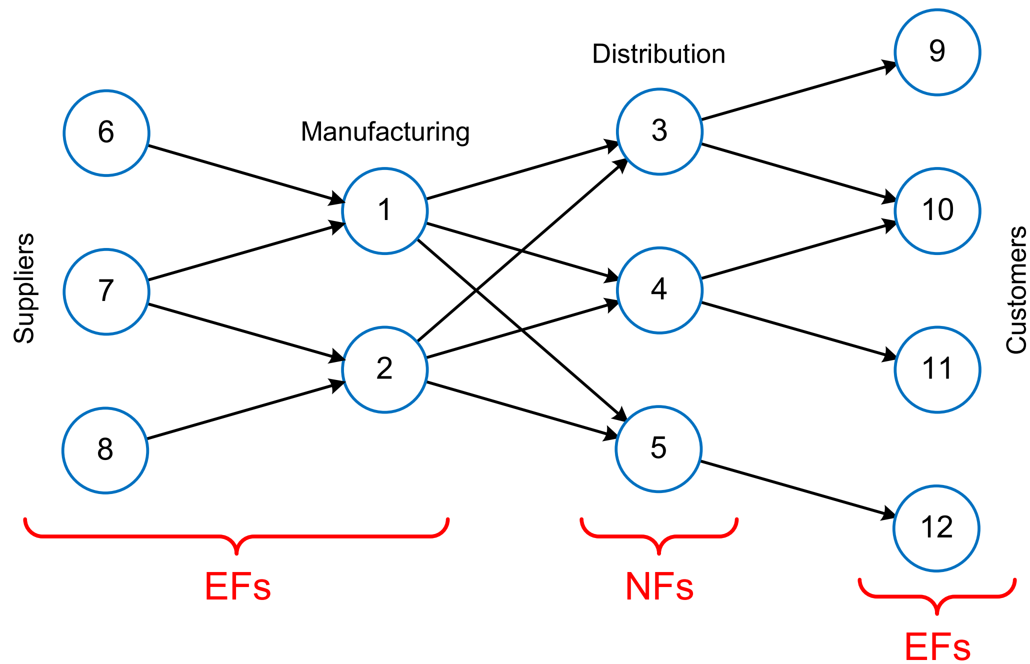
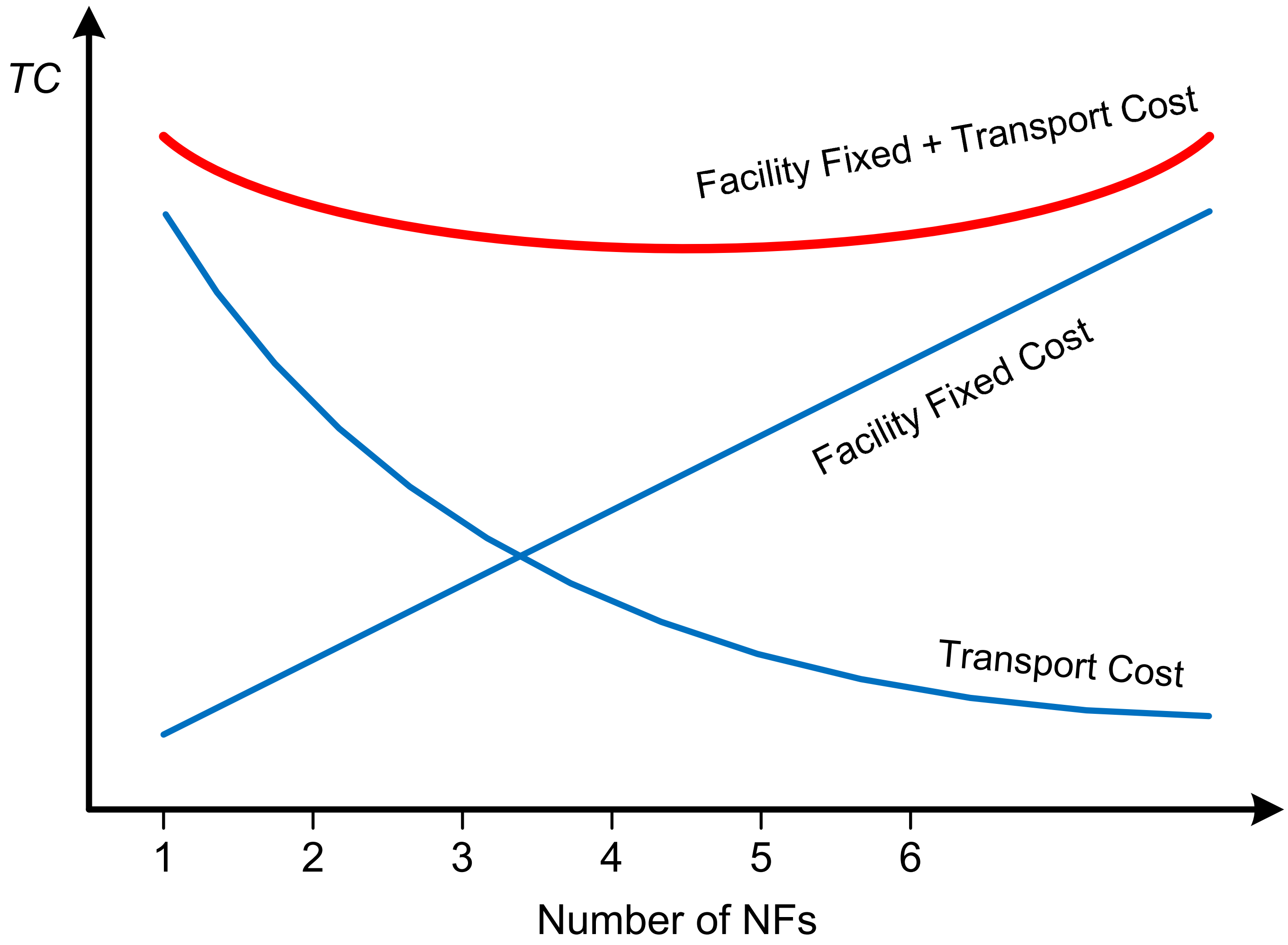
Multiple Single-Facility Location



Best Retail Warehouse Locations

Number of Locations	Average Transit Time (days)	Warehouse Location		
1	2.20	Bloomington, IN		
2	1.48	Ashland, KY	Palmdale, CA	
3	1.29	Allentown, PA	Palmdale, CA	McKenzie, TN
4	1.20	Edison, NJ Meridian, MS	Palmdale; CA	Chicago, IL
5	1.13	Madison, NJ Dallas, TX	Palmdale, CA Macon, GA	Chicago, IL
6	1.08	Madison, NJ Dallas, TX	Pasadena, CA Macon, GA	Chicago, IL Tacoma, WA
7	1.07	Madison, NJ Dallas, TX Lakeland, FL	Pasadena, CA Gainesville, GA	Chicago, IL Tacoma, WA
8	1.05	Madison, NJ Dallas, TX Lakeland, FL	Pasadena, CA Gainesville, GA Denver, CO	Chicago, IL Tacoma, WA
9	1.04	Madison, NJ Dallas, TX Lakeland. FL	Alhambra, CA Gainesville, GA Denver, CO	Chicago, IL Tacoma, WA Oakland, CA
10	1.04	Newark, NJ Palistine, TX Lakeland, FL Mansfield, OH	Alhambra, CA Gainesville, GA Denver, CO	Rockford, IL Tacoma, WA Oakland. CA

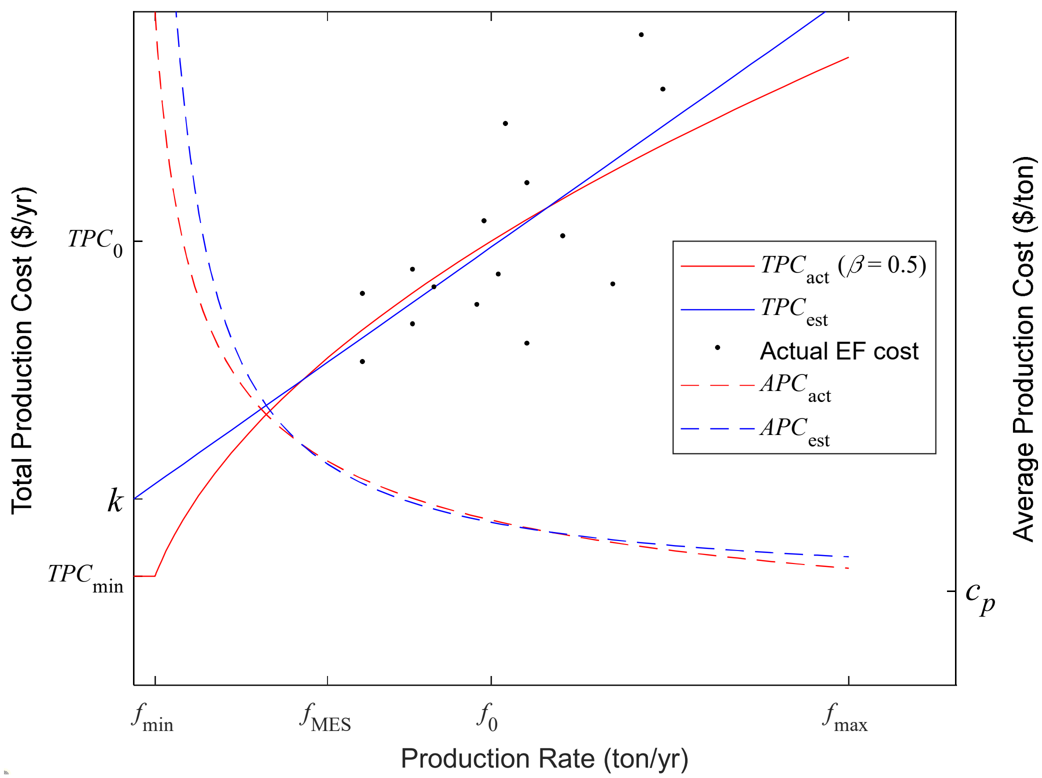
Optimal Number of NFs



Fixed Cost and Economies of Scale

- How to estimate facility fixed cost?
  - Cost data from existing facilities can be used to fit linear estimate
    - *y*-intercept is fixed cost, *k*
  - *Economies of scale* in production

    $\Rightarrow k > 0$ and $\beta < 1$



$$TPC_{act} = \max_{f < f_{max}} \left\{ TPC_{min}, TPC_0 \left(\frac{f}{f_0} \right)^{\beta} \right\}$$

$$\beta = \begin{cases} 0.62, & \text{Hand tool mfg.} \\ 0.48, & \text{Construction} \\ 0.41, & \text{Chemical processing} \\ 0.23, & \text{Medical centers} \end{cases}$$

$$TPC_{est} = k + c_p f$$

$$APC_{act} = \frac{TPC_{act}}{f} = \frac{TPC_0}{f_0^{\beta}} f^{\beta - 1}$$

$$APC_{est} = \frac{k}{f} + c_p$$

$$k = \text{fixed cost}$$

$$c_p = \text{constant unit production cost}$$

$$f_{min} / f_{max} = \text{min/max feasible scale}$$

$$f_{MES} = \textit{Minimum Efficient Scale}$$

$$TPC_0 / f_0 = \text{base cost/rate}$$

MILP

LP: $\max \ \mathbf{c}'\mathbf{x}$

s.t. $\mathbf{Ax} \le \mathbf{b}$

$\mathbf{x} \ge 0$

MILP: some x_i integer

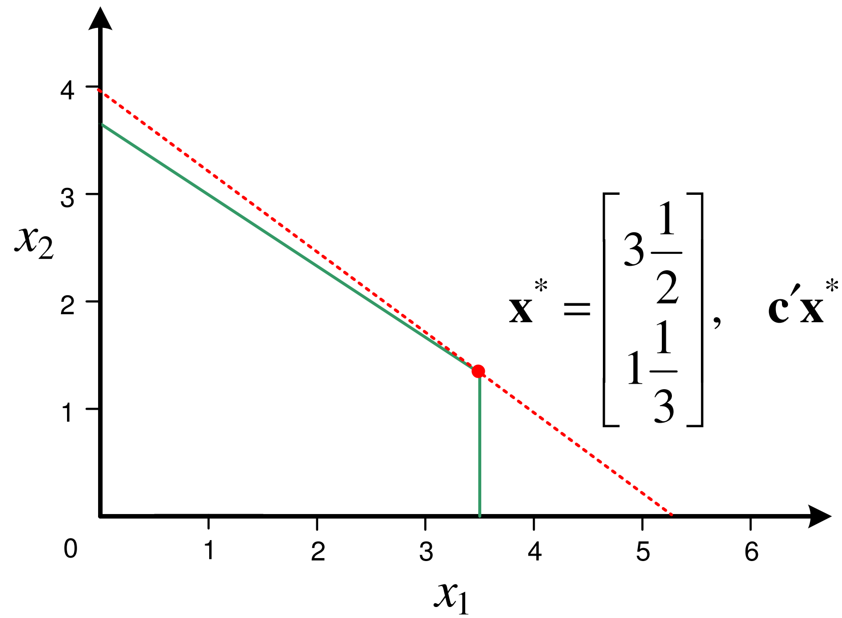
ILP: $\mathbf{x}$ integer

BLP: $\mathbf{x} \in \{0,1\}$

$$\begin{aligned} \max \quad & 6x_1 + 8x_2 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 11 \\ & 2x_1 \qquad\quad \le 7 \\ & x_1, x_2 \ge 0 \end{aligned}$$

$$\mathbf{c} = [6 \quad 8]$$

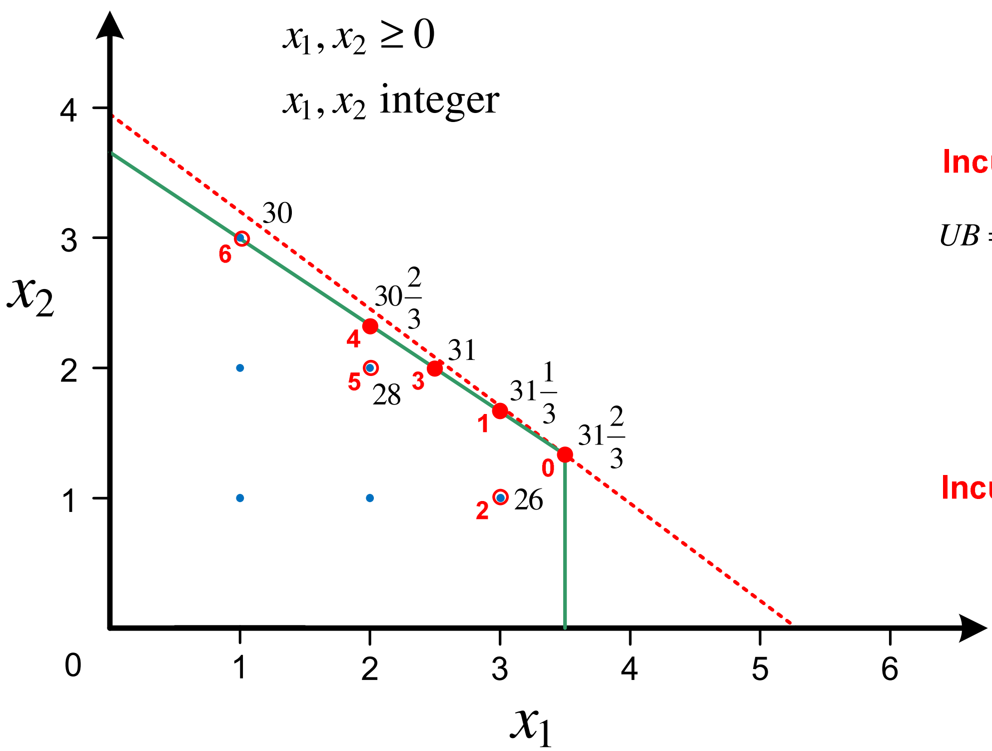
$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}$$

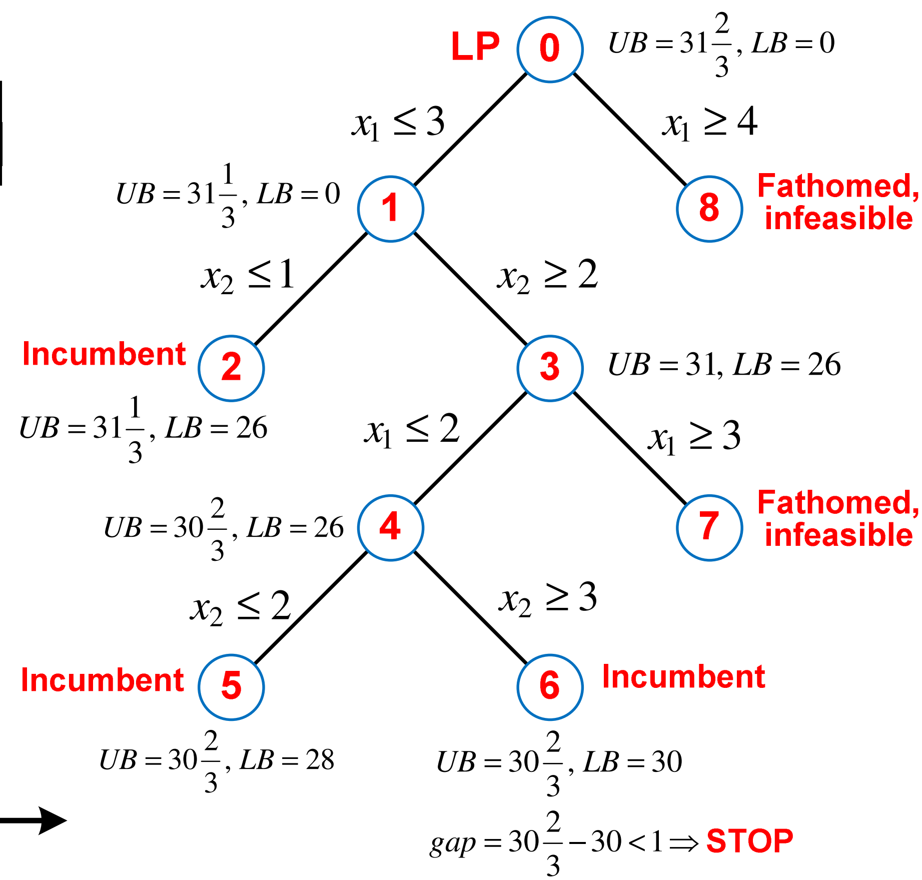


$$\mathbf{x}^* = \begin{bmatrix} 3\frac{1}{2} \\ 1\frac{1}{3} \end{bmatrix}, \quad \mathbf{c}'\mathbf{x}^* = 31\frac{2}{3}$$

Branch and Bound

$$
\begin{aligned}
\max \quad & 6x_1 + 8x_2 \\
\text{s.t.} \quad & 2x_1 + 3x_2 \le 11 \\
& 2x_1 \qquad\;\; \le 7 \\
& x_1, x_2 \ge 0 \\
& x_1, x_2 \text{ integer}
\end{aligned}
\qquad
\mathbf{c} = \begin{bmatrix} 6 & 8 \end{bmatrix}, \quad
\mathbf{A} = \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 11 \\ 7 \end{bmatrix}
$$





MILP Formulation of UFL

$$
\begin{aligned}
\min \quad & \sum_{i\in N} k_i y_i + \sum_{i\in N}\sum_{j\in M} c_{ij} x_{ij} \\
\text{s.t.} \quad & \sum_{i\in N} x_{ij} = 1, && j \in M \\
& y_i \geq x_{ij}, && i \in N,\, j \in M \\
& 0 \leq x_{ij} \leq 1, && i \in N,\, j \in M \\
& y_i \in \{0,1\}, && i \in N
\end{aligned}
$$

where

k_i = fixed cost of NF at site $i \in N = \{1,\ldots,n\}$

c_{ij} = variable cost from i to serve EF $j \in M = \{1,\ldots,m\}$

$$y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}$$

x_{ij} = fraction of EF j demand served from NF at site i.

MILP Formulation of *p*-Median

$$
\begin{aligned}
\min \quad & \sum_{i\in N}\sum_{j\in M} c_{ij}x_{ij} \\
\text{s.t.} \quad & \sum_{i\in N} y_i = p \\
& \sum_{i\in N} x_{ij} = 1, && j\in M \\
& y_i \ge x_{ij}, && i\in N,\ j\in M \\
& 0 \le x_{ij} \le 1, && i\in N,\ j\in M \\
& y_i \in \{0,1\}, && i\in N
\end{aligned}
$$

where

p = number of NF to establish

c_{ij} = variable cost from i to serve EF $j \in M = \{1,\ldots,m\}$

$$y_i = \begin{cases} 1, & \text{if NF established at site } i \\ 0, & \text{otherwise} \end{cases}$$

x_{ij} = fraction of EF j demand served from NF at site i.