Harmonic Balance of Nonlinear RF Circuits

Presented by Michael Steer

Reading:
Chapter 19, Section 19.2


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Harmonic Balance of Nonlinear RF Circuits

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Steady-State Analysis

- Assume that signal can be modeled as the sum of discrete tones.
- Solve for phasors of these tones in the circuit.
  - Everywhere or Just at nonlinear elements.
- Note
  - Cannot consider actual digitally modulated signals.
Modeling Problem

Resistor with passive convention

$i$-v characteristic of a linear resistor.

$i$-v characteristic of a diode.
Properties of RF Signals

- Modulated signals with very narrow modulation bandwidth
  - E.G. 5 MHz WCDMA on a 2 GHz carrier.
    - The amplitude and phase of the carrier change very slowly in time with one symbol transition every 400 RF cycles.
- Biased circuits can take a long time to reach steady-state
- Require high dynamic range in simulation
  - Communications about 100 dB
  - Radar about 160 dB.
What About Transient Simulation?

- Take a very long time to simulate meaningful distortion metrics.
  - E.G. 5 MHz WCDMA on a 2 GHz carrier.
  - In transient simulation must have time steps about $1/20^{th}$ the period of the highest frequency component.
    - Period of a 2 GHz signal is 500 ps. Need $Dt < 25$ps, usually however = 1 ps.
    - Symbol interval is 200 ns. So to look at meaningful signal distortion statistics requires millions of time points.
    - Even worse when considering dynamic range of at least 100 dB required.

- Biased circuits can take a long time to reach steady-state.

- Simply not practical to a traditional SPICE simulation
Modeling of Nonlinear Microwave Circuits

- Two Commercially Available Approaches:
  - Harmonic Balance Simulation
    - Assume that the signals in a circuit are a sum of sinusoids
  - Periodic Steady-State Simulation (PSS)
    - Spice based
    - Assume that there is one large periodic signal
    - Determine dynamic state of the circuit
Harmonic Balance (HB) Analysis

- Assume that the signals in a circuit are a sum of sinusoids

The harmonic balance method partitions the circuit into linear and nonlinear subcircuits.
Example: HB Analysis 1/7

- Assume that the signals in a circuit are a sum of sinusoids

\[ i(t) = v(t) + [v(t)]^2 \]

\[ v(t) = V_0 + V_1 \cos(\omega t) + V_2 \cos(2\omega t) \]

- Linear subcircuit

\[ \overline{I}_0 = V_0 \quad \bar{I}_0 = V_0 + \frac{1}{2}V_2^2 \]

\[ \overline{I}_1 = V_1 - E \quad \bar{I}_1 = V_1 \]

\[ \overline{I}_2 = V_2 \quad \bar{I}_2 = \frac{1}{2}V_1^2 \]

- Nonlinear subcircuit
Example: HB Analysis 2/7

KCL error is

\[ F = |f_0| + |f_1| + |f_2| \]

\[ f_0 = \bar{I}_0 + I_0 \]
\[ f_1 = \bar{I}_1 + I_1 \]
\[ f_2 = \bar{I}_2 + I_2 \]

Newton iteration:

\[ i^{+1} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = i \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} - J \left( \begin{bmatrix} i \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \end{bmatrix} \right)^{-1} \begin{bmatrix} i f_0 \\ i f_1 \\ i f_2 \end{bmatrix} \]

\( J \) is jacobian of \( I \) with respect to \( V \).
Example: HB Analysis 3/7

Newton iteration:

\[
^{i+1} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = ^i \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} - J \begin{bmatrix} ^i V_0 \\ ^i V_1 \\ ^i V_2 \end{bmatrix}
\]

\( J \) is Jacobian of KCL error with respect to \( V \).

\[
J \left( ^i [V_0, V_1, V_2]^T \right) = \begin{bmatrix}
\frac{\partial f_0(^i [V_0, V_1, V_2]^T)}{\partial V_0} & \frac{\partial f_0(^i [V_0, V_1, V_2]^T)}{\partial V_1} & \frac{\partial f_0(^i [V_0, V_1, V_2]^T)}{\partial V_2} \\
\frac{\partial f_1(^i [V_0, V_1, V_2]^T)}{\partial V_0} & \frac{\partial f_1(^i [V_0, V_1, V_2]^T)}{\partial V_1} & \frac{\partial f_1(^i [V_0, V_1, V_2]^T)}{\partial V_2} \\
\frac{\partial f_2(^i [V_0, V_1, V_2]^T)}{\partial V_0} & \frac{\partial f_2(^i [V_0, V_1, V_2]^T)}{\partial V_1} & \frac{\partial f_2(^i [V_0, V_1, V_2]^T)}{\partial V_2}
\end{bmatrix}
\]
Example: HB Analysis 4/7

Now each element of the Jacobian is affected by both the linear and nonlinear subcircuits, so,

\[ f_2 = \bar{I}_2 + I_2 \]

\[
\frac{\partial f_2(\mathbf{i}[V_0, V_1, V_2]^T)}{\partial V_1} = \frac{\partial I_2(\mathbf{i}[V_0, V_1, V_2]^T)}{\partial V_1} + \frac{\partial \bar{I}_2(\mathbf{i}V_2)}{\partial V_1}
\]

\[
\frac{\partial \bar{I}_2(\mathbf{i}V_2)}{\partial V_1} = 0.
\]

\[ I_2 = \frac{1}{2} V_1^2 \]

\[
\frac{\partial I_2(\mathbf{i}[V_0, V_1, V_2]^T)}{\partial V_1} = \frac{\partial}{\partial V_1} \left( \frac{1}{2} \mathbf{i} V_1^2 \right) = \mathbf{i} V_1
\]
Example: HB Analysis 5/7

Similarly

\[
\frac{\partial f_0(i[V_0, V_1, V_2]^T)}{\partial V_0} = 1 + 2iV_0 + 1
\]

\[
\frac{\partial f_0(i[V_0, V_1, V_2]^T)}{\partial V_2} = iV_2
\]

\[
\frac{\partial f_1(i[V_0, V_1, V_2]^T)}{\partial V_1} = 1 + 2iV_0 + iV_2 + 1
\]

\[
\frac{\partial f_1(i[V_0, V_1, V_2]^T)}{\partial V_0} = 2iV_1
\]

\[
\frac{\partial f_2(i[V_0, V_1, V_2]^T)}{\partial V_2} = 2iV_2
\]

\[
\frac{\partial f_2(i[V_0, V_1, V_2]^T)}{\partial V_0} = 2iV_0 + 2
\]

\[
\frac{\partial f_2(i[V_0, V_1, V_2]^T)}{\partial V_1} = iV_1
\]

\[
\frac{\partial f_2(i[V_0, V_1, V_2]^T)}{\partial V_1} = iV_1
\]
Example: HB Analysis 6/7

Enough equations to solve:

\[
\begin{bmatrix}
V_0 \\
V_1 \\
V_2 \\
\end{bmatrix}^i + 1 = \begin{bmatrix}
V_0 \\
V_1 \\
V_2 \\
\end{bmatrix}^i - J \begin{bmatrix}
V_0 \\
V_1 \\
V_2 \\
\end{bmatrix}^{-1} \begin{bmatrix}
if_0 \\
if_1 \\
if_2 \\
\end{bmatrix}
\]
**Example: HB Analysis 7/7**

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**stop**

![Linear subcircuit](image1.png) ![Nonlinear subcircuit](image2.png)
Users Guide to HB Analysis

- Three major factors limit accuracy:
  - The number of tones included in the analysis.
  - The aliasing errors due to a finite transform spectrum. This error can be reduced by considering many tones. The aliasing error is a numerically introduced error. This sets an upper limit on resolution.
  - The final value of the harmonic balance error. The major limiting factor here is how closely the Jacobian describes the actual error function.
    - Both the error function and the Jacobian have truncation error so ideally the Jacobian evaluation reflects the same truncation errors as the error function evaluation. In the end this comes down to the accuracy of the models.
Periodic Steady-State (PSS) Analysis

- **Outline:**
  - Use SPICE analysis for one period of a large tone.
  - Develop linearized dynamic circuit.
  - Consider other tones as operating on dynamic circuit.

- **Detail:**
  - Simulator guesses the initial values of voltages at all terminals, charges on capacitors, and currents through all inductors.
  - Then the circuit is simulated using transient analysis for one period of the exciting waveform. The state variables updated if required.
  - From solution, a model akin to a conversion matrix describing the dynamic circuit is established.

- An advantage of the PSS technique is that a conventional Spice simulator, and the all important transistor models, can be used.
Summary

- Analysis is based on assuming a finite number of tones.
- Not the same as a digitally modulated signal.