

ISE 453: Design of PLS Systems

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Geometric Mean

- How many people can be crammed into a car?
 - Certainly more than one and less than 100: the average (50) seems to be too high, but the geometric mean (10) is reasonable

$$\text{Geometric Mean: } X = \sqrt{LB \times UB} = \sqrt{1 \times 100} = 10$$

- Often it is difficult to directly estimate input parameter X , but is easy to estimate reasonable lower and upper bounds (LB and UB) for the parameter
 - Since the guessed LB and UB are usually orders of magnitude apart, use of the arithmetic mean would give too much weight to UB
 - Geometric mean gives a more reasonable estimate because it is a logarithmic average of LB and UB

Fermi Problems

- Involves “reasonable” (i.e., $\pm 10\%$) *guesstimation* of input parameters needed and back-of-the-envelope type approximations
 - Goal is to have an answer that is within an order of magnitude of the correct answer (or what is termed a *zeroth-order approximation*)
 - Works because over- and under-estimations of each parameter tend to cancel each other out as long as there is no consistent bias
- How many McDonald’s restaurants in U.S.? (actual 2013: 14,267)

Parameter	LB	UB	Estimate	
Annual per capita demand	1	1 order/person-day x 350 day/yr =	350	18.71 (order/person-yr)
U.S. population				300,000,000 (person)
Operating hours per day				16 (hr/day)
Orders per store per minute (in-store + drive-thru)				1 (order/store-min)
Analysis				
Annual U.S. demand		(person) x (order/person-yr) =	5,612,486,080	(order/yr)
Daily U.S. demand		(order/yr)/365 day/yr =	15,376,674	(order/day)
Daily demand per store		(hrs/day) x 60 min/hr x (order/store-min) =	960	(order/store-day)
Est. number of U.S. stores		(order/day) / (order/store-day) =	16,017	(store)

System Performance Estimation

- Often easy to estimate performance of a new system if can assume either perfect (LB) or no (UB) control
- Example: estimate waiting time for a bus
 - 8 min. avg. time (aka “headway”) between buses
 - Customers arrive at random
 - assuming no web-based bus tracking
 - Perfect control (LB): wait time = half of headway
 - No control (*practical* UB): wait time = headway
 - assuming buses arrive at random (Poisson process)

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{8}{2} \times 8} = 5.67 \text{ min}$$

- Bad control can result in higher values than no control

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SELF-COORDINATING BUSES
REDUCE BUNCHING

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A BUS-HEADWAY CONTROLLER

A software system to coordinate buses on a route, based on an [idea](#) by [John J. Bartholdi III](#) and [Donald D. Eisenstein](#). The current version of the software was designed and largely written by Loren K. Platzman. Implementation has been led by [Russ Clark](#), Jin Lee, and David Williamson.



THE IDEA

Delaying buses briefly at certain checkpoints equalizes headways

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PROOF OF CONCEPT

Coordinating trolleys on Georgia Tech's busiest route

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HOW IT WORKS

Tablets, GPS, cellular networks, and web-based control

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Ex 1: Geometric Mean

- If, during the morning rush, there are three buses operating on Wolfline Route 13 and it takes them 45 minutes, on average, to complete one circuit of the route, what is the estimated waiting time for a student who does not use TransLoc for real-time bus tracking?

Answer :

$$\text{Frequency (TH)} = \frac{WIP}{CT} = \frac{3 \text{ bus/circuit}}{45 \text{ min/circuit}} = \frac{1}{15} \text{ bus/min}, \quad \text{Headway} = \frac{1}{\text{Freq.}} = 15 \text{ min/bus}$$

$$\text{Estimated wait time} = \sqrt{LB \times UB} = \sqrt{\frac{15}{2} \times 15} = 10.61 \text{ min}$$

Ex 2: Fermi Problem

- Estimate the average amount spent per trip to a grocery store. Total U.S. supermarket sales were recently determined to be \$649,087,000,000, but it is not clear whether this number refers to annual sales, or monthly, or weekly sales.

Answer : $\frac{\$6.5e11}{3e8} \approx \$2,000 / \text{person-yr}$, $LB = 1 \text{ trips/wk}$, $UB = 7 \text{ trips/wk}$

$$\Rightarrow \sqrt{1(7)} \times 52 \approx 2 \times 52 \approx 100 \text{ trips/yr} \Rightarrow \frac{\$2,000}{100} = \$20 / \text{person-trip}$$

Supermarket / Grocery Store Statistics	Data
Total number of grocery store employees	3,400,000
Total supermarket sales in 2015	\$649,087,000,000
Total supermarket sales in 2012	\$602,609,000,000
Total number of grocery stores / supermarkets	37,053
Median weekly sales per supermarket store	\$384,911
Average grocery store transaction amount	\$27.30
Average number of grocery store trips per week a consumers makes	2.2
Average number of items carried in a supermarket	38,718

Levels of Modeling

0. Guesstimation (order of magnitude)
1. Mean value analysis (linear, $\pm 20\%$)
2. Nonlinear models (incl. variance, $\pm 5\%$)
3. Simulation models (complex interactions)
4. Prototypes/pilot studies
5. Build/do and then tweak it