some cases.

- The curve for NPV$_A$ crosses over the curve for NPV$_B$ at the *crossover rate* of 7.2%.

- If $R < 7.2\%$, then NPV$_B > $ NPV$_A$ and project B is preferred.

- If $R > 7.2\%$, then NPV$_A > $ NPV$_B$ and project A is preferred.
● The *cost of capital*, $k$, is the interest rate at which the individual or firm must borrow to finance an investment.

● If a single independent project is being evaluated without considering other alternatives, then both IRR and NPV lead to the same decision—accept the project if $NPV > 0$ (or equivalently, $IRR > k$) and reject the project otherwise.
• If A and B are mutually exclusive alternatives and $k$ is used as the discount rate,
  
  • then if $k < 7.2\%$, we see from Figure 2 that the NPV method chooses B;
  
  • but the IRR method *always* chooses A regardless of the value of $k$, provided that $k < \text{IRR}_A = 14.5\%$.

• It follows that NPV is the preferred measure of financial performance because IRR can select the inferior alternative when the relevant discount rate is on the “wrong” side of the crossover point.
• Notice also that compared with the curve for $\text{NPV}_A$, the curve for $\text{NPV}_B$ is more sensitive to changes in the discount rate $R$ because of its steeper slope.

  - The greater sensitivity to the discount rate is caused by the longer-term nature of project B compared with project A; and

  - This phenomenon is sometimes interpreted as an indication that B is a higher risk than A.
B. Types of Financial Risk

1. An investment’s *stand-alone risk* is the risk associated with a single investment opportunity, ignoring the fact that the investment may represent only one of several assets held by a firm or individual.

2. *Diversifiable risk* is the type of risk that can be diluted by viewing one investment as part of a group that may contain other investments, of which some may be less risky and some may be more risky compared with the single investment under consideration.
3. **Market, or beta, risk** is the type of risk that cannot be eliminated by diversification because it stems from pervasive factors that typically affect most investments—for example, inflation, recession, and high interest rates.
Concepts Considered in Financial Risk Assessment

1. The three types of risk are closely related.

2. Analysis of individual investments (projects or stock purchases) may consider stand-alone risk.
3. Market risk is usually a greater issue of concern than diversifiable risk, which by definition can be managed with a properly chosen set of investments.

4. Most investors are risk-averse, preferring to avoid risk, and therefore requiring the promise of higher rates of return on risky investments.
C. Measures of Financial Risk

1. Stand-alone risk in the context of comparing alternatives

- **Expected present value** incorporates the value, $v_g$, (gain or loss) from each outcome, $g$, as well as its corresponding probability of occurrence, $p_g$:

$$ EV = \sum_{g=1}^{m} p_g v_g. $$
For a risky investment, the performance measure analogous to rate of return is \textit{expected rate of return}, which includes the rate of return, $k_g$, from each outcome $g$, as well as the probability of its occurrence, $p_g$,

$$\hat{k} = \sum_{g=1}^{m} p_g k_g.$$
Table 2: Payoff Matrix for Stock Investments in Two Companies with Uncertain Demand

<table>
<thead>
<tr>
<th>Demand Level</th>
<th>Probability of Demand Level</th>
<th>Company A Rate of Return for Demand Level</th>
<th>Company B Rate of Return for Demand Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>0.3</td>
<td>100%</td>
<td>20%</td>
</tr>
<tr>
<td>Normal</td>
<td>0.4</td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>Weak</td>
<td>0.3</td>
<td>−70%</td>
<td>10%</td>
</tr>
<tr>
<td>Expected Rate of Return</td>
<td></td>
<td>15%</td>
<td>15%</td>
</tr>
<tr>
<td>Standard Deviation of Return</td>
<td></td>
<td>66%</td>
<td>4%</td>
</tr>
</tbody>
</table>
• When two or more investments have the same expected rate of return, we need another measure of relative riskiness.

• Risk-averse investors typically judge investments with lower variability in expected rate of return, measured by the standard deviation of the rate of return computed over all possible outcomes, as being more dependable and less risky,

\[ \sigma = \sqrt{\sum_{g=1}^{m} p_g \left( k_g - \hat{k} \right)^2} . \]
If the probabilities associated with the various outcomes are not known, then we may estimate the expected rate of return and its standard deviation using the sample mean and sample standard deviation from historical rate-of-return data,

\[ \hat{k}_{\text{est}} = \frac{1}{n} \sum_{t=1}^{n} \bar{k}_t \quad \text{and} \quad \sigma_{\text{est}} = \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (\bar{k}_t - \hat{k}_{\text{est}})^2}. \]
• In the case where one alternative has the higher rate of return and the other has the lower standard deviation, we use the \textit{coefficient of variation}, \( CV \), which gives the risk per unit of return and provides a basis for comparison,

\[
CV = \frac{\sigma}{\hat{k}}.
\]
2. Diversifiable risk and market risk in the context of stock-investment portfolios.

- We begin by determining the expected return of each asset in the portfolio.
- If \( a \) is the number of assets in the portfolio and \( w_i \) is the fraction of the portfolio’s monetary value invested in asset \( i \), then the \textit{portfolio’s expected return} is the weighted average of the expected returns on the individual assets,

\[
\hat{k}_P = \sum_{i=1}^{a} w_i \hat{k}_i.
\]
• Computation of the standard deviation of a portfolio requires estimation of the correlations of the assets.

• If $\mu_X$ and $\mu_Y$ denote the expected returns on two investments, then the covariance between the returns $X$ and $Y$ on the investments is given by

$$\sigma_{XY} = \text{Cov}(X,Y) = E[(X - \mu_X)(Y - \mu_Y)] ;$$

and if $\sigma_X$ and $\sigma_Y$ denote the standard deviations of the two returns, then the correlation between the two returns is

$$\rho_{XY} = \frac{\sigma_{XY}}{(\sigma_X \sigma_Y)}.$$
If $w$ and $1-w$ are the fractions of the portfolio’s value invested in two stocks, then the variance of the portfolio’s return is

$$\sigma_P^2 = \text{Var}\left[wX + (1-w)Y\right]$$

$$= w^2 \sigma_X^2 + (1-w)^2 \sigma_Y^2 + 2w(1-w)\sigma_{XY}. \quad (1)$$
• With perfectly positively correlated returns (\(\rho_{XY} = 1.0\)), diversification would result in **increasing** the risk of a portfolio vs. investing only in the stock with the smaller standard deviation.

• With perfectly negatively correlated returns (\(\rho_{XY} = -1.0\)), diversification could theoretically lead to the elimination of all risk under a certain value of \(w\).
• Correlations between pairs of stocks typically run in the range of +0.5 and +0.7.

• On average the correlation for two randomly selected stocks is about +0.6.
Table 3: Comparison of Expected Returns and Standard Deviations of Two Stocks ($\rho_{CD} = 0.67$) and a Portfolio Consisting of Both Equally

<table>
<thead>
<tr>
<th>Year</th>
<th>Stock C</th>
<th>Stock D</th>
<th>Portfolio 50% Invested in both C and D</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>40.0%</td>
<td>28.0%</td>
<td>34.0%</td>
</tr>
<tr>
<td>2005</td>
<td>–10.0%</td>
<td>20.0%</td>
<td>5.0%</td>
</tr>
<tr>
<td>2006</td>
<td>35.0%</td>
<td>41.0%</td>
<td>38.0%</td>
</tr>
<tr>
<td>2007</td>
<td>–5.0%</td>
<td>–17.0%</td>
<td>–11.0%</td>
</tr>
<tr>
<td>2008</td>
<td>15.0%</td>
<td>3.0%</td>
<td>9.0%</td>
</tr>
<tr>
<td>Average Return</td>
<td>15.0%</td>
<td>15.0%</td>
<td>15.0%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>22.6%</td>
<td>22.6%</td>
<td>20.6%</td>
</tr>
</tbody>
</table>
• If we have a stocks with returns \( \{X_1, \ldots, X_a\} \) where the values of the stocks respectively represent the fractions \( \{w_1, \ldots, w_a\} \) of the portfolio’s total value, then the variance of the portfolio’s overall return is an extension of Equation (1),

\[
\sigma_P^2 = \text{Var}
\left[
\sum_{i=1}^{a} w_i X_i
\right]
\]

\[
= \sum_{i=1}^{a} w_i^2 \text{Var}(X_i) + 2 \sum_{i=1}^{a-1} \sum_{j=i+1}^{a} \text{Cov}(X_i, X_j).
\]
As the number of stocks increases, the risk in the portfolio generally decreases to some limit—namely, the risk in a portfolio consisting of all stocks in the market.

The return $k_M$ of a portfolio consisting of all stocks (a *market portfolio*) has standard deviation $\sigma_M$ of approximately 20.4%.

This is much smaller than the standard deviation of an average stock, which historically has been approximately 35%.
• This market risk, also called *systematic risk*, is the risk that all stock investors must bear.

• For an individual security, the systematic risk is measured by its “coefficient,” which describes how the stock’s return moves in relation to the return of the general market as gauged by some market index such as the Dow Jones Industrial Average.
For the stock in a portfolio, its $\beta$ coefficient may be expressed in terms of the mean and standard deviation of its return as well as its covariance with the market return and the standard deviation of the market return,

$$\beta_i = \rho_{i,M} \sigma_i / \sigma_M = \text{Cov}(k_i, k_M) / \sigma_M^2.$$
Figure 3: Examples of high ($\beta = 2.0$), average ($\beta = 1.0$), and low ($\beta = 0.5$) volatility stocks.
• The risk premium $\text{RP}_M$ measures the additional return above the risk-free rate that is required to compensate for the “average” (that is, market) risk, 

$$\text{RP}_M = k_M - k_{RF}.$$

• $k_M$ is the required rate of return on a portfolio consisting of all stocks and is also the required rate on an “average” stock ($\beta_{\text{avg}} = 1.0$). $k_M$ is typically hard to estimate.

• $k_{RF}$ is the risk free rate of return, usually measured by the return on long-term treasury bonds.
Assuming that we have an estimate for $\text{RP}_M$ then an expression for the risk premium for investment $i$ is

$$\text{RP}_i = \text{RP}_M \beta_i.$$ 

It follows that the required return on investment $i$ is

$$k_i = k_{RF} + \text{RP}_M \beta_i.$$  \hfill (2)
• Equation (2) is known as the Security Market Line (SML) which specifies the relationship of the required rate of return on a risky investment and the investment’s $\beta$ coefficient.

• The SML and the investor’s position on it may change over time because of changes in
  - interest rates
  - the investor’s aversion to risk, and
  - individual investments’ $\beta$ values.
Figure 4: Security Market Line (SML)
• Inflation will affect the \textit{risk-free rate}, $k_{RF}$.

• We may think of $k_{RF}$ as consisting of a “real” inflation-free rate, $k^*$, and an \textit{inflation premium}, $\text{IP}$, required by the investor to compensate for inflation,

$$k_{RF} = k^* + \text{IP}.$$ 

• Typically, $2\% \leq k^* \leq 4\%$. 
• Ultimately, when attempting to assess risk, we must answer the following questions:

1. What do we use as a discount rate when computing NPV?

2. What is the threshold value for the rate of return that is necessary for acceptance of an investment option?

3. How is risk related to the rate of return required to make an investment attractive?
The cost of capital $k$ to the firm or individual is often used as the discount rate for the calculation of NPV.

- Since an investment’s rate of return should exceed this rate in order to be accepted, we refer to it as the “hurdle rate.”

- If the IRR exceeds this rate, then of course NPV $>0$ when computed at the cost of capital.
For the rest of this discussion, we will assume that we have arrived at an appropriate cost of capital to use as the discount.

We will also assume that in the case of diversified investments, we have determined a risk-adjusted rate of return.
II. Applications of Simulation to Financial Risk Analysis

A. Capital Budgeting in Stand-Alone Risk Context

- Decision-making situations suitable for this type of analysis include
  - development of a new product,
  - entrance to a new market,
  - building of a new production facility, and
  - expansion of an existing facility.
This analysis requires the estimation of various input parameters such as:

- costs,
- sales prices, and
- inflation rates.
• Sources of these input parameters include
  • historical data,
  • market research, and
  • “experts’” best estimates.

• These input parameters are subject to uncertainty and variability.
• The analysis will typically begin with a base case—one possible outcome on a decision tree and is often the “most likely.”

• The input parameters shown in Table 4 on the next slide include “best guesses” such as
  • costs, prices, and demand are at current values;
  • demand growth rate is zero; and
  • inflation of costs and product prices is moderate.
### Table 4: Base Case NPV Analysis of Potential Production Facility

| Discount Rate | 10% | Initial Variable Cost/Unit | 1 |
| Initial Annual Production Quantity (000 units) | 21.5 | Variable Cost inflation Rate | 0% |
| Demand Growth Rate | 0% | Initial Product Sale Price | $ 50 |
| Initial Fixed Cost ($000) | $ 200 | Product Price Inflation Rate | 0% |
| Fixed Cost Inflation Rate | 0% | Salvage ($000) | $ 100 |

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Expenditure ($000)</td>
<td>$ 3,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Production Quantity (000 units)</td>
<td></td>
<td>21.5</td>
<td>21.5</td>
<td>21.5</td>
<td>21.5</td>
<td>21.5</td>
</tr>
<tr>
<td>Fixed Costs ($000)</td>
<td></td>
<td>$ 200.0</td>
<td>$200.0</td>
<td>$ 200.0</td>
<td>$ 200.0</td>
<td>$200.0</td>
</tr>
<tr>
<td>Variable Costs ($000)</td>
<td></td>
<td>$ 21.5</td>
<td>$ 21.5</td>
<td>$ 21.5</td>
<td>$ 21.5</td>
<td>$ 21.5</td>
</tr>
<tr>
<td>Total Costs ($000)</td>
<td></td>
<td>$ 221.5</td>
<td>$221.5</td>
<td>$ 221.5</td>
<td>$ 221.5</td>
<td>$221.5</td>
</tr>
<tr>
<td>Revenue ($000)</td>
<td></td>
<td>$ 1,075</td>
<td>$1,075</td>
<td>$ 1,075</td>
<td>$ 1,075</td>
<td>$1,075</td>
</tr>
<tr>
<td>Salvage ($000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$ 100</td>
</tr>
<tr>
<td>Cash Flow</td>
<td></td>
<td>$(3,000)</td>
<td>$ 853.5</td>
<td>$853.5</td>
<td>$ 853.5</td>
<td>$ 853.5</td>
</tr>
<tr>
<td>Disc. CF</td>
<td></td>
<td>$(3,000)</td>
<td>$ 775.9</td>
<td>$705.4</td>
<td>$ 641.2</td>
<td>$ 583.0</td>
</tr>
</tbody>
</table>

NPV @ 10% Discount Rate $297.5  
IRR 13.75%
Applications of Simulation to Financial Risk Analysis

Capital Budgeting Problem

- The outcome however is uncertain and therefore we must be compensated for the risk assumed.

- We require a risk-adjusted discount rate for the NPV analysis and a risk-adjusted threshold for IRR.
Methods historically used to capture and compensate for uncertainty include:

- **certainty equivalent**—subjectively choose and scale down uncertain CFs;
- **risk-adjusted rate of return**—subjectively adjust the threshold cost of capital according to the perceived risk of the project;
- **payoff matrix**—develop various alternate scenarios and assign probabilities to each; and

- **sensitivity analysis**—change input parameters one at a time and plot resulting NPVs and IRRs.
### Table 5: Excel’s Scenario Manager Sample Output

<table>
<thead>
<tr>
<th>Scenario Summary</th>
<th>Current Values:</th>
<th>Worst</th>
<th>Best</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Changing Cells:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount Rate</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Initial Annual Production Quantity (000 units)</td>
<td>21.5</td>
<td>21.5</td>
<td>21.5</td>
</tr>
<tr>
<td>Demand Growth Rate</td>
<td>0%</td>
<td>0%</td>
<td>5%</td>
</tr>
<tr>
<td>Initial Fixed Cost ($000)</td>
<td>$200</td>
<td>$220</td>
<td>$180</td>
</tr>
<tr>
<td>Fixed Cost Inflation Rate</td>
<td>3%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Initial Variable Cost/Unit</td>
<td>$1.00</td>
<td>$5.00</td>
<td>$0.75</td>
</tr>
<tr>
<td>Variable cost inflation Rate</td>
<td>13%</td>
<td>10%</td>
<td>0%</td>
</tr>
<tr>
<td>Initial Product Sale Price/Unit</td>
<td>$50</td>
<td>$30</td>
<td>$75</td>
</tr>
<tr>
<td>Product Price Inflation Rate</td>
<td>3%</td>
<td>0%</td>
<td>3%</td>
</tr>
<tr>
<td>Salvage ($000)</td>
<td>$100</td>
<td>$90</td>
<td>$110</td>
</tr>
<tr>
<td>Initial Capital Expenditure ($000)</td>
<td>$3,000,000</td>
<td>$500,000</td>
<td>$2,700,000</td>
</tr>
<tr>
<td><strong>Result Cells:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NPV @ 10% Disc Rate ($000)</td>
<td>$461.2</td>
<td>−$3,079.2</td>
<td>$8,114.2</td>
</tr>
<tr>
<td>IRR</td>
<td>15.6585%</td>
<td>&lt;0%</td>
<td>93.4354%</td>
</tr>
</tbody>
</table>
Figure 5: Sensitivities of NPV and IRR to Individual Input Parameters
We simulated the production facility problem by:

- fitting Beta distributions to the input parameters using an expert’s estimates of the minimum, most likely, and maximum values; and

- simulating 100,000 independent replications of the NPV and IRR analysis using Crystal Ball; see CapBudgetingSimulation.xlsx
Figure 6: Crystal Ball Frequency Distribution of NPV
Figure 7: Crystal Ball Frequency Distribution of IRR
The advantages of simulation over other methods discussed include the following:

- the ability to simultaneously change all input variables;

- the ability to examine histograms of the probability distributions of NPV and IRR; and

- the ability to determine resulting probabilities of achieving specified levels of NPV and IRR.
B. Value at Risk in Portfolios Comprised of Options

- Two Kinds of Options
  - **Calls**—the right to buy; e.g., a ticket to a game
  - **Puts**—the right to sell; e.g., a money back guarantee
Options terminology

- **Premium** = price of the option
- **Striking price** = price at which you can buy (with calls) or sell (with puts)
- **Underlying asset** – what you have the right to buy or sell
- **Expiration** – when the option’s life ends
- **Exercise** – use the option
- **Write** – create the option
**Black-Scholes Option Pricing Model**

\[ C = \Phi(d_1) - Ke^{-rt}\Phi(d_2), \quad \text{where} \quad \begin{cases} 
  d_1 = \frac{\ln(S/K) + \left(R + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} , \\
  d_2 = d_1 - \sigma\sqrt{t},
\end{cases} \]

where \( C = \) theoretical call value

\( \Phi(x) = \) cumulative normal probability density function

\( K = \) option striking price

\( e = \) base of natural logarithm

\( r = \) riskless interest rate

\( t = \) time until option expiration

\( \sigma = \) standard deviation of returns of underlying asset
• The “Greeks”

- Delta \( \Delta = \frac{\partial C}{\partial S} \)
- Gamma \( \gamma = \frac{\partial^2 C}{\partial S^2} \)
- Theta \( \theta = \frac{\partial C}{\partial t} \)
• **The “Greeks”**

The “position delta” is an indication of the size and directional bias of an option portfolio.

- **Delta** \( \Delta = \frac{\partial C}{\partial S} \)

- **Gamma** \( \gamma = \frac{\partial^2 C}{\partial S^2} \)

- **Theta** \( \theta = \frac{\partial C}{\partial t} \)
• The “Greeks”

The “position delta” is an indication of the size and directional bias of an option portfolio.

- Delta \( \Delta = \frac{\partial C}{\partial S} \)

The “position gamma” is a measure of the robustness of the portfolio’s characteristics to changes in the value of the underlying asset.

- Gamma \( \gamma = \frac{\partial^2 C}{\partial S^2} \)

- Theta \( \theta = \frac{\partial C}{\partial t} \)
The “Greeks”

The investor’s “position delta” is an indication of the size and directional bias of an option portfolio.

- **Delta** \[ \Delta = \frac{\partial C}{\partial S} \]

The investor’s “position gamma” is a measure of the robustness of the portfolio’s characteristics to changes in the value of the underlying asset.

- **Gamma** \[ \gamma = \frac{\partial^2 C}{\partial S^2} \]

Theta measures the change in value of the portfolio due solely to the passage of time.

- **Theta** \[ \theta = \frac{\partial C}{\partial t} \]
• Position Risk

- With *large* movements in the price of the underlying asset, calculus derivatives change dramatically and do not accurately reflect subsequent relationships.

- At a trading company or marketmaking firm, prudent risk management requires someone to be aware of the consequences of low probability events.
Table 6: Example of an Initial Option Portfolio

<table>
<thead>
<tr>
<th>Quantity</th>
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<th>Time</th>
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<th>Psn Delta</th>
<th>Psn Gamma</th>
<th>Psn Theta</th>
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Stock Price: 60
Interest Rate: 2%
Volatility: 40%

6656 -599 467  $288,611
Figure 8: Expert’s Mental Image of the Position of the Initial Option Portfolio

- Rising price of the underlying asset
- Loss
- Falling price of the underlying asset
- Profit

Value at Risk

Applications of Simulation to Financial Risk Analysis
Figure 9: Expert’s Mental Image of the Position of the Initial Options Portfolio

We now write 200 contracts of call # 1
Table 7: Option Portfolio after the 1\textsuperscript{st} Trade

<table>
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<tr>
<th>Quantity</th>
<th>Asset</th>
<th>Strike</th>
<th>Time</th>
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<th>Psn Delta</th>
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**Stock Price**: 60

**Interest Rate**: 2%

**Volatility**: 40%

- Stock: 60
- Interest Rate: 2%
- Volatility: 40%

-842 - 740 - 597 - $177,924
Figure 10: Expert’s Mental Image of the Position of the Option Portfolio after the 1st Trade

- Falling price of the underlying asset
- Rising price of the underlying asset
Figure 11: Expert’s Mental Image of the Position of the Option Portfolio after the 1\textsuperscript{st} Trade

Profit

Falling price of the underlying asset

Rising price of the underlying asset

Now we buy 100 contracts of Call # 2 and 200 contracts of Put # 2
## Table 8: Option Portfolio after 2\textsuperscript{nd} Trade

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<th>Quantity</th>
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**Stock Price:** 60

**Interest Rate:** 2%

**Volatility:** 40%

\[
\begin{align*}
\text{Stock Price} & \quad 60 \\
\text{Interest Rate} & \quad 2\% \\
\text{Volatility} & \quad 40\%
\end{align*}
\]
Figure 12: Expert’s Mental Image of the Position of the Option Portfolio after the 2\textsuperscript{nd} Trade

Profit

Loss

Falling price of the underlying asset

Rising price of the underlying asset
Figure 13: Expert’s Mental Image of the Position of the Option Portfolio after the 2nd Trade

Now we buy 160 contracts of Call# 1

Falling price of the underlying asset

Rising price of the underlying asset

Profit

Loss
Table 9: Option Portfolio after 3rd Trade

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<th>Psn Delta</th>
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Stock Price: 60
Interest Rate: 2%
Volatility: 40%

-20  596  -445  $471,818
Figure 14: Expert’s Mental Image of the Position of the Option Portfolio after the 3rd Trade

- Falling price of the underlying asset
- Rising price of the underlying asset

- Profit
- Loss
• Value at Risk
  
  - Most of the time we do not have three sigma events.
  
  - The honest answer to the question, “What is the worst that can happen?” is often “The wheels come off and we lose almost everything.” This is not a helpful answer.
A better question is “What is a realistic estimate of how much our current position might lose in the next trading day?”

To standardize the answer to this reasonable question, the industry uses a concept known as **Value at Risk** (VAR). This measures the consequences of a 2 standard deviation move over one day.
Crystal Ball demo; see **WINSIM.xls**
IV. Conclusions

- Spreadsheet-based simulation is a powerful tool for performing risk assessment in complex applications.

- Simulation enables the user to do the following:
  - check the validity of the assumptions underlying a financial model;
  - explore the sensitivity of the model results to the input parameters whose values are uncertain or are subject to random variation; and
  - honestly represent the inherent variability of the final results.
IV. Conclusions (Cont‘d)

- Proper accounting for uncertainty and random variability in the inputs and outputs of a financial model is a critical element of financial risk assessment, and simulation provides an effective mechanism for doing this.