Time-Frequency Effects in Wireless Communication Systems

by

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ABSTRACT


Time-frequency effects in wireless communication systems caused by narrowband resonances and coupled with device nonlinearities are revealed as new sources of co-site interference, exploited for the metrology of bandpass circuits, and employed to linearize amplitude-modulated transmissions. The transient properties of bandpass filters are found to last much longer than traditional time/bandwidth rules-of-thumb. The cause of this long-tail behavior is attributed to the coupled-resonator structure of the filter circuit. A solution method which uses lowpass prototyping is developed to reduce, by a factor of two, the complexity of the differential equation set describing a narrowband filter’s transient response. Pulse overlap caused by the frequency dependence of long tails produced by filters is shown to cause intersymbol interference and intermodulation distortion in RF front-ends during frequency-hopped communications. The same properties which cause the ISI and IMD are used to develop three new transient methods for measuring resonant circuit parameters and a one-port method for extracting the operating band of a filter. A new signal-processing technique which combines time- and frequency-selectivity, Linear Amplification by Time-Multiplexed Spectrum, is developed to reduce IMD associated with amplitude modulation. Distortion reduction is demonstrated experimentally for multisines up to 20 tones.
BIOGRAPHY

Gregory James Mazzaro was born in Bronxville, New York. He received his B.S. degree in Electrical Engineering from Boston University in 2004 and his M.S. degree in Electrical Engineering from The State University of New York at Binghamton in 2006. Since 2006, he has worked toward the Ph.D. degree as a research assistant for the Electronics Research Laboratory at North Carolina State University.

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Introduction

1.1 Overview

Time-frequency effects are present in all practical communications systems, though many have been ignored for some time. As modern communications systems push the limits of the information-carrying capacity of wireless devices, however, time-frequency effects have begun to play a larger role in the performance of radio-frequency (RF) and microwave systems.

The extent to which time-frequency effects impact system performance is strongly dependent upon the bandwidths of the components which make up the system and the time-variation of the communications signals themselves. Filters found in RF front-ends are one cause for concern because they are usually designed to have narrow bandwidths. Narrow filters with sharp band-edges are known to cause ringing in circuits that employ them. A second concern is pulsed signaling, commonly found in frequency-hopping systems. Although the modulation bandwidths of individual pulses may satisfy design criteria in steady-state, the transitions between each pulse may not, and such transients may lead to unexpected system behavior. A third concern is the linearity of filtering and amplification, a common pairing in RF front-ends. If pulses of different frequencies are smeared together by a filter before the composite signal is amplified, the simultaneous tones will likely produce
intermodulation distortion (IMD) upon amplification. These examples demonstrate that placing greater emphasis on the time-domain view of signals and systems is required to understand device performance completely.

This work focuses on transient signals in bandpass systems: non-steady-state phenomena which are present in systems designed for a particular steady-state behavior. Properties of resonant circuits are emphasized because they are (a) ubiquitous in wireless devices and (b) most likely to produce unintended behavior in modern communication systems if traditional steady-state design is followed without regard for time-domain consequences. This dissertation aims to explain and model time-frequency effects caused by resonant circuits, to employ these effects in the evaluation of wireless communication circuits and systems, and to exploit these effects to achieve greater linearity in transmitter amplification.

1.2 Motivations

The most common methods of analysis for radio-frequency and microwave systems have historically been steady-state. Frequency-domain tools are powerful for studying such systems because they reduce the coupled differential equations that describe circuits, which are generally unwieldy when analyzing many cascaded components, into algebraic equations which are manageable in matrix forms. Recently, however, several problems have appeared for which steady-state analysis has yielded only limited insight and frequency-domain techniques alone have been unable to solve.

One such problem that has long affected communication systems is co-site interference. It occurs when multiple radios in close proximity inadvertently operate within the same frequency channel at the same time or otherwise produce spurious tones that degrade the received Signal-to-Noise Ratio (SNR). It is known to be caused by circuit-field coupling between nearby radios, and by spectral content produced by nonlinear mixing within nearby radios. Traditional RF and microwave design has used steady-state analysis in mitigating interference sources such as these.

Recent work has uncovered additional sources of co-site interference, such as Passive Intermodulation (PIM) distortion produced by electrothermal interactions [1]. Current research has uncovered additional sources of co-site interference: long-tail pulse decay in
narrowband circuits, and filter-amplifier intermodulation. Steady-state analysis cannot capture these behaviors; such properties must be analyzed in the time domain if their effects on communications systems are to be mitigated.

A second problem is that of testing integrated assemblies. As RF device manufacturers reduce the size, cost, and time-to-market of wireless communications products, higher levels of integration and more sophisticated automated testing procedures are employed. Vertical integration reduces the number of access points that are available for testing individual subsystems, and identification of components which do not meet specifications becomes increasingly difficult. Multilayer filters based on low-temperature co-fired ceramics (LTCC), for example, offer a compaction of the filter structure at the expense of access to its individual resonators [2].

In steady-state, the reflected signal from these filters is at a minimum in-band, and one-port measurements provide limited information about its internal structure. Before reaching steady-state, however, the filter response is not necessarily minimal in-band, and additional information about its internal structure may be obtained by examining its time-domain response.

A third problem is that of reducing distortion when generating high-power amplitude modulated communication signals. The demand for linear and power-efficient transmitters continues to grow as wireless systems migrate towards broadband data and multimedia services. New systems adaptively employ a diverse set of spectrally efficient modulation schemes ranging from Binary Phase Shift Keying (BPSK) to Orthogonal Frequency Division Multiplexing (OFDM) with 64 Quadrature Amplitude Modulation (QAM) symbols on each subcarrier. As a result, transmitter specifications for linearity and signal quality are increasingly stringent to accommodate the higher SNR and adjacent channel interference requirements.

Traditional linearization techniques use predistortion or cancellation to meet such requirements [3–5]. A new technique to improve the linearity of an amplified signal, given a particular amplifier and a set transmit power, employs a fast rotating switch to decrease the signal’s peak-to-average ratio before amplification and a narrowband filter to restore the desired spectrum after amplification. In this way, greater linearity in the desired signal is achieved at the expense of temporarily widening its bandwidth. The conversion of a broadband time-multiplexed spectrum to its narrowband non-multiplexed counterpart is
another time-frequency property of bandpass filtering.

When steady-state techniques are inadequate for explaining adverse system performance, for evaluating component parameters, or for improving device capabilities, time-domain and time-frequency methods must be employed.

1.2.1 When to Use Time-Domain and Time-Frequency Analysis

If it is reasonable to assume that the rate-of-change of the signals which propagate through a system is much slower than the response times of all components which form the system, then steady-state techniques are adequate to determine system behavior. Fourier methods are standard for linear systems; Harmonic Balance analysis is common when nonlinearities are present. A transfer-function description of the system is generally sufficient for analysis. Transient behavior is assumed to be negligible and transient effects on system performance are ignored.

As communication systems evolve, however, the transient behavior of system components becomes significant. Demands for increased communication speeds, improved audio transmission quality, and video transmission require higher data rates, while crowding of licensed communications bands encourages greater spectral efficiency and spurs higher band selectivity. As bit rates increase while frequency-band limits become sharper, the rate-of-change of the signals propagating through the system approaches the response times of the system’s slowest components, which are usually the bandpass filters. For these components, the signal’s transition period between steady states becomes comparable to its time within steady states. In this case, transient behavior is no longer negligible and transient effects on system performance may no longer be ignored.

In order to properly analyze such transient behavior, it is necessary to use time-domain and time-frequency techniques. Only by evaluating circuits in the time-domain may transient properties of communication systems be captured, and only by using a combination of time- and frequency-domain techniques may such transient properties be accurately modeled.
1.2.2 Objective: Explain, Model, & Apply Time-Frequency Effects

There are four main aspects of this work:

- To determine the adverse effects that narrowband circuits produce on the communication systems which incorporate them.

- To model resonator energy dynamics in commonly-used filter architectures and provide closed-form solutions for their time-domain responses.

- To develop time-domain techniques for extracting circuit parameters from systems containing narrowband components.

- To provide a time-frequency technique for improving RF transmitter linearity.

1.3 Original Contributions

To achieve the aforementioned objectives, a number of original research initiatives were undertaken. The author’s contributions to the field of RF and microwave engineering are summarized below.

1.3.1 Evaluation of Filter Transients on Frequency-Hopping

Long-tail effects produced by narrowband filters have been identified as a potential source of co-site interference. Recent work focuses on communication systems that use bursty transmissions because many radios operating in ad-hoc environments use frequency-hopping techniques that are susceptible to co-site interference when transients last longer than expected.

Linear transient effects in frequency-hopping communication schemes are evaluated by measuring SNR degradation caused by the presence of a narrowband filter in a typical communications scenario. Nonlinear transients effects are evaluated by measuring IMD under similar conditions.
1.3.2 Pulse Decay Methods for Estimating Filter Parameters

Bandpass filters, as with many other front-end components that once were produced discretely, are now manufactured as parts of integrated assemblies. Unfortunately, a filter’s frequency selectivity is often skewed by variations in manufacturing due to materials and packaging; thus, manual or automated tuning is often necessary. To enable tuning and to avoid disassembling RF front-ends in order to test component functionality, non-destructive methods of probing integrated filters are sought.

A single-port time-domain pulsed probing method for estimating the loaded quality factor of a filter’s outermost resonator is developed. Whereas frequency-domain techniques are unable to characterize individual filter elements, the time-domain technique is able to isolate a single resonator. Also, a method for estimating the bandwidth of the overall filter structure from the same time-domain decay trace is presented.

1.3.3 Lowpass-Prototyped Filter Transient Response Solution

A number of methods for obtaining closed-form solutions for transient waveforms exist in the relevant literature. Most approaches are solutions to differential equations of physically-realizable filter circuits, and each method requires that either the bandpass circuit values be specified, or that the bandpass transfer function be given, in order to solve for the time-domain output expression. These solutions become increasingly complex for filters of higher order. It is possible, however, to greatly simplify the analysis for filters designed from lowpass prototypes.

An analytical method for determining the transient response of a bandpass filter from its lowpass transient response is developed. Time- and frequency-scaling relationships are used to reduce the complexity of the full differential-equation solution of the bandpass circuit by a factor of two.

1.3.4 Method for Measuring IM Distortion by Switched Tones

When a switched-tone signal is applied to the input of a bandpass filter, a two-tone interference pattern may be produced at the filter output for certain combinations of signal frequencies and switching speeds. Although this pattern can be detrimental to
communications hardware because intermodulation content is created when multitones are present at a nonlinearity, this filter property is advantageous because it allows an amplifier to be tested for intermodulation distortion using a single-frequency source.

A technique for measuring the third-order intercept-point (IP3) of an RF amplifier using switched-tone probes is presented. Conditions are given for choosing filter parameters and tone frequencies which enable distortion measurements to be made using a single synthesizer instead of the standard dual-synthesizer test-bed.

1.3.5 One-Port Nonlinear Method for Filter Passband Extraction

The two-tone interference pattern may also be used to characterize the filter itself. Most techniques for characterizing resonant circuits assume that the structure simplifies to a single, linear resonator in the frequency band of interest [6]. These techniques are insufficient, however, when the circuit contains multiple coupled resonators with overlapping frequency bands. A nonlinear method for determining a filter’s passband from a single input port, which may be extended to any number of coupled resonators, is presented.

1.3.6 Transmission-Line Separation of Coupled Resonators

The cause of long-tail transients in narrowband filters is related to filter structure and the energy interactions that take place within that structure. To describe how filter components affect the time variation of energy flow within — and out of — the overall structure, a circuit technique for dissociating wave transmissions and reflections within a filter is sought.

A method for capturing the dynamics of reactive energy in coupled-resonator filters is developed. The transmission-line expansion technique provides a visualization of interactions between reactive elements within a bandpass Chebyshev architecture.

1.3.7 Linear Amplification by Time-Multiplexed Spectrum (LITMUS)

Intermodulation and harmonic distortion result when amplitude modulation (AM), created by a sum of sinusoids, interacts with the nonlinear characteristics of an RF circuit.
LITMUS prevents interaction between the AM and nonlinearity by applying only one sinusoid at a time to the nonlinear circuit. The spectral components are multiplexed in time, using natural sampling, at a rate much faster than the symbol rate of the information signal. The desired amplified signal is completely recovered at the output using a bandpass filter to remove sampling aliases generated during the time-multiplexing process. Theoretically, LITMUS completely suppresses IMD about the desired signal when an ideal multiplexing switch is used; however, practical switching bandwidths limit experimentally-observed third-order intermodulation distortion (IM3) suppression to a range of 8.8 to 22.7 dB depending on the bandwidth available for the time-multiplexed signal.

1.4 Dissertation Outline

Chapter 2 of this dissertation presents a literature review of time-frequency concepts, narrowband transient analyses, nonlinearities in filter circuits, and a brief discussion of multitone linearization. Several frequency-domain circuit parameters are compared against their time-domain analogues. A number of popular time-frequency analysis techniques are reviewed. The most recent research on narrowband transients and time-domain resonant-circuit parameter extraction is summarized.

Chapter 3 discusses linear transient distortion in wireless communication systems. Pulsed and switched-tone experiments are conducted to show that, as signal transition times approach the response times of system components, it is possible for communication signals to overlap when there are differences in signal propagation times through narrowband components, leading to SNR degradation. A typical frequency-hopping scenario is presented as a case study. Coupled-resonator energy dynamics are discussed and the transmission-line resonator separation method illustrates the interactions between resonators which cause intersymbol interference.

Chapter 4 explores linear metrology of bandpass RF components. The definition of quality factor is shown to be consistent in the time and frequency domains for a single-resonator structure. Time-domain methods for extracting the quality factor of a single
resonator, as well as the bandwidth of the entire structure, are presented. A short-pulse technique for S-parameter extraction is applied to bandpass filters.

Chapter 5 adds nonlinearity to the resonant structures. A method for generating multisines from switched-tone signals is presented, and the multisine signal is used to (a) measure the IP3 of an amplifier and (b) extract the passband of a filter cascaded with an amplifier. A filter-amplifier cascade is found to generate IMD. Another frequency-hopping case study, which demonstrates how frequency-switching in one communications band may interfere with a neighboring band, is presented.

Chapter 6 presents LITMUS as a new high-power linear multitone generation technique. The concept behind LITMUS is explained using a frequency-domain representation of a digitally modulated signal. The theory of LITMUS and experimental results verifying its distortion reduction are presented.

Chapter 7 contains a summary of the research performed and lists the significant results of this work.

1.5 Published Works

1.5.1 Journal papers


1.5.2 Conference papers


1.6 Unpublished Works


2

Time-Frequency Concepts and Prior Research

2.1 Introduction

Before discussing current research on time-frequency effects, it is necessary to present background information to place this dissertation in its proper context. Several mathematical quantities must be defined, fundamental differences between steady-state and time-domain analysis must be explained, and a literature review of the most relevant time-frequency work must be performed.

Section 2.2 defines three important terms which are used throughout this dissertation — frequency, delay, and quality factor — and presents the associated steady-state and transient measure for each. Section 2.3 reviews three time-frequency analysis techniques which are used in later chapters. Sections 2.4 to 2.7 recount prior research performed on narrowband transient distortion, resonant circuit parameter extraction, nonlinearities in narrowband circuits, and multitone amplification. Section 2.8 summarizes the lessons learned from this chapter.
2.2 Important Quantities and their Regions of Applicability

Several mathematical quantities must be reviewed before they can be appropriately applied to time-frequency analysis. These are frequency, linear delay, and quality factor.

2.2.1 Frequency: Fourier vs. Instantaneous

Frequency is defined as the repetition rate of a signal, which need not be sinusoidal [7]. Perhaps the simplest periodic signal is a single sinusoid of the form

\[ x(t) = A \cos(\omega t + \phi) . \]  

(2.1)

Its frequency, in radians per second, is \( \omega \). In general, however, any periodic signal \( x(t) \) can be written as a collection of sinusoids:

\[ x(t) = A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2) + ... + A_n \cos(\omega_n t + \phi_n) , \]

(2.2)

within which there exist a collection of frequencies, \( \omega_1...\omega_n \), which denote the rates of change of each of the component signals that make up \( x(t) \).

When signals are assumed to be constant or periodic for all time, whether they are made up of one or many frequencies, they are said to be in steady-state. The standard continuous-time Fourier Transform given by [8]

\[ X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} \, dt \]

(2.3)

separates \( x(t) \) into complex sinusoids. The value of \( X \) at a particular frequency \( \omega \) equals the magnitude and phase of the component sinusoid with frequency \( \omega \) contained within the signal \( x(t) \).

In steady state, the magnitude and phase values that the Fourier Transform assigns
to each sinusoidal component are intuitive. For signals such as \( x(t) \) given by \( (2.2) \), the magnitudes of each component are \( A_1 \ldots A_n \) and the phases are \( \phi_1 \ldots \phi_n \). These values exactly match the time-domain description of the signal because (a) the components exist for all time and (b) the basis functions of the Fourier Transform are complex exponentials that exist for all time.

When signals are not periodic for all time, which is the case for all practical signals, the transformation between the time-domain and frequency-domain descriptions is less well-defined: Are any signals truly periodic? If a signal is not periodic, is it possible to define a frequency, or set of frequencies, for the signal?

One way to bridge the gap between steady-state and transient descriptions of signals which are not periodic for all time is the concept of instantaneous frequency, defined as

\[
\omega(t) = \frac{d\phi}{dt},
\tag{2.4}
\]

which states that the frequency of a signal is equal to the instantaneous rate of change of its phase. The signal to be analyzed is assumed to be of the form given by \( (2.1) \); the signal is assigned a single frequency value at each point in time.

Although this description is simple, it is a vastly different approach for describing signals from traditional Fourier methods. This difference is highlighted by analyzing a linear chirp, a signal which is commonly used in radar systems:

\[
x(t) = \sin \left( 2\pi \left( f_0 + \frac{k}{2} t \right) t \right).
\tag{2.5}
\]

An example with \( f_0 = 500 \text{ MHz} \) and \( k = 500 \text{ MHz/ns} \) is plotted in Fig. 2.1 for \( t = 0 \) to \( 10 \text{ ns} \). With constant amplitude, the signal changes frequency from \( 500 \text{ MHz} \) to \( 5.5 \text{ GHz} \) over \( 10 \text{ ns} \). The Fourier Transform of this signal, observed between DC and 6 GHz, is plotted in Fig. 2.2(a), and the instantaneous frequency of the chirp signal is plotted in
Fig. 2.2(b).

In this case, the Fourier description does not capture the time-varying nature of the chirp. A plateau of spectral content between 500 MHz and 5.5 GHz, with an average level of $x(t) = 0.14$ and ripples of approximately 0.025 in magnitude. It is unclear from the Fourier Transform whether the signal contains components between 500 MHz and 5.5 GHz that are always present at a magnitude given by the transform, or if the signal merely spends a fraction of its time at each intermediate frequency.

The instantaneous-frequency description, on the other hand, captures the time-varying nature of the chirp. Although no amplitude information is given by (2.4), the plot follows a linear pattern as expected:

$$\phi(t) = 2\pi \left( f_0 + \frac{k}{2} t \right) = 2\pi \left( f_0 t + \frac{k}{2} t^2 \right)$$ (2.6)

$$\omega(t) = \frac{d}{dt} 2\pi \left( f_0 t + \frac{k}{2} t^2 \right) = 2\pi (f_0 + kt)$$ (2.7)

$$f(t) = f_0 + kt$$ (2.8)

This example demonstrates that the Fourier Transform, although a powerful steady-state analysis tool, is sometimes inadequate for capturing the true nature of time-varying signals. Instantaneous frequency is one alternative way of looking at time-varying signals. Additional analysis methods are presented in Section 2.3.

### 2.2.2 Linear Delay: Group vs. Average

Measurements of time delay through circuit elements are central to time-frequency analysis. One such measurement is group delay. For a circuit element whose steady-state
Figure 2.1: Linear chirp example: time-domain signal. The frequency is linearly increasing between $t = 0$ and $t = 10$ ns.

Figure 2.2: Linear chirp example: (a) frequency domain — Fourier Transform, (b) Instantaneous Frequency vs. Time. The time-variation of the signal is not obvious from the Fourier Transform view. Plotting instantaneous frequency reveals the true nature of the signal.
The phase function is given by $\varphi(\omega)$, its group delay $\tau$ is computed as [8]

$$
\tau(\omega) = -\frac{d}{d\omega} \{ \varphi(\omega) \} . \tag{2.9}
$$

The group delay characteristic for a fifth-order bandpass Chebyshev filter, commonly used in Family Radio Service (FRS) radios, is given in Fig. 2.3.

If the frequency content of an input to such a circuit element is sufficiently narrow, group delay may be considered constant over the signal bandwidth. With constant group delay $\tau_0$, the phase function of the system is approximated by

$$
\varphi(\omega) \approx -\int \tau_0 \, dt = -\phi_0 - \omega \tau_0 . \tag{2.10}
$$

Let the incoming signal be denoted $X(\omega)$, and the amplitude transfer function of the component be denoted $|H(\omega)|$. The output from the component when $X(\omega)$ is applied is

$$
Y(\omega) = X(\omega) \ |H(\omega)| \ e^{-j\phi_0} e^{-j\omega\tau_0} , \tag{2.11}
$$
which amounts to a scaling of the amplitude of the incoming signal by $|H(\omega)|$, a phase shift by $-\phi_0$, and a time delay of $\tau_0$. This time delay is the group delay, the common travel time through a system for a small grouping of input frequencies. For the filter example of Fig. 2.3, a packet of frequencies near the center of the passband would take approximately 186 ns to travel from the filter input to the filter output.

If the frequency content of an input to a circuit element is not sufficiently narrow, however, group delay cannot be assumed constant over the signal bandwidth. This is the case with pulsed RF signals whose pulse frequencies approach the system’s communications bandwidth. This condition can occur in a fast-frequency-hopping scheme. An example of a pulsed signal whose travel time cannot be estimated using group delay is given in Fig. 2.4.

Although most of the signal energy is concentrated within a seemingly narrow bandwidth around the fundamental tone, a significant portion of the signal energy is spread beyond this narrow band. (Here, a band of 15 MHz around 465 MHz is required to encapsulate 98% of the signal energy.) With respect to the filter of Fig. 2.3, the group delay cannot be considered constant over the signal bandwidth. Another method for determining linear delay is needed.

An alternative method for determining travel time is average delay. If $x(t)$ is the input signal to a linear component and $y(t)$ is the output signal when $x(t)$ is applied, an
error function can be formed [9]:

\[ e(t, \tau) = y(t) - x(t - \tau) \quad (2.12) \]

The integral of this error squared is

\[ \varepsilon(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} e^2(t, \tau) \, dt \quad (2.13) \]

where \( \tau \) is the delay of the signal.\(^1\) The value of \( \tau \) which minimizes \( \varepsilon(\tau) \) is the average delay. Unlike group delay, which is solely a property of the device and not of the applied signal, average delay depends upon both the transfer function of the device and the input waveform. Whereas the application of group delay is limited to narrowband signals, average delay can be extended to signals of arbitrary bandwidth.

**Settling Time**

Another time-domain quantity that is often used in the context of linear delay is *settling time*, \( t_s \), which is defined as the time required for the output of a circuit to settle within a given percentage of its steady-state amplitude upon application (or change) of a steady-state pulse at its input. Common percentages for this measure are \( 1/e (37\%) \), \( 1/e^3 (5\%) \), and \( 1/e^5 (1\%) \).

Strictly speaking, settling time is not a measure of linear delay because a change of the signal at the output of a lumped-element circuit may appear (almost) instantaneously after a change in the signal at the input of the circuit, while the time required to achieve steady-state is on the order of \( 1/B \) where \( B \) is the circuit’s bandwidth. In other words, settling time cannot discern between a pure-time-delay/wide-bandwidth combination (where no change in the output appears at until after the time delay, some signal appears immediately after the delay, and the signal settles shortly thereafter) and a narrow-bandwidth

\(^1\)The notation of [9] has been retained for clarity.
circuit (where some signal appears immediately after the input is changed but requires approximately $1/B$ to achieve steady-state).

Like average delay, settling time is dependent upon both the circuit and the frequency of the applied signal. This property is illustrated in Chapter 3.

### 2.2.3 Quality Factor: Fractional Bandwidth vs. Energy Decay

Another concept that spans the time and frequency domains is quality factor, $Q$. Quality factor is most relevant when discussing resonant circuits. $Q$ is a measure of the energy storage capability of a resonant structure. In a circuit, it relates the average energy stored in electric and/or magnetic fields to the energy dissipated by the system, scaled by the circuit’s resonant frequency [10]:

$$Q = \frac{\omega \text{(average energy stored)}}{\text{(energy loss / second)}}. \quad (2.14)$$

$Q$ can range from 0 — representing a circuit which stores no energy, up to $\infty$ — representing a resonant circuit which is lossless.

$Q$ is most commonly calculated in the frequency domain as the inverse of the resonant circuit’s half-power percentage bandwidth $\gamma$,

$$Q = \frac{\omega_0}{\Delta \omega} = \frac{1}{\gamma} \quad (2.15)$$

where $\Delta \omega$ is the difference between the upper and lower corner frequencies and $\omega_0$ is the center frequency of the filter. For first- and second-order circuits, the value given by (2.15) is exact, but for higher-order circuits, the value given by (2.15) is an approximation. Using this fractional-bandwidth relationship, it is straightforward to determine the quality factor of a bandpass system from scattering parameters.

Quality factor is also defined in the time domain for low-order resonant circuits. It is calculated as $2\pi$ times the number of RF cycles required for the system’s stored energy
to drop to $1/e$ of its initial value, or [11]

$$Q = 2\pi \frac{\tau}{T_0} = \omega_0 \tau$$

(2.16)

where $\tau$ is the time constant of the energy decay and $T_0$ is the period of the RF excitation at the circuit’s resonant frequency.

A misunderstanding arises when using the fractional-bandwidth relationship given by (2.15) in order to approximate the settling time of the resonant circuit by (2.16). It should be noted that the time-domain definition is strictly valid for first- and second-order resonant circuits only. For higher-order resonant structures, e.g. circuits containing three or more reactive elements, the expression in (2.15) is an approximation which worsens as the number of reactive elements in the circuit increases. The extent of the error in this approximation is evident when applying RF pulses to narrowband filters. This error is addressed in greater detail in Chapter 3.

### 2.2.4 Summary

In this section, a number of time-frequency quantities were defined. The term frequency appears in every chapter of this work. Linear delay is most relevant in Chapter 3 when discussing the transit time of pulses through bandpass filters and in Chapter 4 when discussing the extraction of resonant circuit parameters from pulse-decay waveforms. The quality factor of a coupled-resonator structure is discussed in Chapter 3, while the $Q$ of a single resonator is emphasized in Chapter 4. Chapters 5 and 6 rely less on this terminology and more on nonlinear circuit concepts.

### 2.3 Review of Analysis Techniques

In addition to time-frequency definitions, several methods for analyzing signals across the time and frequency domains should be reviewed. Three such techniques are discussed in this section.
2.3.1 Short-Time Fourier Transform

The Short-Time Fourier Transform (STFT) is a modification of the Fourier Transform that is used for finite time-series data. Multiplying the original signal by a windowing function isolates a portion of the signal to be broken into Fourier components [12]:

\[
X(\tau, \omega) = \int_{-\infty}^{+\infty} x(t) w(t - \tau) e^{-j\omega t} dt
\]  

(2.17)

where \( w(t) \) is a windowing function centered on the time period over which the STFT is to transform the time-series data. The variable \( \tau \) is used to shift the window in time. An example of the STFT is given in Figs. 2.5-2.7. The signal is written

\[
x(t) = \begin{cases} 
1 - e^{-t/100 \text{ ns}} \cos \left(2\pi \cdot 888 \cdot 10^6 t\right) & 0 \leq t \leq 1 \mu s \\
\left[ e^{-(t-1 \mu s)} \right] \cos \left(2\pi \cdot 888 \cdot 10^6 t\right) + \left[ 1 - e^{-(t-2 \mu s)/50 \text{ ns}} \right] \sin \left(2\pi \cdot 900 \cdot 10^6 t\right) & 1 \mu s \leq t \leq 2 \mu s \\
\left[ e^{-(t-2 \mu s)/50 \text{ ns}} \right] \sin \left(2\pi \cdot 900 \cdot 10^6 t\right) & 2 \mu s \leq t \leq 3 \mu s.
\end{cases}
\]  

(2.18)

The signal \( x(t) \) consists of two sinusoids which are active at two different times. The first sinusoid at 888 MHz ramps to an amplitude of 1, following a first-order exponential rise with a time constant of 100 ns. This sinusoid ramps back down to zero after 1 \( \mu s \). As the first sinusoid dissipates, the second at 900 MHz ramps up to an amplitude of 1. This sinusoid reaches its maximum value at a faster rate, a time constant of 50 ns. It ramps back down after another 1 \( \mu s \).

The STFT of \( x(t) \) without any window, which is the standard Fourier Transform, is shown in Fig. 2.6. The STFT of this signal, using a 100-ns rectangular window given by

\[
w(t) = \begin{cases} 
1 & \tau - 50 \text{ ns} \leq t \leq \tau + 50 \text{ ns} \\
0 & \text{otherwise}
\end{cases}
\]  

(2.19)
Figure 2.5: Short-Time Fourier Transform example: signal given by (2.18). The signal ramps up at one frequency, ramps down while ramping up at a second frequency, and ramps down at the second frequency.

Figure 2.6: Short-Time Fourier Transform of the signal in Fig. 2.5: no windowing. Both frequencies are present in this view.

for several different values of $\tau$, is shown in Fig. 2.7.

The Fourier Transform of Fig. 2.6 reveals that the signal contains frequency content concentrated at 888 MHz and 900 MHz. It accurately extracts the signal’s underlying sinusoids, but it does not show that each sinusoid is present for only a fraction of the data collection period. Performing the STFT on the signal using different values of the delay variable $\tau$, however, reveals this information.

When $\tau$ is near to the start of the signal, as is the case in Fig. 2.7(a), the STFT displays a single peak at 888 MHz. When $\tau$ is near to the end of the signal, as with Fig. 2.7(c), the STFT displays a single peak at 900 MHz. Using intermediate values of $\tau$
Figure 2.7: Short-Time Fourier Transform example; rectangular window: (a) $\tau = 0.5 \mu s$, (b) $\tau = 1.0 \mu s$, (c) $\tau = 1.5 \mu s$. The time-varying nature of the signal is captured by windowing at a shorter interval than the data length and moving the window from the start to the end of the signal.
in the neighborhood of the transition time between the two sinusoids, as in Fig. 2.7(b), the STFT displays two peaks, showing that as time progresses, the presence of the first sinusoid is diminished while the presence of the second sinusoid is augmented.

Whereas the full Fourier Transform was unable to capture the time-varying nature of the signal, the Short-Time Fourier Transform (with appropriate windowing) was able to extract the frequency transitions. As evidenced by this example, the STFT is useful for analyzing signals that are sinusoidal over short durations. If the underlying frequency content is not sinusoidal, however, a more general transform is needed to extract time-frequency information.

2.3.2 Wavelet Transform

One such generalized transform which is not restricted to sinusoidal basis functions is the Wavelet Transform. It is written \[ W(\kappa, \tau) = \int_{-\infty}^{+\infty} x(t) \psi(\kappa, \tau, t) \, dt \] (2.20)

where \( \psi \) is the wavelet (the basis function of the transform), \( \tau \) is the position of the wavelet in time, and \( \kappa \) is the scale of the wavelet.

The Wavelet Transform can be thought of as a correlation of a recorded signal, \( x(t) \), with scalings of a pre-defined waveform, \( \psi(t) \). The higher the value of \( W \), the higher the degree of similarity exists between \( x(t) \) and \( \psi(t) \) for a particular scaling \( \kappa \) at a particular location in time \( \tau \). The wavelet must be zero-mean in order for the integral to assume a minimal value when \( x(t) \) and \( \psi(t) \) are uncorrelated.

Although wavelets are similar to sinusoids in that both are zero-mean, wavelets are unlike the sinusoids of Fourier analysis because they have limited duration. The sinusoids of Fourier analysis exist for all time (or over the entire window for the STFT). An example wavelet is shown in Fig. 2.8.

Fourier analysis consists of breaking a signal up into sine waves of various frequencies; wavelet analysis breaks a signal into shifted and scaled versions of the original wavelet.
Wavelet analysis is particularly useful for discerning patterns in a signal which follow a zero-mean shape that is not necessarily sinusoidal.

One such example is wavelet analysis of a reflected wideband linear chirp; such a signal may be found in radar applications. Assume that the received signal consists of some portion of the original transmitted chirp, the original chirp changed frequency linearly at a rate of 500 MHz/µs, the delay of the reflection in time is unknown, and the received signal is buried in white Gaussian noise. Such a signal is shown in Fig. 2.9.

To extract a chirp, the most appropriate wavelet is a chirp. The portion of the chirp to be extracted is from 2 GHz to 4 GHz; thus, the chirp wavelet is nonzero for 2 µs with a starting frequency of 2 GHz and the same chirp rate as the transmitted signal:

Figure 2.8: A Morlet wavelet.

Figure 2.9: Linear chirp from 1 to 8 GHz over 14 µs, starting at \( t = 6 \) µs, with Gaussian white noise; \( |x(t)| = 0.01, \sigma_n = 0.02, \mu_n = 0 \). The chirp signal is not visible within the noise.
Figure 2.10: Location of the 2-to-4 GHz linear chirp, extracted from Fig. 2.9 using chirp wavelet; $\tau = 8 \mu s$. The location of the signal is determined even though the waveform is buried in noise.

$$\psi(\tau, t) = \begin{cases} \sin \left[ 2\pi \left( f_0 + \frac{k}{2} (t - \tau) \right) (t - \tau) \right] & \tau \leq t \leq \tau + T \\ 0 & \text{otherwise} \end{cases} (2.21)$$

$$f_0 = 2 \cdot 10^9 \text{ Hz} \quad k = 500 \cdot 10^6 \text{ Hz} \quad T = 2 \mu s \quad (2.22)$$

When (2.20) is applied to the waveform of Fig. 2.9, the result is Fig. 2.10, which shows a peak at 8 $\mu$s, exactly where the 2-to-4 GHz chirp occurs in the noisy waveform.

Here the parameter $\kappa$ is not used because only one time-scaling of $\psi(\tau, t)$ is necessary to identify the time index of the particular waveform. If the chirp rate were unknown, $\psi(\tau, t)$ could be scaled and $W(\tau)$ could be plotted against a number of time-scalings $\kappa$ in order to extract the chirp rate.

This example shows that the Wavelet Transform is another tool that can be used to capture the time-varying nature of signals. If the signals under scrutiny are believed to follow a regular pattern, the pattern can be turned into a wavelet — the basis function of the transform. The pattern need only be zero-mean; it need not be sinusoidal.
2.3.3 Lowpass Prototyping

Another tool that is useful for analyzing time-frequency properties is lowpass prototyping. Lowpass prototyping is particularly useful in the design and analysis of filters. It is generally discussed in the frequency domain, but its usefulness extends to the time-domain as well.

Frequency Transformation

Typically, bandpass filter design begins with the filter’s lowpass prototype, a network of series inductances and shunt capacitances, realizing specifications of rolloff, passband ripple, and 1-Ω resistive terminations. A frequency transformation is then used to center the filter’s passband at the operating frequency and to accommodate a change in resistance at the filter’s input and output ports. A fifth-order lumped-element Chebyshev filter and its lowpass equivalent are shown in Fig. 4.3(a) and 4.3(b), respectively.

The bandpass transformation substitutes a series LC combination for each $L_N$ and a parallel LC combination for each $C_N$. The relationship between the first resonator of the
bandpass filter and the first capacitor of its lowpass prototype is as follows [9]:

\[
L_1 = R \frac{\gamma}{C_{N1} \omega_0} \quad C_1 = \frac{C_{N1}}{R \omega_0 \gamma} \quad \omega_0 = \frac{1}{\sqrt{L_1 C_1}} \tag{2.23}
\]

where \( R \) is the resistance of the input and output terminations, and \( \omega_0 \) is the center frequency of the resonator as well as that of the overall filter.

**Time Scaling**

It is also noted in [9] that a simple time-scaling relationship exists between (a) the envelope of the transient response of a bandpass filter, when excited by a step-modulated waveform with carrier frequency equal to the filter’s center frequency, and (b) the transient response of the filter’s lowpass equivalent, when excited by a step change in voltage. The envelope of the voltage wave across the input resistor in the bandpass case \( V_R \) is a time-scaled version of the same voltage wave in the lowpass case \( V_1 \):

\[
t = \alpha t_N \quad \alpha = \frac{2}{\omega_0 \gamma} \tag{2.24}
\]

where \( t \) is the time index of the bandpass response and \( t_N \) is the time index of the lowpass equivalent response.

This time-scaling relationship is useful for determining the transient response of a bandpass circuit without solving the circuit’s differential equations directly. If the Laplace-domain transfer function of the system is known, it is possible to use inverse Laplace Transforms in order to solve for the output of the system for any input stimulus. If the transfer function is unknown, it may still be possible to solve for the transient response if certain assumptions can be made about its equivalent lowpass properties. A more general time-scaling method for determining the transient response of a bandpass system, created from lowpass prototyping, to any pulsed RF stimulus, is derived in Chapter 3.
2.3.4 Summary

In this section, several time-frequency analysis tools — which will be used to present a number of results in this dissertation — were presented. The Short-Time Fourier Transform is used to generate all of the spectral plots presented in Chapters 4, 5, and 6. Lowpass prototyping is used in Chapter 3 to derive closed-form time-domain expressions for bandpass filter transients and in Chapter 4 to extract the $Q$ of a single resonator from a coupled-resonator structure.

2.4 Transients in Linear Narrowband Systems

In order to understand how time-frequency effects have impacted communication systems and the extent to which they have been studied, it is necessary to review prior research on narrowband transients.

2.4.1 Discovery of Transients, Quasi-Stationary Behavior

The study of transient effects in narrowband systems can be traced back at least as far as the introduction of frequency-modulation (FM) radio. In 1937, John Carson and Thornton Fry of Bell Laboratories were the first to give a mathematical treatment of distortion in frequency modulation systems [14]. Shortly after the invention of FM, they sought to compare the performance of the new scheme against amplitude modulation (AM). They argued that FM offered a better signal-to-noise ratio, assuming FM could operate over a wider bandwidth than that of traditional AM. Carson and Fry did note the existence of a transient distortion term, but neglected it in favor of analyzing the quasi-stationary signal.

Much of the analysis of frequency-varying signals since Carson and Fry’s time has focused on the quasi-stationary assumption, which states that the transient behavior of a system whose input changes frequency can be approximated by smooth transitions between steady states. Let $x(t)$ be a signal whose frequency changes with time, $\omega(t)$, which is applied to a system with steady-state transfer function $H(\omega)$:

$$ x(t) = A(t) \cos[\omega(t) \cdot t + \phi_0] \quad (2.25) $$
\[ H(\omega) = |H(\omega)| e^{j\varphi(\omega)} \] (2.26)

The quasi-stationary output from the system \( y(t) \) is then

\[ y(t) = |H(\omega(t))| A(t) \cos [\omega(t) \cdot t + \phi_0 + \varphi(\omega(t))] . \] (2.27)

The transient response is scaled in amplitude by \( H(\omega) \) and phase-shifted by \( \varphi(\omega) \) at each point in time, depending upon the input’s instantaneous frequency \( \omega(t) \).

A system operates in the quasi-stationary regime as long as the input signal’s frequency varies slowly with respect to the system’s response time. Fig. 2.12 shows when this is the case and when it is not. Here a linear chirp is applied to a bandpass filter. In Fig. 2.12(a), the chirp rate is much slower than the filter’s transient response. As the input frequency changes, the magnitude of the output waveform changes with the steady-state filter response; the chirp traces out the filter’s frequency-domain transmission characteristic. In Fig. 2.12(b), however, the chirp rate approaches the transient response time of the filter. The output no longer traces out the transmission characteristic because the filter has not been given enough time to assume steady state as the input changes frequency. Because the output response is no longer a smooth transition between steady states, the quasi-stationary assumption is no longer valid.

Concurrent with Carson and Fry’s analysis, Hans Roder, a radio engineer at General Electric, was concerned with the effect of tuned circuits on the reproduction of FM signals [15]. Instead of making the quasi-stationary assumption and using steady-state analysis, he used Bessel functions to represent oscillatory signal components propagating through frequency-to-amplitude converters and tuned amplifiers. Roder determined that, in order to reproduce an audio signal without introducing distortion, a frequency-to-amplitude converter must have linear amplitude and phase characteristics, and an amplifier must have a flat amplitude characteristic and linear phase characteristic over its full operational bandwidth. Unbeknownst to Roder, his analysis implied that a filter would require a constant group delay across its passband for distortionless transmission.

In 1942, Hans Salinger looked at filter characteristics and their impact on AM and FM radio and television applications [16]. Using complex analysis and assuming ideal
Figure 2.12: Output from applying a $-20$ dBm linear chirp to an ideal 3rd-order Chebyshev filter with $f_0 = 1$ GHz and $B = 30$ MHz; chirp from 950 MHz to 1050 MHz: (a) chirp rate = 20 MHz/µs, (b) chirp rate = 400 MHz/µs. At the slower chirp rate, the filter response is approximately a superposition of steady-state responses as the frequency changes, i.e. the response is quasi-stationary. At the higher chirp rate, the filter response can no longer be approximated as a sum of steady states, i.e. it is no longer quasi-stationary.
transmission and reflection properties for a filter’s passband and stopband, Salinger found that the instantaneous frequency response of a bandpass filter slowed with a decrease in filter bandwidth. He noted that the settling times of both AM and FM transients were similar as long as any input frequency swings were well within the filter’s nominal passband. Salinger recommended that a filter’s passband be chosen to enclosed the maximum input swing symmetrically. He also performed back-of-the-envelope calculations to determine the minimum operational bandwidth for FM radio, AM video with FM synchronization, and FM video.

Donald Hess, an associate professor at the Polytechnic Institute of Brooklyn, followed up on Carson and Fry’s quasi-stationary analysis [17]. Hess wanted to know the bandwidth required of single-tuned Butterworth and Chebyshev filters to transmit FM signals with a specified amount of phase distortion. He derived a tighter error bound than Stumpers, and calculated that the experimental requirement of 225 kHz bandwidth for clean FM transmission matched the theoretical requirement for less than 1% phase distortion.

### 2.4.2 Circuit Solutions

Circuit solutions addressing transient distortion began to appear in the late 1940s. Gordon Tucker, a communications researcher for the British Post Office Research in London, presented a mathematical treatment of a commonly-used 6-element (i.e. non-ideal) filter excited by single-tone pulses [18, 19]. Tucker used differential operators (a precursor to Laplace Transforms) to find analytical expressions for the build-up and decay characteristics of the filter’s transmissions at midband. He also provided the first oscilloscope traces of a bandpass filter’s pulse transmissions. Charles Eaglesfield of the Mullard Radio Valve Company in London picked up on Tucker’s work and gave an analytical form for the amplitude of the transmitted pulses, assuming a narrowband case [20, 21].

At approximately the same time as the Tucker and Eaglesfield analysis was formulated, two physicists at Philips Research Labs were studying the production of harmonics by frequency detectors. Balthasar van der Pol determined that, in the quasi-static regime, an FM signal’s distortion is not dependent upon a circuit’s amplitude characteristics; it is completely determined by the circuit’s phase properties [22]. Van der Pol’s student, Frans Stumpers, calculated percent harmonic distortion using Fourier Series, and determined error
bounds for Carson and Fry’s quasi-static solution [23].

Several years later, in 1951, William Hatton at the Massachusetts Institute of Technology was considering FM as a way to transmit video and wanted to compare its performance against AM [24]. Hatton analyzed the response of a parallel RLC circuit to a single discrete frequency step by solving the circuit’s differential equation directly. Using settling time and overshoot of the output’s instantaneous frequency as metrics, he found that FM and AM perform similarly when jumps in the input frequency are small and near the center of a filter’s passband, but FM outperforms AM for larger input frequency swings. He saw that overshoot in the amplitude response could be as high as 10% above steady-state values, even if the operating frequencies are kept within the passband.

2.4.3 Laplace Methods

Laplace Transforms were applied to circuit transients in the 1950s. While engaged in radar-system research at the Hughes Aircraft Company in 1954, R. E. McCoy analyzed FM distortion in stagger-tuned amplifiers [25]. He noticed that, when exciting a tuned circuit by a discrete frequency step, that the shape of the output frequency transient was dependent upon (a) the initial frequency of the step, (b) the magnitude of the step, and (c) the direction of the step. He derived a circuit solution for normalized output frequency deviation using the inverse Laplace Transform, which could be generalized to tuned circuits with any number of stages. McCoy provided detailed computations for large frequency steps in single-tuned circuits. He showed how the frequency transient is longer when using more stages and recommended that stagger-tuning not be used if distortion is to be kept to a minimum.

In 1965, Weiner and Leon of Purdue University set out to find a straightforward relationship between the distortion of AM and FM signals and the characteristics of the linear system causing the distortion [26]. Seeking a more precise solution than the error bounds given by Stumpers and Hess, Weiner and Leon used integration by parts to derive a closed-form expression for AM and FM distortion. With their analytic expressions, they computed bounds on the amplitude overshoot as a function of filter bandwidth and frequency deviation, for $n$ coupled single-tuned circuits and for Butterworth filters up to third order. Weiner and Leon provided a library of traces of measured frequency transients for a
single-tuned circuit and a series of frequency steps.

Harry Hartley of the Westinghouse Electric Corporation in Baltimore presented another simplification of the Laplace Transform method in 1966 [27]. He exploited several properties of Laplace Transforms under narrowband assumptions for both the input signal and the system’s transfer function to determine the instantaneous amplitude, phase, and frequency of output transients.

The Laplace Transform of a system’s output $V_o(s)$ is, in general, the product of a transfer function $H(s)$ with an input excitation $V_i(s)$:

$$V_o(s) = V_i(s) H(s).$$  \hfill (2.28)

$V_o(s)$ may be written as the ratio of two polynomials whose roots are the poles and zeros of $V_o(s)$. It is noted in [27] that, for linear narrowband circuits, all zeroes and poles of the system are (a) clustered in a narrow frequency band, (b) are real or complex conjugates of each other, and (c) have either a zero or negative real part. This allows the system’s output to be written

$$V_o(s) = K \frac{\prod (j\omega_0 + \sigma_{0m}) \prod (j\omega_0 - s_{0n}^*) (s - s_{0n})}{\prod (j\omega_0 + \sigma_{pi}) \prod (j\omega_0 - s_{pj}^*) (s - s_{pj})}.$$  \hfill (2.29)

$$v_o(t) = \Re \left[ \frac{1}{\pi j} \int_{c-j(\omega_0-B/2)}^{c+j(\omega_0+B/2)} V_o(s) e^{ts} ds \right].$$  \hfill (2.30)

where $\sigma_{0m}$ are the real zeros of $V_o(s)$, $s_{0n}$ are the complex conjugate zeros, $\sigma_{pi}$ are the real poles, $s_{pj}$ are the complex conjugate poles, $\omega_0$ is the center frequency of the band of interest, and $B$ is the system bandwidth. The integral of (2.30) is Hartley’s Narrowband Laplace Transform. Although Hartley presented no experimental data, he provided a collection of calculated plots for single- and double-tuned filters and noted that the transient
characteristics could impact frequency-shift-keying communication schemes.

Chohan and Fidler at the University of Essex in England went further in assessing the suitability of bandpass filters for use in frequency- and phase-modulated communications [28]. They focused on the instantaneous frequency of the output and used the $s$-domain shifting property of Laplace Transforms to generalize Hartley's instantaneous frequency result to filters of any order and any $Q$ value. They provided simulated data for second-order Butterworth filters.

2.4.4 Summary & Current Research

Transients in narrowband circuits have been observed as early as the 1930s. Differential equations and Laplace methods have been used to determine the extent to which these transients will distort communication systems, but most work has been limited to lower-order systems for mathematical tractability.

Studies of transient distortion have not yet been extended to higher-order bandpass filters, and no data have yet been presented to evaluate the effects of such distortion on frequency-hopped communications for filters of any design. Because higher-order narrowband filters have become so common to wireless device architectures, and because frequency-hopping systems that implement such filters may fail if circuit transients are longer than anticipated, an evaluation of narrowband linear effects produced by contemporary filter designs is performed in Chapter 3 and an evaluation of narrowband nonlinear effects is performed in Chapter 5.

2.5 Extraction of Resonant Circuit Parameters

It is also necessary to review prior research on resonant circuit parameter extraction if narrowband transients and time-frequency analysis are to be used for linear metrology of higher-order bandpass systems.
Figure 2.13: Single-resonator filter circuit simplification: (a) full structure consisting of multiple resonators, (b) single resonator active when $\omega \approx \omega_0$. When the resonant frequencies of each RLC sub-circuit are spaced apart in frequency, the structure may be reduced from (a) to (b) in the neighborhood of any one of the resonant frequencies.

2.5.1 Single-Resonator Assumption

The earliest metrology performed on RF and microwave resonators assumed that any resonant structure with multiple resonant frequencies could be simplified to a single resonator at a single resonant frequency for the band of interest. This concept is illustrated in Fig. 2.13.

Over the full frequency band of operation of the resonant structure, its equivalent circuit may consist of multiple resonators with multiple resonant frequencies, $\omega_0...\omega_n$, as shown in Fig. 2.13(a). In the neighborhood of one of the resonant frequencies, the equivalent circuit consists of a single resonator, as shown in Fig. 2.13(b). The other resonators are approximated by open- or short-circuits. In Fig. 2.13, for example, the parallel LC’s become open-circuits.

2.5.2 Steady-State Methods

A number of frequency-domain methods have been developed in order to indirectly measure the parameters $L_0$, $C_0$, and $R_0$, the coupling factor $\kappa$, or the quality factor $Q$. Both $Q$ and $\kappa$ are functions of the resistive and reactive components of the resonator as well as external loading. $Q_U$ is the “unloaded” quality factor, which is a property of the resonator
alone, while $Q_L$ is the “loaded” quality factor, which takes into account external loading applied to the resonator’s input and/or output terminals. The quality factors $Q_U$ and $Q_L$ and the coupling coefficient $\kappa$ are given by

$$Q_U = \frac{|X|}{R_0} \quad \quad Q_L = \frac{|X|}{R} \quad \quad \kappa = \frac{P_L}{P_0} \quad (2.31)$$

where $X$ is the total reactance of the LC combination, $R$ is the equivalent resistance of the resonator including external loading, $P_L$ is the power dissipated in the external circuit attached to the resonator, and $P_0$ is the power dissipated in the resonator itself.

In 1957, Edward Ginzton of Stanford University introduced several graphical methods for measuring $Q_L$ for the single-resonator circuit [29]. He showed that, for two-port resonant structures, two transmission-mode methods could be used to estimate $Q_L$: note the half-power bandwidth from the plot transmitted-power versus frequency curve and assume the fractional bandwidth relationship given by (2.15), or note the time constant of the decay of energy out of the structure and use (2.16). He also showed that, for one-port structures, two reflective-mode methods could be used to find $Q_U$: find the half-power points on the VSWR versus frequency curve and again use (2.15), or plot the reflection coefficient in the complex plane and use a series of frequency markers along the circle traced out by the resonance. In 1963, Max Sucher and Jerome Fox at the Polytechnic Institute of Brooklyn reviewed Ginzton’s methods and determined under which coupling and loss conditions each type of measurement should be made [30].

Aps Khanna and Yves Garault, focusing on the case of a dielectric resonator coupled to a microstrip line in a bandstop configuration, built on Ginzton’s graphical methods in order to trace loci of quality factors on the Smith Chart for both transmission-mode and reflection-mode loaded and unloaded quality factors [31]. In 1999, Raymond Kwok and Ji-Fuh Liang introduced a mapping function between insertion loss, coupling coefficient, and $Q_U$ to simplify the process of unloaded quality factor measurement by reflection-coefficient techniques [32]. Also in 1999, Darko Kajfez et. al. at the University of Mississippi derived an equation relating the uncertainty in the transmission-mode measurement of $Q_U$ to given
values of insertion loss and bandwidth [33]. They showed that, as a rule-of-thumb, the transmission-mode method is not suitable for resonant cavities that are overcoupled.

**Augmented RLC Single-Resonator Circuit**

In 1984, Kajfez and Eugene Hwan showed how Ginzton’s reflection coefficient measurement for single-ended cavities could be performed on a network analyzer instead of using a slotted line [34]. To account for the parasitic effects introduced by the network analyzer probes, they augmented the RLC circuit structure to include additional energy-storage and loss terms; these are $R_e$ and $jX_e$ as shown in Fig. 2.14.

Kajfez approached the problem from a different direction in 1986 [35]. He and Perry Wheless mapped the parameters of the augmented RLC circuit to five coefficients that trace out a circle in the impedance plane. In doing so, they were able to determine the circuit’s unloaded resonant frequency, unloaded quality factor, coupling, and parasitic energy storage from least-squares fitting of multiple impedance-plane data points.

**Smith Chart: Critical-Points Method**

Also in 1995, Sun and Chao at the Shan Institute of Science and Technology in Taiwan employed Foster forms of equivalent circuits to more accurately plot the impedance of a cavity operating near resonance [36]. They devised the “critical points” method for calculated quality factor — so named for requiring the identification of the critical points of the input reactance versus frequency curve. This method is illustrated in Fig. 2.15 for a parallel RLC resonance.

A small portion of the Smith Chart is shown. The input impedance of a single-port resonator is measured across a frequency sweep. Sweeps that cross through a resonance
Figure 2.15: The Critical-Points method: the input impedance of the resonator is plotted in the vicinity of $\omega_0$; $\omega_1$ and $\omega_2$ are the frequencies corresponding to the maximum and minimum of the imaginary part of $Z_{in}$ near $\omega_0$.

produce closed-loops on the Smith Chart. As the measurement frequency passes through the resonant frequency $\omega_0$, the resistance of the cavity reaches a maximum where the inductive and reactive impedances of the resonator cancel. Here the real part of the impedance is equal to the resonator impedance $R_0$ plus the external loss factor $R_e$, and the imaginary part is equal to the energy storage correction factor $X_e$. For a nearly-undistorted circle, the frequencies $\omega_1$ and $\omega_2$ are the corner frequencies of the resonator and $Q_U$ can be calculated using

$$Q_U = \frac{\omega_0}{|\omega_2 - \omega_1|} .$$

For an oblate circle, a multiplicative correction factor is added to (2.32).

Lye Heng Chua and Dariush Mirshekar-Syahkal at the University of Essex extended Sun and Chao’s work to extract coupling coefficient and loaded resonant frequency from their equivalent circuit forms [37]. They later refined their work to more accurately describe resonators of low quality factor [38].
2.5.3 Time-Frequency Methods

Until recently, the metrology of resonant structures had been performed exclusively in the frequency domain. Time-frequency methods for extracting quality factor first appeared in the 1990s.

Finite-Difference Time Domain

Exploiting advances in computing power, Chi Wang et al. at the Beijing Institute of Technology created discrete models of cavities and dielectric resonators using Yee’s Finite Different Time Domain (FDTD) method [39] in 1995. By running transient simulations and waiting for the non-resonant modes to disappear, they were able to estimate quality factor by translating the time-stepped data to the frequency domain via the Fast Fourier Transform (FFT) and using the aforementioned relationships between quality factor and fractional-bandwidth.

Prony Analysis

To reduce computation time, Jose Pereda et. al. at the University of Cantabria in Spain used Prony analysis instead of the FFT to translate the time-domain FDTD waveforms into frequency-domain data [40]. Prony analysis fits a recorded data-set to a series of damped complex exponentials:

\[ \tilde{x} (t) = \sum_{i=1}^{p} A_i e^{(\alpha_i + j\omega_i)t} \quad (2.33) \]

where \( i \) is an index which denotes each resonant frequency, \( \omega_i \) is the frequency of each resonance, \( A_i \) are the complex amplitudes of the signal at each resonant frequency, \( \alpha_i \) is the damping factor of each resonance, and \( \tilde{x} (t) \) is the least-squares fit of the original recorded data-set to the Prony form.
2.5.4 Summary & Current Research

Steady-state methods for measuring resonant circuit parameters for simple structures, i.e. those which reduce to a single (possibly lossy) resonator in the neighborhood of a set of resonant frequencies, are well-established. Few methods for measuring circuit parameters of more complex resonant structures are not; time-domain and time-frequency methods are even less common.

Chapter 3 presents several new techniques for resonant circuit metrology. Section 4.2 introduces a time-domain method for extracting the loaded quality factor of the first resonator in a chain of resonators by measuring the time constant of the resonator’s energy decay. The technique is based on the Decrement Method introduced by Ginzton [29], which was originally used to estimate the $Q$ of a two-port resonator from its time-domain transmission response. The waveforms from which $Q$ is extracted are similar to those reported by Joel Dunsmore at Hewlett-Packard Co. [41], although the time-domain data presented here are captured directly from a sampling oscilloscope rather than transformed from frequency-domain network analyzer sweeps. The method is limited by available signal generation hardware to narrowband designs; however, it is theoretically applicable to filters of any bandwidth, and it requires only a single measurement port and a single oscilloscope trace recorded at or near the device’s resonance.

Section 4.3 shows how a single pulse-decay trace can be used to estimate the bandwidth of a bandpass structure, and Section 4.4 shows how a set of short-pulse response traces can be used to extract its S-parameters. These methods have the advantage of enabling broadband frequency-domain measurements using a sampling oscilloscope and Fourier Transform algorithm without requiring a spectrum analyzer.

2.6 Nonlinearities in Narrowband Circuits

Narrowband transients and time-frequency concepts are also applicable to the nonlinear metrology of bandpass systems. One of the most relevant system performance metrics, when discussing nonlinear properties of RF and microwave devices, is intermodulation distortion.
2.6.1 Intermodulation Distortion and Filtering

Intermodulation distortion results when any circuit containing a nonlinearity is subjected to multiple simultaneous input frequencies, or “multisines,” whose interactions at the nonlinearity produce harmonics which are integer combinations of the original frequencies. IMD is particularly vexing to radio-frequency front-ends because (a) some of the intermodulation products fall within the intended communications band, (b) new sources of IMD continue to be discovered, and (c) even when IMD does appear and its source can be identified, it is difficult to mitigate.

IMD in bandpass circuits has several causes. Passive filters themselves exhibit weak nonlinearities which result from conductor defects and uneven charge carrier densities [42,43]. Distortion in active filters which use negative-resistance compensation is produced by transistor nonlinearities [44,45]. Another source of IMD has been observed in amplifier circuits that use bandpass filtering, which disappears if the filter is absent. This IMD can be traced to waveform memory retained as stored reactive energy within the filter.

2.6.2 Current Research

Chapter 5 explains how the IMD resulting from this memory effect may be exploited for circuit metrology. One example is the measurement of amplifier nonlinearity. Because linear filtering effects are known to produce multitone interference when excited by switched tones [46], it is possible to perform nonlinear tests requiring multiple simultaneous frequencies with a single switched-tone source. A switched-tone technique for measuring an amplifier’s third-order intercept point (IP3) is provided in Section 5.2.

The memory effect may also be used to characterize the filter itself. Most techniques for characterizing resonant circuits assume that the structure simplifies to a single, linear resonator in the frequency band of interest [6]. These techniques are insufficient, however, when the circuit contains multiple coupled resonators with overlapping frequency bands. The characterization reported in Section 5.3, which determines a filter’s passband from a single input port, is a nonlinear method which may be extended to any number of coupled resonators.

Distortion related to filtering also has implications for communication system performance. The IMD produced by filters alone is known to cause interference in systems with
multiple simultaneous users operating on different frequencies, as in Orthogonal Frequency Division Multiplexing (OFDM) [43]. The IMD produced by a filter-amplifier cascade may also cause interference in systems with multiple users time-multiplexed to different frequencies, as in Frequency-Hopping Spread Spectrum (FHSS) systems such as ad-hoc networks and WiFi™. Section 5.4 extends the work done on co-site interference produced by linear filtering (in Chapter 3) to nonlinear distortion produced by pairing a bandpass filter with an amplifier. Results show that the transmissions which produce IMD need not originate within the intended communications band.

2.7 Linear Multi-Tone Signal Amplification

Another way to exploit filter memory is to use it to linearize high-power AM transmissions.

2.7.1 High-Power Linear AM Transmission

Because all practical amplifiers are nonlinear (i.e. because all amplifiers reach a saturation point beyond which no more power is output from the device and power-in to power-out is no longer a linear relationship), high-power multisines generate IMD when they are amplified directly. Options for linear transmitter designs include the use of average power backoff with linear transmitters or employing other linearization techniques to achieve spectral emissions requirements. These linearization techniques include predistortion correction, Cartesian feedback correction, or feedforward distortion cancellation [47]. Linearization techniques correct the distorted transmitter signal by either predistorting the input signal [3, 4] or by cancelling the distortion at the output of the transmitter [5]. In either case, knowledge of the nonlinearity being corrected is required either in the form of a look up table, trained model, or feedback circuit, or cancellation calibration is necessary to achieve improvement in linearity.
2.7.2 Current Research

Linear Amplification by Time-Multiplexed Spectrum is a signal-processing linearization technique with the unique property of improving linearity in the desired signal without requiring knowledge of the nonlinear circuit or calibration of a cancellation network. The signal is encoded using time-multiplexed sinusoids such that generation of intermodulation distortion is prevented by applying only one sinusoid at a time to the nonlinear circuit. Ideal switch multiplexing of sinusoids completely prevents generation of intermodulation around the desired signal; however, bandwidth and switching limitations result in finite distortion generation around the desired signal. Reduction of intermodulation distortion using time-multiplexed carriers was demonstrated experimentally for an optical cable-television system [48] and more recently for reducing distortion in a very-high-frequency (VHF) amplifier [49]. Chapter 6 provides the theory of distortion reduction for two time-multiplexed sinusoids, supporting measurements for two tones, and additional measurements which demonstrate consistent distortion reduction for greater numbers of multiplexed sinusoids. LITMUS illustrates how the time-frequency properties presented in Chapters 3 through 5 are directly applicable to the improvement of existing wireless communication systems.

2.8 Summary & Conclusions

In this chapter, a number of time-frequency quantities were defined, and several relevant analysis methods were discussed. Differences between frequency-domain and time-domain measures of frequency, linear delay, and quality factor were presented to clarify what is meant by each term in later chapters. The analysis tools are presented early in this work because they are used to simplify the mathematics of the higher-order bandpass systems involved in this study in Chapters 3 and 4 and to generate snapshots of frequency content over finite periods of time in Chapters 4, 5, and 6.

Prior research on narrowband transients, resonant circuit metrology, bandpass nonlinearities, and AM linearization was summarized in order to place current research in its proper historical context. Prior work has focused on transient distortion in low-order resonant circuits; the results of Chapter 3 extend to higher-order circuits and quantify the
linear distortion that may be caused by bandpass filters in frequency-hopping communication schemes. Most resonant circuit measurement techniques are steady-state and cannot be applied to circuits which contain coupled resonators with similar resonant frequencies; the techniques presented in Chapter 4 are time-domain methods and are applicable to such circuits. Prior work on bandpass nonlinearities has focused on nonlinear properties of the filters themselves; Chapter 5 reveals that the filter-amplifier cascade common to wireless communication architectures is another form of bandpass nonlinearity. Most transmitter linearization techniques require prior knowledge of the system nonlinearity for signal correction; the technique presented in Chapter 6, LITMUS, shows that this knowledge is not necessary to improve the linearity of a signal if the signal may be temporarily time-multiplexed.

Having reviewed a number of time-frequency concepts and past research, the remainder of this dissertation is devoted to original work contributed by the author.
3

Linear Transient Distortion in
Narrowband Systems

3.1 Introduction

Traditional RF and microwave design has been performed primarily in the frequency domain, assuming steady-state conditions and following the rule-of-thumb that transients in a circuit last no longer than approximately $1/B$ where $B$ is the narrowest bandwidth of any component in the system. These design rules have generally been sufficient to ensure proper communication system performance as the narrowest-bandwidth component, usually the receiver’s bandpass filter (BPF), has been of low-order. Low-order filters provide band selectivity, and their passband edges are not steep, which means that even for very narrow nominal filter bandwidths, their transfer functions in the frequency domain are spread considerably wider than their nominal values because their amplitude characteristics decay relatively slowly outside the passband.

A filter’s transfer function drops by approximately 20 dB/decade in power, per filter order, on either side of the passband.\textsuperscript{1} Since a first-order filter’s transfer function

\textsuperscript{1}For low- and high-pass filters, the “order” of the filter equals the number of resonant components. For band-pass and band-stop filters, the “order” is the number of resonant pairs in the design.
decays by 20 dB/decade, the $1/B$ rule-of-thumb is valid. As the order of the filter design is increased, however, this rule-of-thumb is less applicable. The shape of the amplitude characteristic of the filtering function becomes more like a rectangle. For narrow bandwidths, the filter function is approximately an impulse. The time-domain analogue of such a filtering function (i.e. its impulse response) approaches infinite length. By linear system properties, the output of a narrowband filter of very high order equals the input signal convolved with an impulse response which is much longer than $1/B$.

This chapter addresses the distortion introduced into a wireless communication system by high frequency selectivity: narrow bandwidths and high filter orders. Section 3.2 provides simulated and experimental evidence of the longer-than-expected filter transients and explains how these transients derive from resonator energy interactions. Section 3.2.3 introduces a method for separating resonator transmissions and reflections for individual pulses in simulation. Section 3.3 shows how back-to-back pulses transmitted through a filter may produce transient multitone interference patterns. Section 3.4 provides a method for analytically determining the time-domain response of a narrowband filter by solving simultaneous equations with multiplicity equal to the filter order (instead of twice the filter order, as is typically required). Section 3.5 gives an example frequency-hopping scenario which demonstrates the degradation in a receiver’s SNR caused by narrowband filtering. Section 3.6 summarizes this chapter’s results.

### 3.2 Bandpass Response to Single-Tone Pulses

Filters used in communications systems are typically of the Chebyshev type as these provide high band selectivity. In this section, a 900-MHz bandpass filter as used in cellular communications is considered. The impact of ringing (or long-tail response) is examined in simulation and verified experimentally.

#### 3.2.1 Simulation

The pulse response of a seventh-order 34-MHz-wide filter is simulated in fREEDA using Code Listing 3.1. The schematic corresponding to this code is given in Fig. 3.1. When the program is executed, the output of the filter is stored in the file “v.out.out”. The filter
Listing 3.1: fREEDA netlist — filter_pulse.net

* filter transmission response, pulsed stimulus
* 7th-order 900-MHz 34-MHz-wide filter

.tran tstop=5e-7 tstep=1e-11 im=1

.vlfmpulse:v1 1 0 vo=0 va=.7 td=0e-6 fo=895e6 deltaf=0
   chirpdir=-1 phi=-90 tau=25e-8 per=1e-5

ChebyshevBpf:b1 2 0 3 n=7 f0=900e6 bw=34e6 z0=50 ripple=.01

.r:rsourc1 e 2 r=50
.r:rl0ad3 0 r=50

.options gnuplot
.options preamble1="set term x11 font 'helvetica',13';
set title 'Chebyshev Filter Pulse Response';
set xlabel 'Time (microseconds)';
set ylabel 'Vout (volts)"

.out plot term 3 vt 1e6 scalex preamble1 in "v_out.out"

.ends

Figure 3.1: Circuit schematic for Code Listing 3.1 and measurements of Section 3.2.2: the RF source is turned on at $t = 0$ ns and off at $t = 250$ ns.
Figure 3.2: Collection of traces for the simulation of Listing 3.1: pulse RF frequency varied. Waveforms display long-tail responses, frequency-dependent rise and fall times, transmission well outside the filter passband, and peaking when the input to the filter is turned off.
responses to several different RF input frequencies is plotted in Fig. 3.2. Several details in the plots are worth noting:

1. For a signal well within the filter passband (895 MHz), there is still much reactive energy leaving the filter hundreds of RF cycles after the pulse is turned off. This long-tail response dictates that communications pulses propagating through such a narrowband filter, which last for only hundreds of RF cycles, do not achieve steady-state.

2. The energy decay out of the filter is slower as the RF frequency approaches the edge of the passband (883 MHz). The settling time of the filtering circuit is frequency-dependent.

3. For signals outside the passband (880 MHz, 878 MHz), the filter’s transmission response contains a spike (near $t = 0.3 \, \mu s$) before it decays. This spike can only come from reactive energy stored within the filter which remains within the filter in steady-state but eventually escapes under transient conditions.

4. Even when there is very little steady-state transmission for signals far outside the passband (870 MHz, 860 MHz), there is still significant transient transmission while applying and removing the RF pulse. The filter’s transient response is never quick enough to block all of the RF energy that is applied to it; for a finite circuit bandwidth, there will always be a finite amount of time during which transient RF energy passes from input to output.

Measurements which show some of these same effects when applying RF pulses to real bandpass filters are given in the following section.

### 3.2.2 Measurements

The Trilithic 7BC900/27-3-KK filter ($7^{th}$ order, 900 MHz center, 27 MHz nominal bandwidth) is characterized using the circuit of Fig. 3.1. The filter is excited by 1-μs-long single-tone pulses generated by the Agilent E8267C digital synthesizer (at 100 MS/s). The Matlab script for programming the E8267C to output a pulsed waveform is given in Appendix A.2. An example pulse is given in Fig. 3.3. Plots of the filter’s transmission
Figure 3.3: Single-tone RF pulse from the Agilent E8267C synthesizer: the 80-MHz modulation bandwidth provides rise and fall times of approximately 10 ns for each RF pulse. Since 10 ns is much smaller than the response time of the 7BC900/27-3-KK filter, the long tails of Fig. 3.4 are a property of the filter alone.

The rise and fall times of the filtered RF waveforms, as seen in Fig. 3.4, are approximately 200 ns. These times are significantly higher than those expected from fractional-bandwidth rules-of-thumb calculated from the $Q$ of the filter.

The $Q$ of the filter is related to the inverse of the 3 dB fractional bandwidth $\gamma$ from (2.15):

$$Q \approx \frac{1}{\gamma} = \frac{f_0}{(f_2 - f_1)} = \frac{900 \text{ MHz}}{34 \text{ MHz}} = 26.5 . \quad (3.1)$$

and the time constant $\tau$, the time required for the envelope of the filter’s output to reach 63% of its steady-state value, is calculated from (2.16) as

$$\tau_c = \frac{Q}{2\pi f_0} = \frac{26.5}{(2\pi) (900 \text{ MHz})} = 4.7 \text{ ns} ; \quad (3.2)$$
Figure 3.4: Measured step-modulated transmission response envelope as a function of tone frequency. The Chebyshev filter is 7th-order with $f_0 = 900$ MHz, $B = 34$ MHz, passband ripple = 0.01 dB, insertion loss ($IL$) = 3.15 dB. Stimulus tone switched on at $t = 0$ ns, off at $t = 1000$ ns: (a) turn-on response, (b) turn-off response. The settling times of the filter are higher than those calculated from $Q/2\pi f_0$. 
however, both the simulated and measured data show that 63% of the steady-state response is reached between 75 and 125 ns. The apparent measured time constant $\tau_m$ is much longer than that calculated. The tail of the filter’s transmission response is much longer than that calculated from the inverse-bandwidth rule-of-thumb given by (3.1).

For lower-order filters, the approximation given by (3.1) is valid and (3.2) accurately estimates the filter settling time. (This relationship is derived for circuits containing one or two reactive elements in Chapter 4.) From this example, however, it is clear that for higher-order filters, (3.1) is not sufficient and (3.2) does not accurately calculate the settling time.

### 3.2.3 Discussion: Resonator Cascade

To understand why the measured time constant is longer than expected ($\tau_m \gg \tau_c$), the internal structure of the filter must be examined. While much of the analysis in the literature has been performed on filters containing only one or two resonators [23–25, 50, 51], the nature of long-tail filter transients requires an understanding of the time-domain interaction of multiple resonators.

Bandpass filters are made up of coupled resonators. Inside each filter, individual sections with similar resonant properties are chained together to form a larger structure whose frequency selectivity meets a particular design criterion. The lumped-element equivalent of a seventh-order bandpass filter is shown in Fig. 3.5. There are seven resonators in total: four parallel $LC$ tank circuits joined together by three series $LC$ coupling resonators. In order to transmit or reject a signal, a bandpass filter must temporarily store the wave’s energy in a resonant structure similar to this one.
Table 3.1: Time Required to Charge Individual Nodes to 99% of Steady-State Voltage Value, Simulated. The nodes correspond to those shown in Fig. 3.5.

<table>
<thead>
<tr>
<th>Node:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>900 MHz</td>
<td>7 ns</td>
<td>33 ns</td>
<td>57 ns</td>
<td>77 ns</td>
</tr>
<tr>
<td>892 MHz</td>
<td>7 ns</td>
<td>39 ns</td>
<td>83 ns</td>
<td>110 ns</td>
</tr>
<tr>
<td>885 MHz</td>
<td>26 ns</td>
<td>423 ns</td>
<td>462 ns</td>
<td>477 ns</td>
</tr>
</tbody>
</table>

Before a signal is applied to the filter, its inductive branches contain no magnetic energy and its capacitive branches contain no electric energy. After a signal is applied, the resonators begin to charge to their steady-state voltages and currents. Energy is stored in the first resonator with a voltage drop across the first capacitor and a current flow through the first inductor. Series coupling allows this energy to cascade through each of the resonant pairs to eventually reach the filter output. The time required to charge each node is dependent not only on the equivalent resistance seen at the node, but on the charging times of all previous nodes in the cascade.

If the frequency of the applied signal is within the filter’s passband, wave energy enters the filter, the resonators oscillate in-phase, and most of the wave passes through to the filter output. If the frequency is outside the passband, the resonators oscillate out-of-phase and very little energy appears at the output. If the frequency is well outside the passband, most of the energy does not enter the filter at all; instead it is immediately reflected by the first resonator in the filter chain.

The cascade of wave energy through the simulated filter circuit is illustrated by Table 3.1. Displayed are the times required for each of the nodes in Fig. 3.5 to charge to 99% of their steady-state voltage values for three different input frequencies. Because Node 1 immediately follows the 50-Ω source termination of the probe circuit (Fig. 3.1), the resonant pair $L_1C_1$ charges to steady-state the fastest. The other resonant pairs follow in cascade. Because the signal must charge each resonator before moving to the next, the output port requires the greatest amount of time to charge.
Figure 3.6: Filter output: 7th-order Chebyshev design with $f_0 = 900$ MHz, $B = 27$ MHz, ripple $= 0.01$ dB; 1-Volt 150-ns pulse at 890 MHz applied to filter input at $t = 0 \mu s$; the steady-state filter output at 890 MHz is 500 mV. It is unclear from this single pulse which resonator is responsible for which portion of the filter’s transient response.

### 3.2.4 Transient Resonator Interactions

The interactions between charging and discharging coupled resonators are impossible to model in the frequency domain because such behavior is not steady-state, and these interactions are difficult to model in the time domain because there is no obvious way to map sections of a charging or discharging response to individual resonators from time-domain input/output traces.

Fig. 3.6 is another simulated output of a seventh-order bandpass filter when a pulse of RF energy is applied to its input; here the full pulse is shown. The simulator is Advanced Design System 2006. The start of the response shows an S-shaped rise followed by a number of ripples which nearly settle around the steady-state voltage before the input is turned off and the response decays with another set of ripples down to 0 V (much like Fig. 3.4). It is unclear from this time-domain trace which of the 7 coupled resonators govern which portion of the response.
Figure 3.7: Resonator separation circuit in ADS 2006: 7th-order Chebyshev design, 900 MHz center frequency, 0.01 dB passband ripple; lumped-element inductor and capacitor values for 3%, 4%, and 5% bandwidths provided; a transmission line length of 1/6 µs is placed between each resonator. The lossless, multiple-wavelength lines separate the RF pulse transmissions to — and reflections from — each resonator in time.

**Resonator Separation Simulation**

The contributions of individual resonators to the charging and discharging of a coupled structure may be determined by restructuring the filter design to include time delays along the signal path between individual resonators. Fig. 3.7 shows how to perform this restructuring in simulation. The circuit is the same lumped-element Chebyshev design used to generate Fig. 3.6 with transmission-line sections added to it which are

- located between each resonator: 1st and 2nd, 2nd and 3rd, etc.,
- lossless,
- integer numbers of input-tone wavelengths long,
- matched to the input/output port resistance of the filter (e.g. 50 Ω), and
• have time-delays greater than twice the RF pulse length and much greater than the settling time of the filter.

In this configuration, the steady-state voltage at the filter output matches the steady-state voltage of the circuit without the transmission lines (in both amplitude and phase). The unique feature of the transmission-line-separated configuration, however, is that now the pulse-shaping and transmission/reflection characteristics of individual resonators may be discerned.

Fig. 3.8 is a plot of the filter input at an RF carrier frequency of 890 MHz. Fig. 3.9 follows this pulse as it is transmitted through the filter sections to the output. The pulse is transmitted to resonator 1 with an amplitude of 500 mV, half that of the 1.0 V input, because the input impedance of the filter is 50 Ω, initially the rest of the filter appears as 50 Ω because of the first transmission-line $TL_{12}$, and the two form an even voltage divider. As the pulse is transmitted to the internal resonators, it becomes more rounded; its rise- and fall-times become longer as it passes through resonators 2 through 6 before arriving at resonator 7 (which, coupled to the rightmost resistor, is the filter output). Also, the amplitude of the pulse drops as it passes each resonator because the frequency of the input pulse (890 MHz) does not match the resonant frequency of the $LC$ pairs (900 MHz); hence, a portion of the RF energy in this initial forward pulse is reflected back towards the filter input from each of the resonators it encounters.

Fig. 3.10 is a different view of the same simulation: it is a plot of the voltage at the filter output for 16 transmission-line delays (starting when the first pulse arrives after 6 delays). The first pulse of Fig. 3.10 is the last pulse of Fig. 3.9. The second pulse is the RF energy that has experienced a single reflection/re-reflection inside the filter (e.g. reflected from resonator 7 towards the filter input, reflected from resonator 6 towards the filter output, and arrives again at resonator 7). The third pulse is the RF energy that has experienced two reflection/re-reflections, the fourth pulse is the RF energy that has experienced three reflection/re-reflections, and so on.

When the time-separated pulses in Fig. 3.10 are cut in slices of $2\tau$ (where $\tau$ is the unidirectional delay of a single transmission line) and summed together, the resulting trace approaches the filter output without the transmission lines inserted. Fig. 3.11 illustrates this superposition. Fig. 3.11(a) is a comparison of the filter output from Fig. 3.6 against partial sums of the pieces from Fig. 3.10. Fig. 3.11(b) is a zoomed-in view of Fig. 3.11(a).
Figure 3.8: Simulated RF pulse at the filter input: 900-MHz pulse turned on at $t = 0$ ns and off at $t = 150$ ns. The plot on the left shows the overall pulse shape on a longer time scale; the plot on the right shows the carrier sinusoid within the pulse on a shorter time scale.

Figure 3.9: Resonator-separation simulation result: plotting individual resonator voltages for the first pulse transmission through each $LC$ pair for the circuit of Fig. 3.7 using the 900-MHz 5% filter and an 890 MHz pulse. The input pulse becomes increasingly distorted as it passes through each resonator in the filter chain.
at the top of the pulse and at the latter part of its decay. $S_i$ is the sum of the first $i$ pulses that arrive at the filter output in the transmission-line-separated case; $S_T$ is the output in the original case (i.e. the total voltage wave or the infinite sum of the pieces as $t \to \infty$).

From Fig. 3.11 it appears that the partial sums become a good approximation for the true (original) filter output between $S_7$ and $S_{11}$. A plot of the RMS deviation of the traces $S_i$ from $S_T$, for three different carrier frequencies, is given in Fig. 3.12. There is a monotonic decrease in the RMS error as more of the re-reflected pulse contributions are added to the initial filter output. This occurs for frequencies within the passband (890 and 900 MHz), at the edge of the passband (877 MHz), and outside the passband (870 MHz).\textsuperscript{2}

The error approaches a low value which is greater than 0 V due to the finite time period assigned to each pulse contribution. The true tail response of each resonator decay is infinite, and truncating each piece to contribute to a partial sum introduces error (for each piece) because its tail is shortened to $2\tau$.

Fig. 3.13 contains simulation results for two RF frequencies other than 890 MHz. Fig. 3.13(a) is the filter output for 877 MHz, which is at the edge of the passband, and

\textsuperscript{2}It is interesting to note that the highest RMS error, across all partial sums, occurs near the edge of the filter passband.
Figure 3.11: Comparison of partial sums $S_i$ of the pulses from Fig. 3.10 against the waveform $S_T$ from Fig. 3.6. As more pulse transmissions/reflections are included in the partial sum, $S_i$ approaches $S_T$. 
Figure 3.12: RMS error, partial sums $S_i$ from Fig. 3.10 compared to total voltage $S_T$ from Fig. 3.6 for four different carrier frequencies. Generally, the closer the RF frequency is to midband, the quicker the pulses sum to the true transient pulse response.

Fig. 3.13(b) is the filter output for 870 MHz, which is well outside the passband. It is worth noting that, in both of these cases, the pulse with the greatest amplitude is not the first (un-reflected) signal; in fact, at 870 MHz, the greatest amplitude is in the signal path which contains two re-reflections.

3.2.5 Summary

In this section, the pulse response of narrowband filters was studied. High-order filters were found to have a tail response that lasts much longer than the settling time calculated from bandwidth considerations. The length of the tail was found to be dependent upon frequency, both inside and outside of the filter band. Even for frequencies outside the filter band, there is still transient transmission from input to output. The spike in the filter response corresponding to the end of the input pulse was attributed to the escape of reactive energy stored within the filter which is built up when applying RF frequencies inside or outside the filter bandwidth. Each of these effects were observed in simulation, verified experimentally, and were linked to the bandpass filter’s coupled-resonator structure. The resonator-separation method was developed to separate individual resonator transmissions and reflections in time so that the distortion produced by each filter section may be observed.
Figure 3.13: Resonator-separation simulation results for the circuit of Fig. 3.7 and RF frequencies of (a) 877 MHz and (b) 870 MHz. The largest pulse transmission/re-reflection at the filter output is not necessarily the first to arrive.
by probing the nodes around each resonator.

### 3.3 Bandpass Response to Switched-Tone Signals

In the previous section, it was noted that the rise and fall times of the pulsed filter response are not only dependent upon the filter circuit, but on the applied frequency. In this section, it is shown that this frequency-dependence may cause pulse overlap and multitone interference when the filter input is switched (quickly) between two tones.

From Fig. 3.4(a) it is apparent that the time required to charge the output of the filter to steady-state generally increases as the applied tone is moved away from midband and from Fig. 3.4(b) it is apparent that the time required for the filter’s output to discharge to zero follows the same trend. This trend implies that it is possible to produce a two-tone interference pattern at the filter output if its input is switched between a tone whose response falls slowly and another tone whose response rises quickly. This “switched-tone response” is examined in simulation and measured experimentally.

#### 3.3.1 Switched-Tone Transmission Simulation

The response of the 34-MHz-wide filter to an input which has a constant amplitude but switches frequency instantaneously — with constant phase across the transition — is simulated using Code Listing 3.2. The schematic is the same as that of Fig. 3.1, except that the source is broken into two RF pulses: one is active between \( t = 0 \, \mu s \) and \( t = 0.5 \, \mu s \), and one is active between \( t = 0.5 \, \mu s \) and \( t = 1 \, \mu s \).

The output of the filter for several different RF input frequencies is plotted in Fig. 3.14. Several details in the plots are worth noting:

1. There is a discernable two-tone interference pattern for many different combinations of input frequencies (890 to 900 MHz, 880 to 890 MHz, 884 to 910 MHz). This result is expected because of the frequency-dependence of the filter charge/discharge, but it is counter-intuitive because only one frequency is applied to the filter at one time.

2. The interference pattern is evident when switching from a frequency away from the center of the passband to a frequency near to the center of the passband (884 to
Figure 3.14: Collection of traces for the simulation of Listing 3.2: switched RF frequencies varied. Transient multitone patterns are observed at the filter output when switching quickly between particular combinations of frequencies at the filter input. Two-tone interference is observed although only one tone is applied to the filter at any tone time.
900 MHz). This result confirms the trends of Fig. 3.4 and proves that the effect is brought about by frequency-switching and not phase or amplitude transients.

3. There is little two-tone interference when switching from a frequency near to the center of the passband to a frequency away from the center of the passband (900 to 890 MHz). Almost no interference is produced by switching from a quickly-discharging tone to a slowly-charging tone; the first tone discharges almost completely before the second tone charges, so there is little overlap.

4. The greatest peaking in the pattern occurs when switching from a frequency near the passband edge to the passband center (880 to 900 MHz). Tones just inside the passband edge are transmitted through the filter with little attenuation but are the slowest to discharge (of the tones within the passband), while tones at midband are transmitted almost completely and are the fastest to charge. This combination of input tones produces the greatest degree (i.e. amplitude and time-length) of two-tone
interference at the filter output.

Measurements which show the two-tone interference pattern when applying switched-tone signals to real bandpass filters are given in the following section.

### 3.3.2 Switched-Tone Transmission Measurements

To generate two-tone interference, the synthesizer is reprogrammed with a stimulus that switches its frequency between two tones at regular (1-µs) intervals while maintaining a constant amplitude envelope and a constant phase across the transition. The measurement setup is the same as in Section 3.2.2. The Matlab script for programming the Agilent E8267C to output a switched-tone waveform is given in Appendix A.2. Plots of the 7BC900/27-3-KK filter’s switched-tone transmissions as recorded by the Tektronix TDS684B are shown in Fig. 3.15 along with transient fREEDA simulations of the same envelope responses (again using Code Listing 3.2). The two-tone interference pattern is clearly visible in the recorded oscilloscope traces.
3.3.3 Pulse Overlap & Two-Tone Interference

Because the fall time of a pulse response with frequency near to the passband is slow and the rise time of a pulse well within the passband is fast, an interference pattern can be produced by exciting the bandpass filter with a waveform that switches frequency abruptly from a slowly-decaying tone to a quickly-rising tone. The slowly-decaying output at the first frequency overlaps with the quickly-charging output at the second frequency, producing two-tone interference for approximately as long as it takes a single tone to decay completely.

This phenomenon can be explained physically by reading Table 3.1 up-and-down instead of left-to-right. Because wave energy couples into the resonators more efficiently at frequencies near the center of the filter’s passband, the time required for the internal nodes to charge to steady-state generally increases as the applied tone is moved away from midband. This trend implies that pulses of wave energy do not travel through cascaded resonators at equal rates.

Pulses with frequencies well-inside the filter passband travel through the filter faster than pulses with frequencies nearer to the passband edge. Thus, if two consecutive pulses are applied at the filter’s input port — a slower-traveling pulse before a faster-traveling one — it is possible for the later pulse to partially overtake the earlier pulse, producing an interference pattern at the filter’s output port.

3.3.4 Pulsed & Switched Reflection Simulations

In addition to the filter’s transmission response, it is worthwhile to study the filter’s reflection response to see if long tails and multitone interference may be returned from the filter’s input port without needing the signal to propagate through the entire coupled-resonator structure.

Since the signals that are reflected from a filter are not independent of the signals that are transmitted through the filter, long tails and two-tone interference are likely to appear in the filter’s reflected pulsed and switched responses as well. A fREEDA simulation which investigates the reflected cases is given in Code Listing 3.3\(^3\). The reflected responses are stored in the file “v_refl.out”.

\(^3\)Documentation for the “vlfmpulse” and “chebyshevbp” elements is given in Appendix B.
Listing 3.3: freeda netlist — switch refl.net

* filter reflection response, switched-tone stimulus
* 7th-order 900-MHz 34-MHz-wide filter

.tran tstop=1e-6 tstep=1e-11 im=1

vlfmpulse:v1 1 9 vo=0 va=.7 td=0e-7 fo=870e6 deltaf=0 chirpdir=-1
phi=-90 tau=5e-7 per=2e-6
vlfmpulse:v2 9 0 vo=0 va=.7 td=5e-7 fo=860e6 deltaf=0 chirpdir=-1
phi=-90 tau=5e-7 per=2e-6

circulator:c1 2 3 5 0 0 0

chebyshev:b1 3 0 4 n=7 f0=900e6 bw=34e6 z0=50 ripple=.01

r:r1 1 2 r=50
r:r2 4 0 r=50
r:r3 5 0 r=50

.options gnuplot
.options preamble1="set term x11 font ’helvetica,13’;
set title ’Chebyshev Filter Transient Response’;
set xlabel ’Time ( microseconds )’;
set ylabel ’Voltage ( volts )”

.out plot term 5 vt 1e6 scalex preamble1 in ”v.refl.out”

.end

Figure 3.16: Circuit schematic for Code Listing 3.3.
The schematic corresponding to this simulation is that of Fig. 3.16. The main difference between this simulation and that of Listings 3.1 and 3.2 is the addition of the circulator. The circulator is used in Listing 3.3 in order to separate the backward-traveling filter reflection from the forward-traveling input waveform. The filter reflection is captured as $V_{\text{refl}}$.

Several pulsed and switched-tone filter reflections are shown in Fig. 3.17.\(^4\) From these plots, several observations can be made:

1. Long tails are present in some reflected pulses (870 MHz, 878 MHz, 895 MHz) and a two-tone interference pattern is present for some frequency pairs (870 to 860 MHz, 878 to 868 MHz). The transient effects seen in the filter’s transmitted response are indeed present in the filter’s reflected response as well.

2. There is significant distortion for pulses well outside the passband of the filter (870 MHz). Much like the transmitted case, the pulse decay in the reflected response is slower as the RF frequency approaches the edge of the passband (878 MHz).

3. For signals near or inside the passband (878 MHz, 895 MHz), the filter’s reflected response contains a spike before it decays. This is similar to the transmitted response near or outside the passband.

4. Even when there is very little steady-state reflection for signals well-inside the passband (895 MHz), there is still noticeable transient reflection while applying and removing the RF pulse. A practical filter never responds quickly enough to transmit all of the RF energy immediately upon application; some energy is always reflected.

5. The interference pattern is evident when switching from a frequency \textit{near to} the passband to a frequency \textit{away from} the passband (870 to 860 MHz, 878 to 868 MHz). This condition for pulse interference is the inverse of the transmitted case.

6. There is no two-tone interference when switching from a frequency \textit{away from} the center of the passband to a frequency \textit{near to} the center of the passband (880 to 900 MHz). This condition for minimum interference is also the inverse of the transmitted case.

\(^4\)To simulate the pulsed response, the amplitude of source $v_2$ is set to 0 Volts and the time-duration of source $v_1$ is set to 250 ns.
Figure 3.17: Collection of traces for the simulation of Listing 3.3: pulsed and switched RF frequencies varied. Long tails and multitone interference are also observed in a narrowband filter’s reflected response.
7. The greatest peaking in the interference pattern occurs when switching from a frequency near the passband edge to a frequency well-outside the passband (878 to 868 MHz). Tones just outside the passband edge are reflected from the filter with little attenuation but are the slowest to decay, while tones far away from the passband are reflected almost completely, immediately. This combination of applied tones produces the greatest degree of two-tone interference in the filter’s reflected waveform. Measurements which demonstrate that two-tone reflections may be captured from real filters are given in the following subsection.

### 3.3.5 Wireless Switched-Tone Reflection Measurements

The block diagram of the measurement setup used to capture switched-tone filter reflections in a wireless environment is given in Fig. 3.18. The Agilent E8267C produces the switched-tone waveforms. Two pairs of antennas, the ETS Lindgren 3164-03 wideband horn antenna and the Pasternack 51014 cellular-band dipole antenna, form the wireless channel. The Raditek RADC-650-1000M circulator separates the transmitted and reflected waveforms. The MiniCircuits ZRL-1150LN amplifier is used to boost the filter reflection to a level that is above the noise floor of the system at the oscilloscope, such that no additional signal-processing is required to extract the two-tone waveforms.

Two Trilithic 7BC900 filters are characterized (with 27 and 36 MHz nominal bandwidths). Plots of the RF envelopes recorded by the Tektronix TDS684B oscilloscope are shown in Fig. 3.19. This data confirms that the interference pattern seen in simulation can be measured using a cellular-band (900 MHz) filter and a high-speed digitizer over a wireless channel.

Fig. 3.19(a) shows that the pattern is indeed a two-tone interference phenomenon, as larger input frequency steps result in (a) faster oscillations of the reflected envelope, and (b) greater initial peaking caused by a larger steady-state amplitude for the second tone. Fig. 3.19(b) shows that the pattern is both a function of filter bandwidth and stimulus frequency. If the stimulus frequencies are held constant, a greater degree of peaking and a longer rippling period are evident when using a smaller filter bandwidth. Both plots of Fig. 3.19 indicate that a typical cellular antenna is fairly broadband and will not destroy the characteristic two-tone pattern.
Figure 3.18: Block diagram of the wireless switched-tone measurement system. The filters tested are the Trilithic 7BC900/27-3-KK and the 7BC900/36-3-KK; all antennas transmit and receive on vertical polarization.

Figure 3.19: Filter reflection measurements for the setup of Fig. 3.18 as recorded by the TDS684B: (a) $B = 27$ MHz nominal, switched frequencies varied, (b) 920 to 940 MHz switch, $B = 27$ MHz (3%) and $B = 36$ MHz (4%). The input frequency switches at $t = 0$ ns; the RF carriers of the waveforms have been removed for clarity. Two-tone interference in filter switched-tone reflections is captured over a wireless channel.
3.3.6 Summary

In this section, the switched-tone response of narrowband filters was studied. The time-dependence of the charge/discharge of filter resonators on the applied frequency produced multitone interference patterns in the transmitted and reflected waveforms for certain combinations of frequency steps applied to the filter input. The interference was attributed to the variation in speed of pulses through the filter’s resonant cascade across the spectrum of applied frequencies. These effects were observed in simulation and captured over a wireless channel.

3.4 Mathematical Modeling of Filter Transients

Closed-form time-domain expressions for filter transients are desired in order to predict and mitigate their effects on narrowband circuits. In this section, a new method for obtaining such expressions is presented. This technique exploits the way filters are usually designed in order to simplify the analysis required to determine their transient responses.

3.4.1 Prior Approaches

Prior methods for obtaining closed-form solutions for bandpass transients were presented in Section 2.4. While Salinger performed his analysis using complex integration and assuming ideal filters [16], the rest of the approaches directly solved the differential equations of physically-realizable filter circuits.

Hatton solved the full differential equation in the time domain for a single resonator [24]. Tucker and Eaglesfield used differential operators, a precursor to Laplace Transforms, to obtain solutions to excitations at midband for commonly-used third-order filters [18–21]. McCoy applied Laplace Transforms to generalize the response to any number of resonators, assuming ideal coupling between them [25]. Hartley used assumptions of narrow bandwidth, high $Q$, and $s$-domain symmetry to reduce the complexity of the Laplace method [27]. Chohan and Fidler exploited the $s$-domain shifting property to generalize Hartley’s results to filters of any $Q$ value [28].

Each of the aforementioned methods solves a differential equation of order $2N$,
where $N$ is the number of resonators contained in the filter. Also, each of these methods requires that either the bandpass circuit values be specified, or that the bandpass transfer function be given, in order to solve for the time domain output expression. These solutions become increasingly complex for filters of higher order. For example, the time-domain expression for a seventh-order filter response requires solution of a $14^{\text{th}}$-order differential equation. It is possible, however, to greatly simplify the analysis for bandpass filters that are designed from lowpass prototypes. This derivation is presented in the next section.

3.4.2 Response Derived from Lowpass Prototyping

Bandpass filter design usually begins with a filter’s lowpass prototype. The concept of lowpass prototyping was introduced in Section 2.3.3, and the scaling rules used to transform lowpass inductance and capacitance normalized values into their bandpass de-normalized values are given in (2.23) and listed in Appendix C.3. The lowpass equivalent of the filter of Fig. 3.5 is given in Fig. 3.20.

The same frequency transformation that is used to obtain the bandpass circuit values from their corresponding lowpass equivalent values can be used to obtain the bandpass time-domain response from its lowpass equivalent response. The Laplace domain frequency translation in the narrowband case is given by [9]:

$$ s = \frac{\omega_2 - \omega_1}{2} S \pm j\omega_0 $$ (3.3)

where $\omega_1$ and $\omega_2$ are the lower and upper passband frequencies, $\omega_0$ is the center frequency of the filter, $s$ is the bandpass Laplace variable, and $S$ is the lowpass Laplace variable. With respect to their lowpass counterparts, the poles and zeros of the filter are scaled by half of
the radian bandwidth of the filter and shifted up to $+j\omega_0$ and down to $-j\omega_0$.

Let the input voltage stimulus and its Laplace Transform be

$$V_0 e^{j(\omega_0 + \Delta \omega) t} u(t) \Leftrightarrow \frac{V_0}{s - j(\omega_0 + \Delta \omega)} \quad (3.4)$$

where $\Delta \omega$ represents the deviation in the excitation frequency from the center of the filter’s passband, $V_0$ is the amplitude of the applied RF pulse, and $u(t)$ is the Heaviside unit step function. The lowpass equivalent of this waveform and its inverse Laplace Transform are found by shifting and scaling the single pole in the denominator of (3.4) according to (3.3):

$$\frac{V_0}{s - j \left( \Delta \omega \frac{2}{\omega_2 - \omega_1} \right)} \Leftrightarrow V_0 e^{j \left( \Delta \omega \frac{2}{\omega_2 - \omega_1} \right) t} u(t). \quad (3.5)$$

Assuming filter poles with no greater multiplicity than one, the lowpass equivalent response can be written [9]

$$V_{\text{lp out}}(t) = \sum_{i=1}^{n} A_i e^{S_i t} + B e^{j \left( \Delta \omega \frac{2}{\omega_2 - \omega_1} \right) t} \quad (3.6)$$

where $A_i$ and $B$ are the residues at each lowpass pole. The bandpass output is given by

$$V_{\text{bp out}}(t) = \sum_{k=1}^{2n} A_k e^{s_k t} + \frac{B}{2} e^{j (\Delta \omega + \omega_0) t} + \frac{B}{2} e^{j (\Delta \omega - \omega_0) t} \quad (3.7)$$

where the transformation of (3.3) splits each lowpass pole into two bandpass poles with equal residues. Rearranging terms and simplifying gives

$$V_{\text{bp out}}(t) = \sum_{i=1}^{n} \frac{A_i}{2} e^{\frac{\omega_2 - \omega_1}{2} S_i t} (e^{j \omega_0 t} + e^{-j \omega_0 t}) + \frac{B}{2} e^{j (\Delta \omega) t} (e^{j \omega_0 t} + e^{-j \omega_0 t}) \quad (3.8)$$

$$= \cos (\omega_0 t) \left[ \sum_{i=1}^{n} A_i e^{S_i \frac{\omega_2 - \omega_1}{2} t} + B e^{j \Delta \omega t} \right] \quad (3.9)$$
\[ = \cos(\omega_0 t) \frac{V_{\text{lp}}^{\text{out}}(\omega_2 - \omega_1 t)}{2} \quad (3.10) \]

which states that the time-domain response of a narrowband bandpass filter to an RF pulse step is equal to a time-scaled version of its lowpass response that modulates a sinusoid at the center frequency of the bandpass filter.

From (3.6) and (3.10), the method for determining the bandpass time-domain response can be summarized:

- scale the deviation frequency by \(2 / (\omega_2 - \omega_1)\),
- find the lowpass response at this new frequency,
- time-scale the lowpass response by \((\omega_2 - \omega_1)/2\), and
- multiply the time-scaled response by the filter center frequency.

The filter response to the end of the input pulse is found by determining the circuit’s response to a delayed and inverted version of the input stimulus given by (3.4) and adding it to the original response. The switched-tone output is found by superimposing two single-tone pulse responses, with the turn-on time of the second pulse equal to the turn-off time of the first pulse. Fig. 3.21 compares two single-tone time-scaled lowpass prototype envelopes with RF pulse outputs from another cellular-band filter. The RF envelopes show very good agreement.

### 3.4.3 Summary

The time-scaled lowpass prototype method does not require the solution of a \(2N^\text{th}\) order differential equation. Traditional Laplace methods can be used to solve the \(N^\text{th}\) order circuit equation for a filter’s lowpass prototype, and the aforementioned frequency- and time-scaling relationships can be used to upconvert the lowpass response to the frequency band of interest. The technique presented in this section cuts the complexity of solving the full differential equation solution in half.

The filter transients discussed here are entirely linear. Therefore, the complete pulse response is found by summing transient responses of the same frequency with opposite
3.5 Case-Study: Frequency-Hopped Communications

The aforementioned interference patterns are caused by the overlapping of pulses within bandpass filters. This smearing-together of RF pulses has implications for communications systems which implement such filters, particularly those systems which hop between user pulses at a fast rate. This section investigates the intersymbol interference (ISI) caused by narrowband filtering in a typical frequency-hopping communication scenario.

3.5.1 Settling Time vs. Group Delay

Due to the extended period of time required for a single-tone pulse response to decay, it is clear that a communication scheme which implements fast frequency switching between user pulses has an upper limit to its frequency-switching rate as a result of filtering.
Figure 3.22: Time for the filter output to settle to 5% of its steady-state value after its input has been switched off, compared with group delay, simulated; filter same as in Fig. 3.4. The group delay curve peaks and falls on either side of the filter passband, while the settling-time curve monotonically increases on either side of the passband.

Fig. 3.22 gives the 95% settling time plotted against stimulus frequency for the aforementioned 34-MHz-wide filter. It is apparent from the simulated data that a communications scheme containing this filter cannot switch frequencies faster than once every 80 ns, even at midband, without causing a significant portion of each pulse to bleed into its neighboring time-slots.

The group delay of the simulated filter is also plotted in Fig. 3.22. As can be seen in the figure, there is very little correlation between the settling time of the pulse responses and the group delay of the filter. The pulse response curve is approximately parabolic both inside and outside of the filter’s passband, whereas the group delay displays similar curvature solely within the passband. Outside this region, the group delay is monotonically decreasing beyond its peaks near each passband edge, a phenomenon which is true of all practical multi-pole bandpass filters [9].

Since group delay is a steady-state quantity, it is generally not relevant for describing transient behavior. If the signal bandwidth (considering a single pulse) is small, corresponding to longer pulse periods and slower rise and fall times, then the group delay characteristic may indeed be used to estimate the time required for a single pulse to traverse
the filter. In this case, the system is said to be quasi-stationary: transient effects between pulses are minimal and steady-state analysis is adequate to describe system behavior. A system designer can easily estimate the transmission times for operating frequencies by reading values from the group delay curve. If the signal bandwidth is large, corresponding to short periods and faster transitions, however, the group delay characteristic cannot be used to estimate pulse travel time because the system is not near steady-state. To a system designer, it is unclear what the transmission time of a pulse will be when the bandwidth of the pulse is a significant fraction of the filter bandwidth; this information is not readily available from the group delay curve.

3.5.2 Impact of Filtering on Signal-to-Noise Ratio

An example of the impact that filter transients can have on a multi-user frequency-hopping system is given in Fig. 3.23. The reception of two data streams which share the same frequency in adjacent time-slots is shown. Fig. 3.23(a) presents the two filtered user transmissions as separable, but with a small overlap near the start of the second pulse. Fig. 3.23(b) is an expansion of the envelope from Fig. 3.23(a) near the point of the overlap. Fig. 3.23(b) also shows the result of the superposition of the two pulses.

The superposition shown in Fig. 3.23(b) is the signal that a frequency-hopping basestation would receive. Since the frequency that the first user has just transmitted is the same as the frequency that the second user is currently transmitting, the basestation cannot separate the two data streams until the transient response due to the first pulse has become negligible with respect to the second. The extended tail of User 1’s transmission appears as interference during User 2’s transmission, even if the frequency of operation is well within the filter’s passband.

Fig. 3.24 shows the Signal-to-Interference ratio (SIR) during reception of the second user’s transmission as a function of guard-band length. It is worth noting that the frequency of operation is the center frequency of the filter, the condition for minimum pulse overlap.

The trend in Fig. 3.24 is intuitive: a larger guard-band reduces the signal power that spills over from User 1’s transmission into User 2’s time slot. One result from the plot is surprising, however: even for guard bands that last a hundred RF cycles, the SIR,
Figure 3.23: Filtered frequency-hopping pulses: 900-MHz 7th-order 4% (7BC900/36-3-KK) filter, User 1 at 10 dBm, User 2 at -20 dBm, 900 MHz frequency, 100 ns guard band: (a) individual pulses, larger voltage and time scale, User 1’s pulse overlaps User 2’s pulse near $t = 0.7 \mu s$; (b) pulse envelopes, voltage and time scale focused on the overlap region, RF carriers removed for visual clarity, User 1’s residual tail is clearly visible within User 2’s timeslot.
considering only the transient response of the bandpass filter, may be only 5-10 dB. This data shows that an additional degree of caution is necessary when designing a bandpass frequency-hopping scheme such that the transients produced by the basestation filter do not result in ISI between user transmissions.

3.5.3 Summary

Narrowband filtering has been identified as another source of co-site interference in wireless communication devices. Since pulses of different frequencies travel through bandpass circuits at different speeds, faster pulses can overtake slower pulses within an RF front-end filter. Certain combinations of pulse frequencies will cause intersymbol interference. Frequency-hopping systems are most likely to be effected by this type of linear distortion. Measurements were taken which show SNR degradation by narrowband filtering in an example communication scenario.
3.6 Conclusions

The analysis of RF systems has been performed primarily in the frequency domain. Since steady-state is assumed, the transient behavior of RF networks is generally ignored. As systems with narrower bandwidths and sharper band-limiting characteristics are implemented, however, it becomes necessary for resonant structures to store more field energy to produce more rigid frequency-domain characteristics, and the time required to charge and discharge the structures is extended. Measurements show that narrowband transients in cellular-band communications filters can last for hundreds of RF cycles, a time period that is significantly longer than conventional time/bandwidth rules-of-thumb. The transient behavior of narrowband components can no longer be ignored.

In this chapter, simulations were performed which reveal long-tail behavior in high-order narrow-bandwidth filters. The cause of this transient effect was tied to the physical structure of the filter’s resonant cascade. The time to charge and discharge the filter was found to depend upon the frequency of the applied signal. A circuit separation method using lossless transmission lines was developed to illustrate the interactions of coupled resonators in simulation.

Interference patterns were produced at the output of a filter by switching the applied frequency faster than the settling time of its resonators to show that pulse-overlap is possible within a narrowband device. Measurements confirmed that two-tone interference is present in the transmissions of a narrowband filter when the frequency of a single input tone is stepped closer to the center of the filter’s passband. Measurements also confirmed that two-tone interference is present in the reflections of a narrowband filter when the frequency is stepped away from the center of the passband. Switched-tone-interference reflections were captured wirelessly.

It was noted that the systems most likely to be affected by longer-than-expected transient behavior are those which use time-division schemes with pulse lengths approaching the duration of these transients, such as fast-frequency-hopping communications. The long-tail response produced by filtering stronger signals has the potential to overlap the reception of weaker signals, leading to a reduced signal-to-interference ratio. Measurements confirmed the SNR degradation that is possible by linear filtering alone.

If narrowband filtering should produce intolerable ISI, several techniques can be
used to mitigate the problem. For a fixed filter architecture, an increase in guard banding
between pulses would reduce pulse overlap, thereby directly reducing ISI. For a fixed data
rate, a decrease in filter order would also reduce pulse overlap by shortening each pulse’s
residual tail. If neither the filter architecture nor the data rate can be changed, the ambi-
guity of overlapping pulses may yet be avoided by coding hopping sequences that do not
repeat frequencies between users in adjacent time-slots. In any case, it is the responsibility
of the system designer to evaluate these options so that the basestation filter — a component
whose behavior is not widely known to produce co-site interference — does not degrade the
communication system’s performance.
4

Linear Metrology of Bandpass RF Components

4.1 Introduction

Time-frequency techniques can be employed in the measurement and characterization of resonant circuits. For circuits with few resonant sections, the short time-period over which transient phenomena occur (on the order of nanoseconds) dictates that observing the ringing, spiking, and multitone interference produced by a band-limited circuit requires capturing time-domain traces using a digitizer capable of recording tens of billions of samples per second, with tens of gigahertz of analog bandwidth. For circuits with many resonant sections, however, the longer time-period over which the same effects occur enables similar observations to be made using equipment with less stringent hardware requirements.

The long-tail properties of narrowband filters, while possibly detrimental to system performance as described in Chapter 3, are advantageous to device metrology because (a) they are observable with readily-available laboratory equipment and (b) the transient behavior of a circuit is linked to its steady-state properties. It is possible to extract steady-state parameters of a circuit from time-domain traces, and this chapter explains how narrowband
circuit phenomena may be used to do so.

Section 4.2 explains how a filter’s quality factor \((Q)\) may be extracted from its RF pulse decay response. Section 4.3 shows how a filter’s bandwidth \((B)\) may be determined from the same pulse waveform. Section 4.4 demonstrates how a circuit’s S-parameters may be estimated by exciting it with a short (broadband) pulse. Section 4.5 summarizes this chapter’s results.

### 4.2 Extraction of Resonator Q from RF Pulse Decay

Non-destructive methods for testing in-circuit components are invaluable to RF device manufacturers when attempting to identify non-functional components of integrated assemblies. The ability to extract circuit parameters of individual resonators from a coupled structure, for example, would be particularly useful to filter designers when implementing filters with high band selectivity, which requires cascading many coupled resonators. In this section, a method for extracting the \(Q\) of the outermost (i.e. first or last) resonator in a chain is presented.

#### 4.2.1 Simulation

Simulations have shown that if the input port of a Chebyshev filter is excited by a single-tone signal of constant amplitude, the loaded \(Q\) of the filter’s first resonator may be approximated by measuring the rate of energy decay immediately after the stimulus tone is turned off [6]. The fREEDA netlist for such a simulation is given in Listing 4.1. When the program is executed, the filter response is stored in the file “\(v_{\text{r.out}}\)”.

The simulated circuit is that of Fig. 4.1 with voltage measured at Node 2. The input waveform is a single-tone sinusoid at the filter’s center frequency \(\omega_0\) with constant amplitude \(2V_0 = 1.0\) V. The source is turned off at time \(t = 400\) ns. A plot of the simulated voltage envelope for three filters with different bandwidths is given in Fig. 4.2. The plot shows characteristic decay forms: the shapes of the three filter-discharge waveforms are nearly the same, but the narrower bandwidths are more stretched in time, i.e. they decay more slowly.

The time constant of the energy decay is found by approximating the beginning
Listing 4.1: fREEDA netlist — pulse_decay.net

- filter pulse-decay response
- 7th-order 900-MHz 36-MHz-wide filter

.tran tstop=8e-7 tstep=1e-11 im=1

vlmpulse:v1 1 0 vo=0 va=1 td=0e-7 fo=900e6 deltaf=0
  chirpdir=-1 phi=-90 tau=400e-9 per=2e-6

chebyshev_bpf:bl 2 0 3 n=7 f0=900e6 bw=36e6 z0=50 ripple=.01
r:r1 1 2 r=50
r:r2 3 0 r=50

.options gnuplot
.options preamble1="set term x11 font 'helvetica,13';
set title 'Chebyshev Filter Transient Response';
set xlabel 'Time (microseconds)';
set ylabel 'Voltage (volts)"

.out plot term 2 vt 1e6 scalex preamble1 in "v_r.out"
.end

---

Figure 4.1: Circuit corresponding to the simulation of Code Listing 4.1.
Figure 4.2: Simulated envelope decay at the filter input port: 7th-order Chebyshev designs, 900-MHz center frequency, 0.01 dB passband ripple, varied bandwidth, 900-MHz stimulus tone turned off at $t = 400$ ns (RF carrier removed). The beginning of each voltage decay can be approximated by a first-order exponential drop.

of the voltage decay by the exponential function

$$V_R(t) = V_0 e^{-t/\tau}$$  \hspace{1cm} (4.1)

where $V_R$ is the magnitude of the voltage wave across the input resistance $R$, and $V_0$ is the magnitude of the steady-state voltage at the input port immediately before the input stimulus is turned off. Sinusoidal variation at the filter’s center frequency is implied; only the envelope of the voltage wave is analyzed. Justification for this first-order exponential approximation of the envelope response is provided in Section 4.2.2.

The energy decay is equal to the power dissipated in the source termination,

$$P(t) = \frac{V_R^2(t)}{R} = \frac{V_0^2}{R} e^{-t/\tau}.$$  \hspace{1cm} (4.2)

Taking the natural logarithm of both sides and rearranging terms gives

$$\tau = \frac{t}{\ln(V_0^2) - \ln(V_R^2(t))}.$$  \hspace{1cm} (4.3)

where $t$ must be chosen close to the start of the decay. The $Q$ factor can then be estimated
Table 4.1: Simulated Q-Value Estimation, 900-MHz Chebyshev Designs. The loaded Q-value of several filter designs is calculated from steady-state considerations and compared against the Q-value estimated using (4.3).

<table>
<thead>
<tr>
<th>Filter Order</th>
<th>BW</th>
<th>Passband Ripple (dB)</th>
<th>Loaded Q Value</th>
<th>Estimated Q Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3%</td>
<td>0.01</td>
<td>21.0</td>
<td>20.5</td>
</tr>
<tr>
<td></td>
<td>3%</td>
<td>0.10</td>
<td>34.4</td>
<td>33.2</td>
</tr>
<tr>
<td></td>
<td>4%</td>
<td>0.01</td>
<td>15.7</td>
<td>15.3</td>
</tr>
<tr>
<td></td>
<td>4%</td>
<td>0.10</td>
<td>25.8</td>
<td>25.0</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>0.01</td>
<td>12.6</td>
<td>12.2</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>0.10</td>
<td>20.6</td>
<td>20.0</td>
</tr>
<tr>
<td>5</td>
<td>3%</td>
<td>0.01</td>
<td>25.5</td>
<td>24.9</td>
</tr>
<tr>
<td></td>
<td>3%</td>
<td>0.10</td>
<td>38.2</td>
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<tr>
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<td>0.01</td>
<td>19.1</td>
<td>18.7</td>
</tr>
<tr>
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<td>4%</td>
<td>0.10</td>
<td>28.7</td>
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<td>0.01</td>
<td>15.3</td>
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<td>0.10</td>
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</tr>
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<td>0.01</td>
<td>26.6</td>
<td>25.9</td>
</tr>
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<td></td>
<td>3%</td>
<td>0.10</td>
<td>39.4</td>
<td>37.0</td>
</tr>
<tr>
<td></td>
<td>4%</td>
<td>0.01</td>
<td>19.9</td>
<td>19.4</td>
</tr>
<tr>
<td></td>
<td>4%</td>
<td>0.10</td>
<td>29.5</td>
<td>28.8</td>
</tr>
<tr>
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<td>5%</td>
<td>0.01</td>
<td>15.9</td>
<td>15.5</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>0.10</td>
<td>23.6</td>
<td>22.6</td>
</tr>
</tbody>
</table>

from (2.16). A slower rate of decay corresponds to a higher value of $Q$, and vice versa.

Table 4.1 lists this estimated $Q$ for several simulations of commonly-used filter orders, bandwidths, and passband ripples. Alongside the estimated $Q$ is the loaded $Q$ of the first resonator calculated using the capacitance and inductance of the lumped-element Chebyshev simulation model and a 50 Ω resistance. Although the estimated $Q$ consistently under-approximates the resonator $Q$, the maximum deviation between the estimated and exact values is less than 6%.

### 4.2.2 Equivalence of Quality Factor in Time & Frequency Domains

To show why this one-port $Q$ estimation is valid, lowpass prototyping is invoked. A seventh-order lumped-element Chebyshev filter and its lowpass equivalent are shown in Fig. 4.3(a) and 4.3(b), respectively. The bandpass transformation substitutes a series $LC$
combination for each \( L_N \) and a parallel \( LC \) combination for each \( C_N \). The relationships between the element values of the \( LC \) pairs of the bandpass filter and the reactive elements of the lowpass prototype are given by (2.23) and listed in Appendix C.3.

There is also a time-scaling relationship between (a) the envelope of the transient response of a bandpass filter when excited by a step-modulated waveform with carrier frequency equal to the filter’s center frequency, and (b) the transient response of the filter’s lowpass equivalent when excited by a step change in voltage [9]. An illustration of this relationship is shown in Fig. 4.4. The envelope of the voltage wave across the input resistor in the bandpass case is a time-scaled version of the same voltage wave in the lowpass case:

\[
t = \alpha t_N \quad \alpha = \frac{2}{\omega_0\gamma}
\]  

where \( t \) is the time scale of the bandpass response and \( t_N \) is the time scale of the lowpass equivalent response. This time-scaling relationship will be used at the end of the next subsection.

**Q of a Single Resonator**

The following analysis shows that the \( Q \) factor derived from considering the filter’s bandpass envelope is equal to the \( Q \) factor derived from steady-state considerations, with the approximation that the decay of the lowpass equivalent is a first-order exponential drop.

From (2.14), the quality factor of the first resonator from steady-state considerations alone is

\[
Q_1 = \omega_0 \frac{1}{2} C_1 V_1^2 \left( \frac{1}{2} \frac{1}{V_1^2 / R} \right) = \frac{R}{2} \sqrt{\frac{1}{C_1 L_1}} = \frac{R}{2} \sqrt{\frac{C_1}{L_1}}
\]  

with two parallel resistive branches \( R \) attached to the \( RLC \) tank circuit.

To determine \( Q \) in the time domain, the differential circuit equation involving the first reactive component in the lowpass prototype is considered. The voltage \( V_1 \) measured
Figure 4.3: 7th-order Chebyshev filter: (a) lumped element model, (b) lowpass equivalent circuit. In the lowpass equivalent circuit of a bandpass structure, series $LC$ pairs become series inductors and parallel $LC$ pairs become shunt capacitors.

Figure 4.4: Relationship between bandpass filter and lowpass prototype transient responses: 7th-order 0.01-dB ripple Chebyshev designs, simulated. The lowpass filter has a corner frequency of 1 rad/s and the bandpass filter has a center frequency of 900 MHz and a bandwidth of 27 MHz. The source voltage $2V_0$ is deactivated at $t = 0$. The bandpass response is a time-scaled version of the lowpass response.
Figure 4.5: Equivalent lowpass circuit after the source is zeroed: (a) for all time, (b) immediately after zeroing. Across the switch, inductor current remains constant and capacitor voltage remains constant; the reactive energies stored in the outermost capacitors are the first to decay.

across the input resistance $R_N$ in Fig. 4.3(b) is found by solving

\[
\frac{V_1(t_N)}{R_N} + C_{N1} \frac{dV_1(t_N)}{dt} + I_2(t_N) = 0.
\]  

(4.6)

It should be noted that this expression is strictly valid for all odd-order filters. For even-order filters, a sum-of-voltages replaces (4.6) and the same result for $Q$ is obtained. There is no loss of generality.

When the source voltage is turned off at $t_N = 0$, the circuit becomes that of Fig. 4.5(a). Since the current through inductor $L_{N2}$ cannot change instantaneously, its value immediately before and after the input is turned off is a constant, and since current must flow through $L_{N2}$ before reaching the circuit components nearer to the output, initially only capacitor $C_{N1}$ reacts to the change. Thus, as a consequence of the cascaded structure, initially only the voltage across $C_{N1}$ decays while the other reactive elements retain their “charged” voltage and current values. From the standpoint of $C_{N1}$, the circuit immediately after the switch is that of Fig. 4.5(b), and (4.6) becomes

\[
\frac{dV_1(0)}{dt_N} = - \frac{V_1(0)}{R_NC_{N1}} - \frac{I_2(0)}{C_{N1}} = - \frac{2}{R_NC_{N1}} V_1(0)
\]

(4.7)
whose solution, in the neighborhood of $t_N = 0$, is

$$V_1(t_N) = V_0 e^{-2t_N/R_NC_{N1}}. \quad (4.8)$$

It is important to note that the same exponential factor is obtained by analyzing the transient behavior at the beginning of the voltage step rather than at the end; therefore, this same technique can be used to extract $Q$ from the *start* of the RF pulse.

The power dissipated in the 1-Ω resistor $R_N$ is

$$P(t_N) = \frac{V_0^2}{R_N} e^{-4t_N/R_NC_{N1}} = V_0^2 e^{-t_N/\tau_N} \quad (4.9)$$

with a lowpass time constant

$$\tau_N = \frac{C_{N1}}{4}. \quad (4.10)$$

Time-scaling (4.10) by (4.4), the bandpass time constant is

$$\tau = \alpha \tau_N = \frac{C_{N1}}{2\omega_0 \gamma} \quad (4.11)$$

which, when substituted into (2.16) and expanded by (2.23), gives

$$Q = \frac{C_{N1}}{2\gamma} = \frac{1}{2\gamma} R \left( \frac{1}{\sqrt{L_1 C_1}} \right) \gamma C_1 = \frac{R}{2} \sqrt{\frac{C_1}{L_1}}. \quad (4.12)$$

This value is equal to the steady-state quality factor given by (4.5). The time-domain envelope-decay calculation of $Q$ gives the same result as the traditional steady-state frequency domain calculation of $Q$. 
4.2.3 Measurements

Measurements confirm that the loaded $Q$ of the first resonator may be extracted using the estimation method described by probing the bandpass filter with an RF pulse generator and capturing the necessary time-domain data with a sampling oscilloscope. It should be noted that this measurement technique is valid only if (a) the rate of decay of the stimulus pulse is much greater than the rate of decay of the resonator under test, and (b) the oscilloscope used to record the reflected waveform is able to produce a faithful image of the resonator’s decay. These conditions imply that the RF bandwidths of both the pulse generator and oscilloscope must be significantly greater than the bandwidth of the filter.

Using a vector signal generator with a closed-loop bandwidth of 70 MHz, two filters are analyzed: the Trilithic 7BC900/36-3-KK with a bandwidth of 36 MHz, and the Trilithic 7BC900/45-3-KK with a bandwidth of 45 MHz. The nominal center frequency of both filters is 900 MHz. A block diagram of the measurement system is given in Fig. 4.6.

With the Agilent E8267C digital synthesizer as the waveform source, the input of each filter is pulsed with 1-µs bursts of a 1.0-Volt sinusoidal source at 900 MHz. The Matlab script for programming the E8267C to output a pulsed waveform is given in Appendix A.2. A circulator is inserted between the source and the filter input port so that the reflected voltage wave from the filter is redirected to the oscilloscope. The Tektronix TDS684B digitizing oscilloscope records the raw voltage waveforms. A Matlab routine is then used to remove the 900-MHz carrier from the raw oscilloscope data, leaving only the characteristic envelope of each waveform.\(^1\) The envelope-extraction routine is provided in Appendix C.1.

\(^1\)Alternatively, a rectifier and lowpass filter could be used to capture the signal’s envelope.
Fig. 4.7 shows two sample traces of these waveform envelopes. The traces are truncated so that only the envelope response immediately following the sinusoidal burst is seen. These traces are analogous to the simulated filter responses of Fig. 4.2.

The equation form at the start of the voltage decay is assumed to be that of (4.1). $V_0$ is equal to the value of the first voltage sample for each trace. Time $t = 0$ corresponds to the moment that this first sample is recorded. The energy decay time constant $\tau$ is determined from (4.3), where $t$ is the time that the second sample is recorded.$^2$ $Q$ is then calculated using (2.16).

Table 4.2 compares the one-port $Q$-value estimation against the two-port fractional-bandwidth $Q$-value measurements of two Chebyshev filters with different bandwidths. The one-port estimations were found to be within 10% of their corresponding two-port measurements. These values are consistent with simulation results for seventh-order filters with a passband ripple of 0.01 dB.

4.2.4 Discussion: Coupled Resonators

This quality factor extraction is only possible because of the structure inherent to a coupled-resonator filter. Many bandpass filters are constructed this way. Inside each

---

$^2$For $Q$ calculation, $V_0 = 500$ mV and $t = 200$ ps.
Table 4.2: Measured $Q$-Value Estimation, 900-MHz Chebyshev Designs. The measured 2-port $Q$-value of two filter designs is compared against the $Q$-value estimated using (4.3) with measured pulse-decay data.

<table>
<thead>
<tr>
<th>Filter Order</th>
<th>Passband Ripple (dB)</th>
<th>2-Port $Q$ Value</th>
<th>1-Port $Q$ Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4%</td>
<td>0.01</td>
<td>23.3</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>0.01</td>
<td>18.4</td>
</tr>
</tbody>
</table>

filter, individual sections with similar resonant properties are chained together to form a larger structure whose frequency selectivity meets a particular design criterion. When an RF pulse is first applied to the filter input, the resonators charge to their steady-state voltages and currents. For frequencies outside the passband, the pulse returns from the input port unchanged, little energy is stored in the resonant structure, and very little energy is transmitted through to the output. For frequencies inside the passband, most energy enters the filter, the maximum amount of energy is temporarily stored in the resonators, and most energy is eventually transmitted through to the output port.

When the pulse is removed (whether the frequency is inside or outside the passband), the resonators release their stored energy to the terminal resistances, as well as to internal parasitics. The rate of energy decay is set by both the energy-storage capacity of the filter (e.g. the number and size of the reactive elements) and the resistances of the paths by which the resonators discharge. Since the first resonator in the filter chain is physically nearest the input port, the energy decay measured immediately after the source is shut off will most nearly follow the circuit time constant of the first resonator. As time progresses, energy from the internal resonators also passes out of the input port; their different $Q$ factors — and therefore different rates of reactive energy discharge — may help explain the ripple pattern seen after the initial decay.

4.2.5 Summary

In this section, a one-port time-domain method for estimating the quality factor of a bandpass filter’s outermost resonator was introduced. Simulations showed that the decay of reactive energy when an RF source is removed from a narrowband filter may be approximated by a first-order exponential form. This technique under-estimates the actual
resonator $Q$, but not by more than 6% for commonly-used filters.

This estimation method was proven valid using the time-scaling relationship between the transient response of a bandpass filter and that of its lowpass prototype. Measurements on two 900-MHz-band filters confirmed that the $Q$ of a single resonator could be extracted from an oscilloscope capture of the tail of the filter’s RF pulse response. These measurements are made possible by the coupled-resonator structure of a narrowband filter.

### 4.3 Estimation of Filter Bandwidth from RF Pulse Response

Another tool that would be useful to a filter designer is the ability to determine the bandwidth of a resonant circuit from inspection of its RF pulse response. This section (a) presents the theory behind this technique, (b) simulates the estimation method, and (c) demonstrates its validity with measurements on three cellular-band filters.

#### 4.3.1 Circuit Bandwidth & Rippling in its Pulse Response

The relationship between a filter’s bandpass transfer function $H_{bp}(\omega)$ and its equivalent lowpass prototype transfer function $H_{lp}(\omega)$ is [9]

$$H_{bp}(\omega) = H_{lp}\left(\frac{2\omega}{\omega_2 - \omega_1}\right) \ast \pi \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]$$

(4.13)

where $H_{lp}$ for a high-order filter with linear phase across the passband is approximated by the rectangular bandpass function

$$H_{lp}(\omega) \approx e^{-j\omega_0} \times \begin{cases} 1, & \omega \leq 1 \\ 0, & \omega > 1 \end{cases}.$$  

(4.14)

The Fourier Transform of this lowpass transfer function is the lowpass impulse response (sinc function)

$$h_{lp}(t) = \frac{\sin(t-t_0)}{\pi(t-t_0)}.$$  

(4.15)
The lowpass step response $g_{lp}$ is computed by integrating the lowpass impulse response:

$$g_{lp}(t) = \int_{-\infty}^{t} h_{lp}(\tau) \, d\tau . \quad (4.16)$$

Since the relationship between a narrowband filter’s output at RF may be found by time-scaling and modulating the filter’s lowpass response (following the procedure given in Section 3.4.2),

$$v_{out}^{bp}(t) = v_{out}^{lp} \left( \frac{\omega_2 - \omega_1}{2} t \right) \cos(\omega_0 t) , \quad (4.17)$$

a narrow filter’s step-modulated response $g_{bp}$ may be computed from its step response $g_{lp}$ using the same time-scaling relationship:

$$g_{bp}(t) = g_{lp} \left( \frac{\omega_2 - \omega_1}{2} t \right) \cos(\omega_0 t) \quad (4.18)$$

where the input to the filter is the modulated step function $u^{bp}$ given by

$$u^{bp}(t) = u^{lp} \left( \frac{\omega_2 - \omega_1}{2} t \right) \cos(\omega_0 t) = u(t) \cos(\omega_0 t) . \quad (4.19)$$

By substituting (4.16) into (4.18), the narrow filter’s step-modulated response may be written

$$g_{bp}(t) = \cos(\omega_0 t) \int_{-\infty}^{t} h_{lp} \left( \frac{\omega_2 - \omega_1}{2} \tau \right) \, d\tau \quad (4.20)$$

whose RF envelope (i.e. removing the carrier at $\omega_0$) is

$$\text{env} \left\{ g_{bp}(t) \right\} = \int_{-\infty}^{t} h_{lp} \left( \frac{\omega_2 - \omega_1}{2} \tau \right) \, d\tau . \quad (4.21)$$
The time-derivative of this expression is thus the time-scaled lowpass impulse response:

\[
\frac{d}{dt} \left[ \text{env} \left\{ g^{bp} (t) \right\} \right] = h^{lp} \left( \frac{\omega_2 - \omega_1}{2} t \right). \tag{4.22}
\]

When this \((\omega_2 - \omega_1)/2\) time-scaling is applied to (4.15), the result is

\[
h^{lp} \left( \frac{\omega_2 - \omega_1}{2} t \right) = \frac{\sin \left[ (t - t_0) (\omega_2 - \omega_1)/2 \right]}{\pi (t - t_0) (\omega_2 - \omega_1)/2}, \tag{4.23}
\]

whose nulls occur at

\[
(t - t_0) \left( \frac{\omega_2 - \omega_1}{2} \right) = n \pi \quad t \left( \frac{\omega_2 - \omega_1}{2} \right) = n \pi + k_0 \tag{4.24}
\]

where \(n\) is any integer > 0 and \(k_0\) is the offset for the first null. The distance between each time value for which \(h^{lp} = 0\) is thus

\[
\Delta t \frac{\omega_2 - \omega_1}{2} = n \pi \quad \Delta t = \frac{2\pi}{\omega_2 - \omega_1} = \frac{1}{f_2 - f_1} = \frac{1}{B} \tag{4.25}
\]

which states that the nulls in the time-derivative of a narrowband filter’s step-modulated response \(g^{hp}\) occur at intervals of \(1/B\) where \(B\) is the filter bandwidth in Hz. In other words, the bandwidth of a narrowband circuit may be extracted from a single time-domain trace of the envelope of its pulse response near its resonant frequency.

4.3.2 Simulation

A simulation of the bandwidth estimation method is contained in Fig. 4.8 and Code Listing 4.2. The simulated filter is a seventh-order Chebyshev design with a center frequency of 900 MHz and bandwidth of 36 MHz. Its input and output ports are terminated in 50 \(\Omega\). The voltage source is a 900-MHz sinusoid which switches on at \(t = 0 \mu s\). Samples of the filter output are stored in the file “v_out.out”.

Matlab plots of the filter output and its time-derivative are given in Fig. 4.9. Like the filter pulse responses observed in Chapter 3, the filter output begins to charge,
overshoots its final voltage, and rings around its steady-state value. Contained within the
ringing is information about the filter’s frequency-domain characteristics, e.g. the distance
from one ripple to another is approximately $1/B$.

The first time at which the derivative $dv_{out}/dt$ goes to zero is $t_1 = 78.3$ ns. The
second time at which the derivative goes to zero is $t_2 = 107.1$ ns. By (4.25),

$$\Delta t = t_2 - t_1 = 28.8 \text{ ns} \quad B = \frac{1}{\Delta t} = \frac{1}{28.8 \text{ ns}} = 34.7 \text{ MHz}.$$  (4.26)

This 34.7 MHz estimate is less than 4% away from the true 36 MHz filter bandwidth.

4.3.3 Measurements

Measurements of several 900-MHz Chebyshev bandpass filters demonstrate this
bandwidth estimation method for real circuits. In the experiment, a pulse of 900-MHz at
0 dBm (at 50 Ω) is applied to three Trilithic 7BC900 filters.

The RF source is the Agilent E8267C. The filter response is captured by the
Tektronix TDS684B sampling oscilloscope at 5 GS/s. The Matlab scripts for programming
the E8267C to output a pulsed waveform and for directing the TDS684B to capture a
time-domain trace are given in Appendix A.3.

The filter is charged to steady-state with the 0 dBm sinusoid, the pulse is turned
off at $t = 1 \mu$s, and the filter’s step response at discharge is analyzed. The time-derivatives of the step responses at turn-on and turn-off are equal in magnitude and opposite
in sign. The spacing of their nulls is the same.

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off at $t = 1 \mu$s, and the filter’s step response at discharge is analyzed. The time-derivatives of the step responses at turn-on and turn-off are equal in magnitude and opposite
in sign. The spacing of their nulls is the same.
Listing 4.2: fREEDA netlist — bw_extract.net

* filter step-modulated response
* 7th-order 900-MHz 36-MHz-wide filter

.tran tstop=3e-7 tstep=5e-12 im=1

.vlfmpulse:v1 1 0 vo=1e-20 va=1 td=0e-6 fo=900e6
deltaf=0e6 chirpdir=-1 phi=-90 tau=5e-6 per=10e-6

.chebyshev bpf: bpf1 2 0 3 n=7 f0=900e6 bw=36e6 z0=50 ripple=.1

.r: r1 1 2 r=50
.r: r2 3 0 r=50

.out plot term 3 vt in "v_out.out"

.end

Figure 4.9: Simulated step-modulated response and associated RF envelope time-derivative: 900-MHz 4% Chebyshev filter, RF source at 900 MHz turned on at $t = 0$. The interval between the times for which $dv/dt = 0$ is approximately $1/B$. 
The first null in the filter response occurs at \( t_1 = 1059 \text{ ns} \). The second null occurs at \( t_2 = 1092 \text{ ns} \). This gives

\[
\Delta t = t_2 - t_1 = 33 \text{ ns} \quad \quad B = \frac{1}{\Delta t} = \frac{1}{33 \text{ ns}} = 30.3 \text{ MHz}
\] (4.27)

which is less than 5% away from the true 31.7 MHz bandwidth.

Measurements on three different filters are summarized in Table 4.3. The measured bandwidth values are in good agreement with the values provided by the filter manufacturer. The estimation method is confirmed.

### 4.3.4 Summary

This section introduced a time-domain technique for estimating the bandwidth of a narrowband circuit from its RF pulse response. This estimation method showed that it is not always necessary to apply the Fourier Transform to collect basic frequency-domain information from a time-domain trace.

Sharp and narrow frequency-domain responses correspond to slowly-decaying, rippling time-domain responses. The rectangular shape of a high-order narrowband response corresponds directly to the distance between lobes in its time-domain analogue. This relationship was evident in simulation. The bandwidths of several 900-MHz-band filters were correctly estimated by recording their pulse responses at resonance and taking the inverse of the distances between nulls in their envelopes: \( B = 1/\Delta t \).

### 4.4 Extraction of S-Parameters from Short-Pulse Responses

It is also possible to extract wideband two-port frequency-domain data from a device under test (DUT) using pulsed stimuli, time-domain traces, and a Fourier Transform algorithm. This section shows how this is done (a) mathematically, (b) in simulation, and (c) on the laboratory bench.
Figure 4.10: Measured pulse decay response and RF envelope: Trilithic 7BC900/27-3-KK filter, $IL = 3.15$ dB, 0-dBm 900-MHz pulse removed at $t = 1 \mu s$. The interval between the nulls of $v_{out}$ is approximately $1/B$.

Table 4.3: Filter Bandwidth Estimation Measurements. The bandwidth of three different filters is estimated using (4.25) and compared against the bandwidth printed on the manufacturer’s datasheets.

<table>
<thead>
<tr>
<th>Filter Model (Trilithic)</th>
<th>$B$ from Datasheet (MHz)</th>
<th>$\Delta t$, Measured, 1st 2 Nulls (ns)</th>
<th>$B$, Calculated (MHz)</th>
<th>% Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>7BC900/27-3-KK</td>
<td>31.7</td>
<td>33</td>
<td>30.3</td>
<td>4.4</td>
</tr>
<tr>
<td>7BC900/36-3-KK</td>
<td>38.7</td>
<td>26</td>
<td>38.5</td>
<td>0.5</td>
</tr>
<tr>
<td>7BC900/45-3-KK</td>
<td>48.8</td>
<td>21</td>
<td>47.6</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Figure 4.11: Passband extraction circuit: node numbers correspond to Code Listing 4.3.
4.4.1 Narrow Pulses, Wideband Spectra

The circuit which accomplishes the frequency-domain measurement from time-domain excitation is given in Fig. 4.11. Let the input to the circulator be the RF pulse given by

\[ v_i(t) = A_{RF} \cos (\omega_{RF}t + \phi_{RF}) s(t) \]  

(4.28)

with \( s(t) \) the periodic switching signal given by

\[ s(t) = u(t) - u(t - \tau) = s(t + T) \quad \tau < T. \]  

(4.29)

The Fourier Transform of the input to the filter is thus

\[ V_i(\omega) = A_{RF} e^{j\phi_{RF}} \sum_{k=-\infty}^{+\infty} \frac{\sin(k\pi\tau/T)}{k} \left[ \delta(\omega - k\omega_s - \omega_{RF}) + \delta(\omega - k\omega_s + \omega_{RF}) \right] \]  

(4.30)

where \( \omega_s = 2\pi/T \). An example input pulse is shown in Fig. 4.12(a) and its Fourier Transform is given in Fig. 4.12(b).

It should be noted that, in the frequency domain, the energy of the pulse is spread over a wide bandwidth. The main lobe of the square pulse contains 90% of the signal’s energy, and its nulls occur at \( f_{RF} \pm 1/\tau \); thus, the input pulse of duration \( \tau \) and carrier frequency \( f_{RF} \) (effectively) applies wave energy to the DUT in a bandwidth of \( 2/\tau \) centered at \( f_{RF} \).

The relationships of the measured voltages (in Fig. 4.11) to the input voltage, in the frequency domain, are

\[ V_t(\omega) = S_{21}(\omega)V_i(\omega) \quad V_r(\omega) = S_{11}(\omega)V_i(\omega) \]  

(4.31)

where \( V_t \) and \( V_r \) are transformed (via the Fast Fourier Transform or another FT algorithm) from the time-domain traces \( v_t \) and \( v_r \). The input \( V_i \) is transformed from \( v_i \), which can be
Figure 4.12: Simulated filter input pulse for \( \omega_{RF} = 465 \text{ MHz}, A_{RF} = 0.5 \text{ V}, \phi_{RF} = -90^\circ, \tau = 25 \text{ ns}, T = 4 \mu\text{s} \): (a) time domain, (b) frequency domain (amplitude spectrum). A short time-length pulse contains frequency content across a wide band. The pulse’s spectrum is centered around its carrier frequency.

recorded at Port 4 by short-circuiting the BPF. Thus, two of the two-port S-parameters may be extracted by rearranging (4.31),

\[
S_{21}(\omega) = \frac{V_t(\omega)}{V_i(\omega)}, \quad S_{11}(\omega) = \frac{V_r(\omega)}{V_i(\omega)},
\]

and the S-parameters \( S_{12} \) and \( S_{22} \) may be measured in the same manner as \( S_{21} \) and \( S_{11} \), respectively, but with the DUT’s input and output ports reversed.

4.4.2 Simulation

A fREEDA netlist which simulates the \( S_{21} \) and \( S_{11} \) measurement is given in Code Listing 4.3. The DUT is a fifth-order 465-MHz Chebyshev bandpass filter with 5 MHz of bandwidth and 0.1 dB of passband ripple. The stimulus pulse is a 25 ns burst of a 465 MHz sine wave at an amplitude of 1 V. The simulation runs for 4 \( \mu\text{s} \) in 20-ps increments.

The time domain traces are recorded as follows: \( v_i \) in “v_in.out”, \( v_t \) in “v_trans.out”,
and $v_r$ in “v_refl.out”. The data from these traces is imported into Matlab, the Short-Time Fourier Transform creates the vectors $V_i$, $V_t$, and $V_r$, and (4.32) is used to obtain $S_{21}$ and $S_{11}$. The magnitude spectra of the voltages are given in Figs. 4.13 and 4.14.

The (approximate) shape of the filter’s passband is contained within the odd rippling waveforms of Figs. 4.13(a) and 4.14(a). The Fourier Transform extracts this information. It is worth noting that the shorter the input RF pulse is, (a) the more broadly the input RF energy is spread across frequency, (b) the flatter the input spectrum is near $f_{RF}$, and (c) the more $|V_t|$ and $|V_r|$ resemble $|S_{21}|$ and $|S_{11}|$ before being normalized by $|V_i|$.

For comparison of these time-domain-extracted S-parameters against the steady-state S-parameters, another fREEDA netlist is written. Code for the steady-state simulation is given in Listing 4.4. The differences in Listing 4.4 from Listing 4.3 are (a) the voltage source is changed from a pulsed RF stimulus to a constant 1 V AC, (b) the analysis is changed from transient to AC, and (c) the S-parameters are computed within fREEDA and stored in the files “s_21.out” and “s_11.out”.

The comparison of the time-domain-extracted (TD) S-parameters to the frequency-domain-measured (FD) S-parameters is shown in Fig. 4.15. The two amplitude traces match
Figure 4.13: Simulated filter transmission for the RF pulse of Fig. 4.12 applied to a 5th-order 465-MHz low-ripple Chebyshev design: (a) time domain, (b) frequency domain (amplitude spectrum). The amplitude spectrum of a pulse transmission recorded near the circuit’s operating frequencies resembles its $|S_{21}|$.

very well both inside and outside the filter’s passband.

4.4.3 Measurement

A block diagram of the measurement setup used to validate the time-domain S-parameter extraction is given in Fig. 4.16. The RF pulse source is the Agilent E8267C, which outputs $f_{RF} = 450$ MHz with $A_{RF} = 0$ dBm (at $R = 50$ Ω) for $\tau = 20$ ns, repeating at $T = 3$ µs. The circulator is the Raditek RADC-400-650M-N23-100WR; its operational bandwidth is between 400 and 650 MHz. The filter is the Trilithic 5BC465/5-3-KK; it is a 50-Ω 5th-order Chebyshev tubular bandpass filter with $f_c = 465$ MHz, $B = 5$ MHz, a passband ripple less than 0.05 dB, and an insertion loss of 5.7 dB.
Figure 4.14: Simulated filter reflection for the RF pulse of Fig. 4.12 applied to a 5th-order 465-MHz low-ripple Chebyshev design: (a) time domain, (b) frequency domain (amplitude spectrum). The amplitude spectrum of a pulse reflection recorded near the circuit’s operating frequencies resembles its $|S_{11}|$.

The short pulses are output from the E8267C synthesizer using the same Matlab script (from Sections 4.2.3 and 4.3.3) in Appendix A.2. Data traces are again captured with the Tektronix TDS684B oscilloscope. The TDS684B ports are terminated in 50 Ω, and voltage samples are recorded at a rate of 5 GS/s for a period of 3 µs. The filter transmission is recorded on Channel 1 and the filter reflection is recorded on Channel 2.

The results of the test are given in Fig. 4.17. The time-domain captures of the filter transmission and reflection are shown in Fig. 4.17(a), and the extracted S-parameter magnitudes are shown in Fig. 4.17(b). The filter’s steady-state $S_{21}$ and $S_{11}$, as measured by the Agilent N5030A network analyzer, are the frequency-domain traces for the experimental comparison.
Listing 4.4: fREEDA netlist — pb_compare.net

* filter transmission & reflection response, steady-state
* 5th-order 465–MHz 5-MHz-wide filter

.ac start=450e6 stop=480e6 n_freqs=3001

vsource:v1 1 0 vdc=1.0 vac=1.0 phase=0 delay=0 tr=0

chebyshevbpf:bpf1 3 0 4 n=5 f0=465e6 bw=5e6 z0=50 ripple=.1

circulator:c1 2 3 5 0 0 0

r:r1 1 2 r=50
r:r2 4 0 r=50
r:r3 5 0 r=50

.out plot term 4 vf term 2 vf div mag dB in "s_21.out"
.out plot term 5 vf term 2 vf div mag dB in "s_11.out"

.end

Figure 4.15: Simulated comparison of the time-domain and steady-state S-parameter extractions. $S_{21}$ and $S_{11}$ from both methods match well both inside and outside the circuit’s operating band.

Figure 4.16: Passband extraction measurement system.
Figure 4.17: Measured pulse transmissions and reflections for the 5BC465/5-3-KK filter: (a) time domain, (b) frequency domain (amplitude spectrum). The filter’s transmission and reflection coefficients match well in the neighborhood of $f_{RF}$. $S_{21}$ and $S_{11}$ are unreliable outside of $2/\tau$. 
The traces match well within the frequency band of the stimulus pulse, which is

\[ f_{RF} \pm 1/\tau = 450 \text{ MHz} \pm 1/(20 \text{ ns}) = 450 \text{ MHz} \pm 50 \text{ MHz} \]. \hspace{1cm} (4.33)

The time-domain S-parameter extraction is unreliable outside of this range, however, because the spectral energy of the pulse drops near/under the noise floor of the oscilloscope beyond the $2/\tau$ pulse bandwidth.

It should be noted that this measurement traces out the passband of the filter (between 462 and 468 MHz) even though the carrier frequency of the stimulus pulse is well outside the filter passband ($f_{RF} = 450$ MHz). This result is achieved for any choice of $f_{RF}$ and $\tau$ such that $f_{RF} \pm 1/\tau$ encompasses the filter band.

### 4.4.4 Summary

This section introduced a way to extract two-port S-parameter data from a DUT using time-domain captures of the device’s response to short RF pulses. The method exploits the link between narrow traces in the time domain and wide bands in the frequency domain. A time-domain stimulus for probing a DUT for its S-parameters within a finite bandwidth was constructed from a sinusoidal carrier modulated by a short rectangular envelope. Fourier Transform relationships were used to derive steady-state parameters from time-domain traces of this short-pulse’s reflection and transmission from/through the DUT. Both simulations and measurements on 465-MHz-band filters show that two-port S-parameters may be estimated in this manner over the bandwidth of the applied pulse.

### 4.5 Conclusions

This chapter introduced three linear time-frequency methods for the metrology of resonant circuits. The three techniques are directly applicable to coupled-resonator band-pass structures, although bandwidth estimation is applicable to any bandpass circuit, and S-parameter extraction is applicable to all linear devices.

The first method extracted the loaded quality factor of a filter’s outermost res-
onator(s). Previous techniques were frequency-domain based and were unable to isolate a single resonator. The most relevant methods for calculating $Q$ still required measurements at multiple frequencies. The method presented here characterizes a single resonator while requiring only a single pulse-decay trace recorded at a single excitation frequency. This technique was derived from the equivalence of the time and frequency domain definitions of $Q$. A comparison of the natural responses of a bandpass filter and its lowpass prototype showed that a resonator’s envelope response may be written in a simple exponential form, and its quality factor may be determined in the time domain. Measurements on cellular-band filters confirmed these results.

The second method estimated the operational bandwidth of a bandpass filter. This bandwidth was extracted from the time interval between the nulls in the envelope (at the beginning or at the end) of its pulse response. This technique was derived from the relationship between the time-domain responses of filters and their associated lowpass prototypes. Simulations and measurements on 900-MHz-band filters validated the estimation method.

The third method extracted a device’s two-port S-parameters over a finite bandwidth. A short RF pulse, which contains a continuous band of excitation energy in the frequency domain, was applied to a DUT whose ports were terminated in the system impedance. Parts of this frequency band were transmitted through the DUT and other parts of it were reflected; the ratios of the Fourier Transforms of the transmitted and reflected waveforms to the Fourier Transform of the original stimulus pulse were found to equal the S-parameters of the device within the excitation band. Simulations on narrow filters matched these results exactly. Measurements on real filters matched these results well, and highlighted that the technique is only valid within the effective frequency band of the applied stimulus pulse.

Perhaps the greatest advantage of these three methods is that each enables frequency domain circuit properties to be measured using time-domain traces. Each measurement can be made with a sampling oscilloscope. The first two methods extract the steady-state parameters $Q$ and $B$ without any Fourier analysis, and the third method extracts frequency-dependent transmission and reflection coefficients without using a spectrum analyzer. Filter designers have three additional fast, low-power, inexpensive quality assurance tests at their disposal.
Nonlinear Metrology of Bandpass RF Systems

5.1 Introduction

In addition to linear time-frequency effects, the transient properties of narrowband circuits result in nonlinear effects. Among these nonlinear effects, intermodulation distortion is readily observed. When pulses of different frequencies are applied to a filter, they are smeared together in time by the filtering process and multiple frequencies exist simultaneously at the filter output. If such a multitone pattern encounters a nonlinearity in the circuit following the filter, a mixing of tones will occur which distorts the otherwise linear signal.

Let the input to a nonlinear circuit element be the multitone signal

\[ v_{in} = A_1 \cos (2\pi f_1 t + \phi_1) + A_2 \cos (2\pi f_2 t + \phi_2) \]  \hspace{1cm} (5.1)

where \( A_1 \) and \( A_2 \) are the amplitudes of two sinusoids, \( f_1 \) and \( f_2 \) are the tone frequencies, and \( \phi_1 \) and \( \phi_2 \) are their initial phases. If this signal is applied to a memoryless nonlinearity
whose input-output model is given by

\[ v_{\text{out}}(t) = \sum_{m=1}^{M} a_m v_{\text{in}}^m(t), \]  

(5.2)

where \( a_m \) are the complex power-series coefficients of the element’s response, the output consists of components at harmonics (integer multiples) of the original frequencies \( (f_1, f_2, 2f_1, 2f_2, 3f_1, 3f_2, ...) \), sum-combinations \( (f_1 + f_2, 2f_1 + f_2, 2f_2 + f_1, ...) \), and difference-combinations \( (f_2 - f_1, 2f_1 - f_2, 2f_2 - f_1, ...) \). These sum- and difference-combinations of the original tones are “intermodulation” frequencies. Whereas the harmonics are present if \( f_1 \) and \( f_2 \) are applied to the nonlinearity either simultaneously or one-at-a-time, the intermodulation frequencies exist only if \( f_1 \) and \( f_2 \) are applied to the nonlinearity simultaneously.

The harmonics and sum-combinations of the original tones generally occur well outside the communications band of interest. Since they do not interfere with the signal frequencies, they are usually ignored. Many of the difference-combinations occur at baseband and are also ignored; however, some of the difference-combinations occur at frequencies near to the original tones. Of note are the third-order intermodulation (IM3) tones \( 2f_1 - f_2 \) and \( 2f_2 - f_1 \), which may be produced by a non-zero \( a_3 \) (and/or a higher-order odd \( a_m \)). These intermodulation frequencies are separated from the fundamental tones by only \( \Delta f = f_2 - f_1 \), such that they often appear within the intended communications band. Because these frequencies are the closest intermodulation tones to the original input frequencies, and as such they are most likely to disrupt the operation of the circuit through which they propagate, the amount of RF power generated at these IM3 frequencies is a common measure of the degree of nonlinearity of an RF circuit.

Section 5.2 demonstrates how a switched-tone signal, when converted to steady-state tones by a filter, may be used to measure the nonlinearity of an RF circuit. Section 5.3 gives a method for extracting the passband of a filter by recording the IM3 products reflected back through the filter from a nonlinearity attached to its output. Section 5.4 shows how a filter-amplifier cascade can generate its own IM3 power, which is another potential source of co-site interference. Section 5.5 summarizes this chapter’s results.
5.2 Generation of Multisines from Switched Tones

The resonant properties of bandpass filters produce transient effects that are not widely known. One such effect is the output of multitone interference from the application of switched-tone pulses [46]. Another effect is the output of steady-state multisines from the application of repetitive switched tones. In this section, the mathematics which model the switched-tone to multisine conversion are presented, along with measurements which confirm the effect, a discussion of filter energy storage which explains the observed phenomenon, and a demonstration of how switched tones may be used to measure amplifier nonlinearity.

5.2.1 Switched-Tone to Steady-Tone Theory

Let the input to a bandpass filter (BPF) be the switched-tone signal given by

\[ v_i(t) = \sum_{n=1}^{N} A_n \cos(\omega_n t + \phi_n) s(t - T[n - 1]/N) \]  

where \( \omega_n \) are the frequencies of the tones, \( A_n \) are their amplitudes, \( \phi_n \) are their initial phases, and \( s(t) \) is the periodic switching waveform given by

\[ s(t) = u(t) - u(t - T/N) = s(t + T) \]

where \( T \) is the switching period. The circuit for \( N = 2 \), which converts a two-tone switched signal into a two-tone multisine, is given in Fig. 5.1.

The Fourier Transform of \( s(t) \) is

\[ S(\omega) = \sum_{k=-\infty}^{+\infty} \frac{2\sin(k\pi/N)}{k} \delta(\omega - k\omega_s) \]

where \( \omega_s = 2\pi/T \) is the switching frequency; thus, the Fourier Transform of the input
Figure 5.1: Circuit for converting two switched tones into a two-tone multisine. Only one tone is applied to the filter at any one time.

The voltage $v_i(t)$ is

$$V_i(\omega) = \sum_{k=-\infty}^{+\infty} \frac{\sin (k\pi/N)}{k} \sum_{n=1}^{N} \left\{ A_n e^{j\phi_n} e^{-j\omega T[n-1]/N} \times \left[ \delta (\omega - k\omega_s - \omega_n) + \delta (\omega - k\omega_s + \omega_n) \right] \right\}.$$  \hspace{1cm} (5.6)

Let $S_{21}(\omega)$ be the filter’s forward transmission characteristic. If the switching frequency is much greater than the filter bandwidth, $\omega_s \gg B$, such that only the $k = 0$ terms from (5.6) are passed, then the filtered sum reduces to

$$V_a(\omega) = \sum_{n=1}^{N} \frac{A_n}{N} e^{j\phi_n} S_{21}(\omega_n) \left[ \pi \delta (\omega - \omega_n) + \pi \delta (\omega + \omega_n) \right]$$ \hspace{1cm} (5.7)

whose time-domain equivalent is

$$v_a(t) = \sum_{n=1}^{N} \frac{A_n}{N} |S_{21}(\omega_n)| \cos \left[ \omega_n \left( t - \frac{T}{N}[n-1] \right) + \phi_n + \theta(\omega_n) \right]$$ \hspace{1cm} (5.8)

where $\theta(\omega) = \arg \{ S_{21}(\omega) \}$. The filtered output of the switched-tone input is a multisine signal. Each tone’s amplitude is proportional to its original amplitude $A_n$, reduced by the switching duty cycle $1/N$, and multiplied by the filter’s amplitude characteristic at $\omega_n$. Each tone’s phase is equal to its original phase $\phi_n$, plus the switching delay $T[n-1]/N$, plus the filter’s phase characteristic at $\omega_n$. 

5.2.2 Measurements

A measurement system which demonstrates the switched-tone to multisine conversion is shown in Fig. 5.2. The Agilent E8267C functions as the tone generator and switch; it outputs two tones, spaced 100 kHz apart, centered at \( f_0 = 465 \text{ MHz} \), with a switching frequency of \( f_s = 40 \text{ MHz} \). The Matlab script for programming the Agilent E8267C to output a switched-tone waveform is given in Appendix A.2. The filter is the Trilithic 7BC465/5-3-KK, a low-ripple seventh-order Chebyshev design with a center frequency of 465 MHz, a bandwidth of 5 MHz, and an insertion loss of 9.75 dB. The Tektronix TDS684B oscilloscope records the time-domain waveforms. The Matlab script for programming the TDS684B to capture a time-domain trace is also given in Appendix A.3. Measured data is shown in Fig. 5.3.

The switched-tone input to the filter is shown in Fig. 5.3(a). The waveform switches between 464.95 MHz and 465.05 MHz at 25-ns intervals. It has a nearly-constant envelope, with some amplitude modulation. The residual modulation is due to the 80 MHz modulation bandwidth of the E8267C synthesizer, i.e. this waveform is a digitized version of the ideal switching waveform \( v_a \) with a bandwidth of only 80 MHz. A wider modulation bandwidth would contain a greater number of switching harmonics from (5.6) and produce a more constant envelope.

The multisine output from the filter is shown in Fig. 5.3(b). This plot displays a classic two-tone multisine pattern, whose amplitude has been attenuated from the input waveform by the 50% duty cycle of the switching mechanism and by the insertion loss of the filter. The filter, with a bandwidth of 5 MHz, has attenuated all of the switching harmonics (at 465±40 MHz, 465±80 MHz, etc.), leaving only the fundamental tones around 465 MHz at its output. The switched-tone to multitone conversion is confirmed.

5.2.3 Mechanism for Conversion: Filter Energy Storage

The mathematics of Section 5.2.1 do not imply the physical mechanism of the switched- to multitone conversion. The multisine output can be explained, however, by studying filter energy storage. Several simulations in Advanced Design System (ADS) 2006 provide insight into bandpass filter behavior. Fig. 5.4 is a sample of these simulations. The results of the simulation are given in Fig. 5.5.
Figure 5.2: Test setup for switched-tone to multisine conversion.

Figure 5.3: Measured time-domain illustration of switched-tone to multisine conversion using 7th-order 465-MHz 1% filter: (a) input to filter, (b) output of filter. The filter’s insertion loss is 9.75 dB. A switched-tone waveform is applied to the filter; a multisine waveform emerges from the filter.
Figure 5.4: ADS 2006 simulation of fast-switching tones applied to a bandpass filter. This layout simulates the circuit of Fig. 5.1 and the measurement setup of Fig. 5.2.

Short (25 ns) single-tone pulses are applied to a seventh-order 465-MHz 3%-bandwidth filter. The first simulation is performed with only one tone active \((A_2 = 0)\). The filter input is shown in Fig. 5.5(a), while the filter output corresponding to this input is shown in Fig. 5.5(b).

The pulsed filter response is somewhat counter-intuitive. Whereas the input displays clear intervals where the signal is absent, the filtered output shows no time during which the signal is absent. Although the input to the filter is sometimes switched completely off, the filter output never decays.

The lack of decay in the output can be attributed to the speed at which the input signal changes relative to the speed at which the bandpass filter responds. The response time of the filter is on the order of \(1/B\), while the switching period is \(1/f_s\). Since the switching frequency is much higher than the filter bandwidth, the response time of the filter is much slower than the switching speed. The input switches on and off faster than the time required to charge/discharge the resonant sections of the filter.
Figure 5.5: Simulated fast-switched pulse response of 7th-order 465-MHz 3% Chebyshev filter: (a) input to filter with only $f_1$ active, (b) output from filter with only $f_1$ active, (c) output from filter with $f_1$ and $f_2$ active. $P_i = 0$ dBm/tone in each case, $f_1 = 463$ MHz, $f_2 = 467$ MHz, $f_s = 20$ MHz, filter IL = 9.75 dB. A single-frequency on-off pulse quickly switched at the filter input produces an always-on single-tone wave at the filter output (after the initial transient). A two-frequency switched-tone pulse switched at the same rate produces a steady two-tone multisine.
Applying a series of pulses in this manner forces the filter sections to build resonance, albeit at a slower rate than using longer pulses, without giving the resonant sections time to discharge between each pulse. For this reason, applying a single on-off tone at the filter input produces a steady single tone at its output (after its initial transient) as shown in Fig. 5.5(b). A second on-off tone applied to the input, shifted in time by half of the switching period, produces a similar steady single tone at the output, with a phase shift of half the switching period by (5.8). Adding both inputs produces a constant-envelope switched-tone stimulus, while adding both outputs produces a two-tone multisine, as shown in Fig. 5.5(c). The filter charges to a steady state at any passband frequency, regardless of the time-offset of its input, as long as the input switches faster than its transient response; thus, by rotating through switched tones at its input, any number of multitones may be produced at its output.

5.2.4 IP3 Measurement by Switched Tones

One useful feature of the switched-tone to multisine conversion is the ability to perform multitone measurements using single-frequency synthesizers. In those instances when a multisine source is not available, multiple single-tone sources, a switch, and a narrowband filter will suffice.

An example of such a measurement is that of output IP3. This type of measurement is typically performed on an amplifier or mixer by applying a two-tone source at the DUT input with a small tone separation (1 MHz or less), sweeping the input power at a fixed frequency, and plotting the linear output power and intermodulation power on the same logarithmic axes [52]. The IP3 is found by extrapolating the linear data from the region where the amplifier provides a constant gain over power for the fundamental tones, extrapolating the nonlinear data from the region where the amplifier provides 3 dB of additional nonlinear power for every 1 dB of linear power, and finding the intersection of the two extrapolations. An example of this technique is shown in Fig. 5.6. The intersection of the 1:1 and 3:1 projections takes place at $P_o = 53$ dBm, so this is the measured output-referred third-order intercept point (OIP3) of the amplifier.

The measurement given in Fig. 5.6 is unique, however, because the data is generated using a switched-tone setup as given in Fig. 5.7. The Agilent E8267C provides the
Figure 5.6: Output third-order intercept measurement for the Ophir 5162 amplifier: switched tones with $7^{th}$-order 900-MHz 3% filter, $f_1 = 899.5$ MHz, $f_2 = 900.5$ MHz, $f_s = 40$ MHz. This measurement records the amplifier’s IP3 at 53 dBm; the amplifier datasheet reports its “typical IP3” as 54 dBm.

switched tones, the Trilithic 7BC900/27-3-KK filter performs the switched-tone to multi-sine conversion, and the RF frequencies are chosen to be within the amplifier’s operational bandwidth. A 20-dB coupler is used to tap off a portion of the amplifier’s output signal so as not to damage the spectrum analyzer by feeding it excessive power. The script for capturing frequency-domain traces from the Agilent E4445A spectrum analyzer is given in Appendix A.4.

Table 5.1 lists the IP3 values provided by the manufacturer’s datasheets, measured IP3 using the traditional steady-state technique, and measured IP3 using the switched-tone technique for three different amplifiers. The steady-state technique gives values within 9% of the datasheet values, while the switched-tone technique gives values within 10%. The switched-tone technique can under- or over-estimate the true amplifier IP3, but it is nearly as accurate as the steady-state technique.

1The MiniCircuits ZRL3500 amplifier is measured with a $5^{th}$-order 2-GHz 1% filter with tones at $f_1 = 1999.5$ MHz and $f_2 = 2000.5$ MHz. The RFMD RF2320 amplifier is measured with a $5^{th}$-order 1-GHz 2% filter with tones at $f_1 = 999.5$ MHz and $f_2 = 1000.5$ MHz.
Figure 5.7: Test setup for output IP3 measurement using switched tones: \( f_2 - f_1 = 1 \text{ MHz}, f_s = 40 \text{ MHz} \). The E8267C outputs a switched-tone waveform, the BPF converts the switched tones to multisines for the amplifier (the DUT), and the 20-dB coupler reduces the amplifier output so as not to damage the input port of the E4445A. The 50-Ω termination is capable of dissipating 2 W.

Table 5.1: IP3 Measurement: Two-Tone vs. Switched-Tone. The third-order intercept for three amplifiers are obtained in three different ways: from the manufacturer’s datasheet, from a two-tone steady-state measurement, and from the new switched-tone method.

<table>
<thead>
<tr>
<th>Amplifier</th>
<th>IP3 from Datasheet (dBm)</th>
<th>IP3 from Two-Tone (dBm)</th>
<th>IP3 from Switched-Tone (dBm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MiniCircuits ZRL3500</td>
<td>42</td>
<td>43.1</td>
<td>42.8</td>
</tr>
<tr>
<td>Ophir 5162</td>
<td>54</td>
<td>53.8</td>
<td>52.9</td>
</tr>
<tr>
<td>RFMD RF2320</td>
<td>34</td>
<td>36.8</td>
<td>37.2</td>
</tr>
</tbody>
</table>

5.2.5 Summary

This section demonstrated how to convert switched-tone waveforms to multisine signals using a bandpass filter. The rule of thumb for the conversion was found to be \( \omega_s \gg B \): the switching frequency is much greater than the filter bandwidth. Measurements on 465-MHz-band filters confirm these results.

The waveform conversion was attributed to the speed of energy buildup in the filter relative to the on-off switching of any one tone. At a high switching frequency, the filter does not have enough time between each switch to discharge reactive energy and consequently emits a tone at its output which does not switch off. Applying multiple on-off tones in succession produces a multitone pattern at the filter output.

This multitone pattern was applied to the measurement of amplifier nonlinearity. Bandpass filters in the operational bands of several different amplifiers were employed to
measure their output IP3s. An accurate measure of each amplifier’s IP3 was achieved. This data was taken without a multitone source.

5.3 One-Port Filter Passband Extraction

Another consequence of the multisine conversion is the ability to perform a two-port measurement on a bandpass filter using a single input port. Because multiple simultaneous tones may be generated at the filter output, it is not only possible to transmit intermodulation to the output of a nonlinearity in cascade with the filter, it is also possible to reflect intermodulation from the input of such a nonlinearity, transmit it back through the filter, and measure it when it returns to the filter’s input port.

The nonlinear reflection is present only in the retransmission, whereas linear reflections are generated by both the filter and the amplifier. The linear reflections are not separable at the filter input port. For this reason, only the nonlinear frequency content is able to provide two-port data from the single-port measurement [53]. This section presents the mathematics which govern the nonlinear reflection and provides measured data which show that it is possible to extract the operational bandwidth of a filter using an appropriate frequency sweep in wireline and wireless environments.

5.3.1 Transfer Function Magnitude from IM Products

The circuit used to extract bandpass filter transmission frequencies is shown in Fig. 5.8. The signal path from $v_i$ to $v_r$ resembles the filter-nonlinearity-filter path from prior studies of bandpass nonlinearities [54, 55]. Here, the filters on either side of the nonlinearity (NL) are the same circuit, with the input and output reversed.

The signal power transmitted through the filter to the nonlinearity is

$$P_a(\omega) = |S_{21}(\omega)|^2 P_i(\omega). \quad (5.9)$$

Let the amount of linear power transmitted to the nonlinearity which is converted to re-
Figure 5.8: Circuit for extracting filter passband from nonlinear reflections. Switched tones \( (v_i) \) are sent through the circulator to the filter and converted to multisines by the filter \( (P_a) \); some of the multisine energy is converted to nonlinear energy, reflected by the nonlinear device, and travels back through the filter \( (P_b) \); this backward wave is redirected by the circulator to the measurement port \( (v_r) \).

Reflected intermodulation power in the fundamental zone be

\[
\tilde{P}_b(\omega) = 10^\alpha P_a(\omega)^\beta
\] 

(5.10)

where \( \alpha \) is a logarithmic scaling factor, \( \beta \) is the order of the nonlinearity, and the tilde above \( P_b \) denotes intermodulation. For a two-tone signal, \( P_a(\omega) \) is non-zero only at \( \omega_1 \) and \( \omega_2 \), while \( \tilde{P}_b(\omega) \) is non-zero only at \( \omega_{\text{avg}} \pm \Delta \omega (2m + 1)/2 \) where

\[
\omega_{\text{avg}} = \frac{\omega_1 + \omega_2}{2}, \quad \Delta \omega = \omega_2 - \omega_1, \quad m = 0, 1, 2, \ldots
\] 

(5.11)

If \( S_{12}(\omega) \) is the filter’s reverse transmission characteristic, then the nonlinear power retransmitted through the filter is given by

\[
\tilde{P}_r(\omega) = \lvert S_{12}(\omega) \rvert^2 \tilde{P}_b(\omega).
\] 

(5.12)

It is assumed that the filter itself is linear. For filters that are also reciprocal, \( S_{12} = S_{21} \). For many front-end bandpass filters, the approximation \( \lvert S_{12} \rvert \approx \lvert S_{21} \rvert \) is sufficient. Using this approximation and substituting (5.9) into (5.10) and (5.10) into (5.12) gives

\[
\tilde{P}_r(\omega) = 10^\alpha \lvert S_{21}(\omega) \rvert^{2(\beta+1)} P_t(\omega)^\beta
\] 

(5.13)
which states that the intermodulation power reflected from the filter-nonlinearity cascade is the product of the input linear power raised to the order of the nonlinearity $\beta$, the filter’s power transmission raised to $\beta + 1$, and a logarithmic scaling factor $\alpha$.

The values of $\alpha$ and $\beta$ may be determined by rewriting (5.13) as

$$\log \tilde{P}_r(\omega) = \beta \log P_i(\omega) + (\beta + 1) \log |S_{21}(\omega)|^2 + \alpha.$$  \hspace{1cm} (5.14)

By plotting $\log \tilde{P}_r(\omega)$ at different input power levels $\log P_i(\omega)$, the rightmost two terms in (5.14) remain constant, and $\beta$ is the slope of this plot. If the filter is assumed to be lossless, then $\alpha$ will be the y-intercept of this same plot when the input tones are within the filter passband, i.e. when $S_{21} = 1$ such that the middle term of (5.14) vanishes. If the filter cannot be assumed lossless, then $\alpha$ is not easily determined; however, since the goal of this procedure is to extract the filter passband and not its insertion loss, the value of $\alpha$ is not critical.

The filter passband is extracted by plotting $\log \tilde{P}_r(\omega)$ over frequency at a constant input power. For constant input power, the first and third terms of (5.14) contribute a DC offset to the trace, while the second term traces out the power transmission characteristic of the filter (scaled by $\beta + 1$).

### 5.3.2 Wireline Measurement

A measurement system which demonstrates the passband extraction is shown in Fig. 5.9. The same Agilent E8267C unit is the switched-tone source, and the Ophir 5162 broadband amplifier is used to boost the linear power to a suitable level to excite the nonlinearity. A circulator transmits the switched-tone probe to the filter input port and redirects reflections from the filter-amplifier cascade to the Agilent E4445A spectrum analyzer. The filter output port is connected to the Philips BGA2648 amplifier, which is terminated in 50 $\Omega$ (not shown). The Philips amplifier is chosen as the nonlinearity so that its relatively low input IP3 accentuates the intermodulation response. Three 900-MHz filters (the Trilithic 7BC900/27-3-KK, the 7BC900/36-3-KK, and the 7BC900/45-3-KK) were tested.

An equivalent test setup for the passband extraction is given in Fig. 5.10. Two synthesizers, the Agilent E8267C and the Agilent E4438C, output constant-amplitude single
Figure 5.9: Test setup for filter passband extraction: the nonlinearity is the BGA2748 amplifier which has an input IP3 of $-24$ dBm at 900 MHz and is powered by a 4-Volt supply; the circulator is the Raditek RADC-650-1000M-N23-100WR; $f_s = 40$ MHz. The E8267C outputs switched tones; the Ophir 5162 amplifies the switched tones; the circulator directs the switched tones to the bandpass filter; the bandpass filter converts the switched-tones to multisines; the Philips BGA2748 reflects nonlinear energy back through the filter; the circulator redirects the backward wave; the Agilent 4445A captures the nonlinear reflection.

Figure 5.10: Alternative test setup for filter passband extraction: the two synthesizers are time-multiplexed with a hardware switch; a third waveform generator provides the time-multiplexing control signal.
tones at different frequencies. The two tones are time-multiplexed through a hardware switch, the MiniCircuits ZFSWA-2-46, which is controlled by a third signal generator, the Tektronix AWG2021.\textsuperscript{2} The remainder of the signal path is the same as before.

Fig. 5.11 shows a plot of average reflected IM3 ($m = 2$) power versus frequency for the three filters. Each subfigure of Fig. 5.11 contains a scaled, attenuated version of the filter’s transmission characteristic $S_{21}$ for comparison to the theory of Section 5.3.1. With the exception of the dips at the center of the passbands for the 3% and 5% filters, the shapes of the reflected intermodulation power traces and the scaled filter transmission characteristics match very well in the expected communications band. The passband of each filter is readily identified from these frequency sweeps. Similar identification may be performed on filters in other communications bands so long as the frequency sweeps can be made to encompass the passbands.

5.3.3 Wireless Measurement

A block diagram of the measurement setup used to capture switched-tone intermodulation products in a wireless environment is given in Fig. 5.12. The experiment is much like that of Fig. 5.10; here, the circulator is replaced with a pair of antennas. The “transmit antenna” is the horizontal polarization of an ETS Lindgren 3164-03 dual-polarized broadband horn antenna, and the “receive antenna” is the vertical polarization of the same physical antenna. Because the two polarizations are decoupled (by more than 25 dB), the transmission and reflection paths are sufficiently isolated.

The “target antenna” is another ETS Lindgren 3164-03 using only its vertical polarization.\textsuperscript{3} The target antenna is aligned on the same propagation axis as the transmit/receive antenna, but is tilted at 45° with respect to the transmit/receive H-V polarizations. This arrangement is used so that the target receives the linear signal and retransmits the nonlinear frequency content on the same polarization (i.e. one antenna, if the target were not capable of using dual polarizations), while the probe antenna may still separate the transmit and receive paths using orthogonal polarizations.

Plots of the average IM3 power as recorded by the Agilent E4445A spectrum

\textsuperscript{2}Because the ZFSWA-2-46 switch requires a differential control signal, two complementary synchronized outputs from the AWG2021 (square waves, $V_{\text{low}} = -5$ V, $V_{\text{high}} = 0$ V) are employed.

\textsuperscript{3}The target antenna’s horizontal polarization is terminated in 50 $\Omega$.\n
Figure 5.11: Passband extraction for three 7th-order 900-MHz Chebyshev filters: (a) 3% bandwidth, (b) 4% bandwidth, (c) 5% bandwidth. $P_i = 18$ dBm/tone, $f_{avg} = \omega_{avg}/2\pi$, $f_2 - f_1 = 100$ kHz, $\alpha = -12$, $\beta = 1$. $P_r$ is the average IM3 power. Scaled, attenuated $S_{21}$ for each filter included for comparison. With the exception of the dip in power at midband, the shape of the nonlinear reflected IM3 trace matches that of $|S_{21}|$ near the filter passbands.
**Figure 5.12:** Block diagram of wireless nonlinear switched-tone reflection measurement system: two ETS Lindgren 3164-03 antennas are oriented at 45° to each other; the distance from the back-plane of one antenna to the back-plane of the other is 2.0 m; the bandpass filter is the Trilithic 7BC900/27-3-KK. The measurement setup is the same as Fig. 5.10 except that the circulator is replaced with a pair of antennas.

**Figure 5.13:** Wireless switched-tone reflection measurement results: $P_i = 41$ dBm/tone, $f_2 - f_1 = 7$ MHz, $f_s = 40$ MHz, $\alpha = -60$, $\beta = 1$. Scaled, attenuated $S_{21}$ for the 7BC900/27-3-KK filter included for comparison. The passband is not as well-defined as in Fig. 5.11, but may yet be estimated from the recorded frequency sweep.
analyzer for a switched-tone sweep between 865 MHz and 935 MHz are shown in Fig. 5.13. The data shows a trace that is similar to the wireline data of Fig. 5.11, although the passband is not as well-defined as in the well-controlled coaxial setup. Still, this data does confirm that the wireline filter passband characterization of Figs. 5.9 and 5.10 may be extended to a wireless channel.

5.3.4 Summary

This section presented a method for determining a filter’s passband using the non-linear properties of the circuit attached to its output. The switched-to-multisine conversion was used to generate intermodulation at the input of the nonlinearity and the reflection of this nonlinearity back through the filter was used to characterize the filter.

Power calculations showed how the order of the nonlinearity could be extracted from a power sweep and the shape of the filter’s transmission coefficient could be extracted from a frequency sweep in the neighborhood of the filter passband. Measured traces of the nonlinear power and $|S_{21}|$ match well in a wireline, coaxial environment; these same traces still match, but to a lesser degree, in a wireless environment.

5.4 IM Distortion in Frequency-Hopping Systems

Although filter memory enables multisine testing and filter characterization, there are notable negative consequences to the multisine conversion. One such consequence is the signal-to-noise degradation in a frequency-hopping scheme which implements a high-order filter in its receiver [46]. Another consequence is the IMD produced in a similar scheme which amplifies the signal after filtering it. This section explains the origin of this distortion and presents measurements which confirm its existence in a frequency-hopping scenario.

5.4.1 Origin of IMD in Frequency-Hopping

A switched-tone signal consisting of a single switch between two frequencies, when applied to a bandpass filter, is known to produce two-tone interference at the filter output for particular input frequency combinations. This is because wave energy at different fre-
frequencies travels through the filter at different speeds, so that an interference pattern may be produced at the filter output by inputting an frequency that travels slowly through the filter and switching to a frequency that travels quickly through the filter [46].

Despite the transitory nature of the single switching event, a multisine waveform will still exist at the filter output and this multisine signal will be applied to an amplifier that follows the filter, which in turn produces IMD at its output because of its inherent non-linearity. Similar to the way the steady-state two-tone excitation produced the steady-state intermodulation discussed in Sections 5.2.4 and 5.3, here a transient two-tone pattern produces transient intermodulation. Such intermodulation is detrimental to a communications system because the IMD frequencies are often inside or nearby the intended communications band.

5.4.2 Measurements

A measurement system which demonstrates IMD generated in a frequency-hopping scenario is shown in Fig. 5.14. The Agilent E4438C synthesizer acts as User 1 transmitting data at carrier frequency $f_1$ while the Agilent E8267C synthesizer acts as User 2 transmitting data at carrier frequency $f_2$ (or, equivalently, User 1 transmitting data after having hopped to frequency $f_2$ from $f_1$). The MiniCircuits ZFSWA-2-46 switch time-multiplexes the two users to the Trilithic 7BC900/27-3-KK filter and Ophir 5162 amplifier which act as the RF front-end of the receiver. The average IM3 power (divided by the average fundamental tone power) recorded by the Agilent E4445A for several combinations of user frequencies is plotted with a grayscale colormap in Fig. 5.15.

A number of results in Fig. 5.15 are worth noting. First, the intermodulation generated by users whose frequencies are within or near the filter band is always above $-120$ dBc and is sometimes as high as $-83$ dBc. The highest levels of distortion are recorded for starting frequencies near the filter passband edges and ending frequencies well inside the passband (e.g. $f_1 = 887$ MHz, $f_2 = 900$ MHz). This trend is expected, though, because of the aforementioned dependence of signal speed on input frequency: by switching to a “faster” frequency after a “slower” one, a greater degree of multitone interference is produced at the filter output, resulting in a greater amount of distortion generated by the amplifier. Second, the IMD is no less than $-110$ dBc for frequency combinations well
within the filter’s rated passband. This result means that users transmitting inside the communications band generate distortion in the receiver front-end at frequencies within the communications band. This distortion is then termed co-site interference. Third, there is considerable distortion recorded for frequencies outside the filter passband. The gray stripes in the lower-left and upper-right corners of Fig. 5.15 contain frequency combinations well outside the filter passband whose IMD is generally above $-100$ dBc. This result means that users transmitting outside the communications band still generate distortion in the receiver front-end at frequencies outside the communications band, despite the expected attenuation of these frequencies by filtering. Fourth, there exist many combinations of frequencies, one outside the filter passband and one inside the filter passband, which will generate IMD in the receiver front-end at frequencies within the communications band (e.g. $f_1 = 882$ MHz, $f_2 = 890$ MHz, $2f_2 - f_1 = 898$ MHz). This result means that it is possible for a transmitter operating in one channel to cause IMD within the communications band of a receiver operating in an adjacent channel if the pulse transitions within the two channels take place at approximately the same time.

5.4.3 Summary

This section identified intermodulation distortion caused by the coupling of a band-pass filter with an amplifier — a common pairing in RF front-ends — as another source of co-site interference in wireless communication systems.

Data taken using cellular-band filters showed that a communications scheme which implements a filter-amplifier cascade and whose received data comes from different frequen-
Figure 5.15: Measured intermodulation distortion for a single switching event: Tone 1 is applied for 112 ns, then tone 2 is applied for 112 ns, then both are switched off for 226 ns; $P_i = -11$ dBm/tone. The passband of the filter is 885 MHz to 915 MHz. Significant IM3 content is visible inside the communications band for tone combinations both inside and outside the filter band.

cies in adjacent time slots, whether they are within the communications band or not, is subject to IMD caused by filter memory. The degree of distortion was related to the location of user frequencies relative to the filter passband and to the order in which the frequencies are used.

5.5 Conclusions

Although circuit measurements have traditionally been performed using linear and steady-state methods, additional information may be obtained by using nonlinear and transient methods. Resonant energy storage, while generally not an intended behavior, is always present when signals are applied to bandpass filters. This energy storage, along with both time- and frequency-domain analysis, enables circuits to be measured in ways that are not possible in linear steady-state.

This chapter presented a novel method for measuring circuit nonlinearities using switched-tone waveforms. By increasing the switching frequency on the switched-tone
stimuli used in Chapter 3, steady-state multisine waveforms were generated from the same bandpass filters. These multisine waveforms were applied to the measurement of amplifier IP3 and to the one-port extraction of a filter’s passband.

A serious drawback of the switched-to-multisine waveform conversion was discovered which can cause problems for systems that employ frequency-switching in addition to filtering, as in frequency-hopping communications. Switching between frequencies may lead to distortion at the transitions between pulses due to the pulse-smearing (in time) caused by filter memory and the inherent amplifier nonlinearity.

The data presented was a worst-case scenario for a frequency-hopping scheme which places no guard-band between user pulses. One way to reduce the amount of distortion is to place a waiting period between user transmissions which will lessen the pulse overlap caused by the filter transient and cut down on the multitone interference presented to the amplifier input, thereby reducing the distortion generated at the amplifier output. Care must be taken so that a bandpass filter, which is designed to block extraneous frequency content, does not itself create interference within its intended communications band.
Linear Amplification by

Time-Multiplexed Spectrum

6.1 Introduction

As explained in Section 5.1, the interactions between multitone signals and the nonlinear characteristics of an RF circuit produce harmonic and intermodulation distortion. The amplitude modulation in the RF signal (i.e. its slowly-varying envelope magnitude) mixes down to zero-frequency, up to harmonics of the fundamentals, and up/down to integer combinations of the input tones.

This IMD is a problem for wireless transmitters when generating high-power communication signals. A tradeoff exists between power level and linearity. Generally, as the RF power generated within a circuit increases, the nonlinear portion of the signal increases. Under low power, memoryless RF components behave in a linear fashion; at a given frequency, each component may be analyzed as a superposition of linear circuit elements. These same components, however, cannot pass an unlimited amount of RF power. When they begin to saturate, their power-in-to-power-out curves level off; the input-output characteristic is no longer a linear relationship.
The input-output curve of a memoryless device may be modeled by a power-series representation as in (5.2). The $a_m$ terms for which $m > 1$ produce harmonic distortion when individual frequencies are present and intermodulation distortion when multiple frequencies are present. The method presented in this chapter, Linear Amplification by Time-Multiplexed Spectrum (LITMUS), is a new time-frequency signal processing technique to reduce intermodulation distortion produced by multitone amplification by ensuring that only one frequency is applied at the input of a nonlinear amplifier at any one time.

Section 6.2 presents the theory of time-multiplexed sinusoids passed through a nonlinearity for two input tones. Section 6.3 provides measured data to confirm the distortion reduction of the time-multiplexing process predicted by Section 6.2 and extends LITMUS to higher numbers of tones and random phases experimentally. Section 6.4 shows how the time-multiplexing process is undone and the desired high-power amplitude-modulated signal is restored by narrowband filtering. Section 6.5 summarizes this chapter’s results.

6.2 Distortion Reduction Theory

In this section, the theory of LITMUS is described, beginning with an analysis of time-multiplexed sinusoids passed through a nonlinearity. Analysis proceeds with a time-multiplexed two-tone signal to simplify the analysis and clearly illustrate the principals behind LITMUS. The analysis reveals similarities between time-multiplexed sinusoids and cancellation linearization schemes in the sense that distortion is reduced, in comparison to not using the technique, by generation of intermodulation components that cancel the intermodulation distortion. In contrast, time-multiplexed sinusoids do not correct for specific nonlinear characteristics; rather, intermodulation distortion generation is suppressed as a result of the technique regardless of the specifics of the nonlinear circuit.

6.2.1 Time-Multiplexed Sinusoids

The theory of time-multiplexed signals [56] is used to establish a description of a sum of time-multiplexed sinusoidal signals. Let the input to the amplifier of Fig. 6.1 be the
Figure 6.1: LITMUS circuit architecture. The switch applies only one tone to the input of the amplifier at one time. Using ideal switching and equal amplitudes $A_n = A$, the peak-to-average ratio of $v_i$ is 1.

Figure 6.2: Example switched-tone spectrum from (6.5) as generated by the Polyphase Microwave QM3337A modulator, amplified by the MiniCircuits ZHL-1042J amplifier, and recorded by the Agilent E4445A spectrum analyzer, $N = 10$, $P_i = -25.4$ dBm/tone: (a) wideband view, sinc envelope evident; (b) narrowband view, individual tones evident. Replicas of the narrow spectrum from (b) appear at every $\pm k\omega_s$ offset from the fundamental tones $\omega_1...\omega_N$ in (a). The first switching lobe is between 3.48 and 3.63 GHz.
switched-tone (time-multiplexed sinusoidal) signal given by

\[ v_i(t) = \sum_{n=1}^{N} A_n \cos (\omega_n t + \phi_n) s \left( t - T_s \frac{n-1}{N} \right) \]  \hspace{1cm} (6.1)

where \( N \) is the number of tones, \( A_n \) are the amplitudes of each tone, \( \omega_n \) are the frequencies of each tone, \( \phi_n \) are the initial phases of each tone, and \( s(t) \) is the periodic switch waveform given by

\[ s(t) = u(t) - u(t - T_s/N) = s(t + T_s) \]  \hspace{1cm} (6.2)

where \( T_s \) is the switching period. The Fourier Transform of \( s(t) \) is

\[ S(\omega) = \sum_{k=-\infty}^{+\infty} \frac{2 \sin (k\pi/N)}{k} \delta (\omega - k\omega_s) \]  \hspace{1cm} (6.3)

where \( \omega_s = \frac{2\pi}{T_s} \) is the switching frequency and each \( \pm k \) value is a harmonic of this frequency. Including only \( k = -1...1 \) in this expansion produces sinusoidal switching at \( \omega_s \); more ideal switching is achieved with higher values of \( k \).

The frequency-domain representation of \( v_i(t) \) equals the convolution of this switching signal with the fundamental tone set:

\[ V_i(\omega) = \frac{1}{2\pi} \sum_{n=1}^{N} \left\{ A_n e^{j\phi_n} \left[ \pi \delta (\omega - \omega_n) + \pi \delta (\omega + \omega_n) \right] \ast S(\omega) e^{-j\omega T_s[n-1]/N} \right\}. \]  \hspace{1cm} (6.4)

Performing the convolution gives

\[ V_i(\omega) = \sum_{n=1}^{N} A_n e^{j\phi_n} \sum_{k=-\infty}^{+\infty} \left\{ \frac{\sin(k\pi/N)}{k} e^{-j\omega T_s(n-1)/N} \times \left[ \delta (\omega - k\omega_s - \omega_n) + \delta (\omega - k\omega_s + \omega_n) \right] \right\}. \]  \hspace{1cm} (6.5)

An example switched-tone spectrum for \( N = 10 \) is given in Fig. 6.2. When transformed
back into the time domain, \( V_i(\omega) \) becomes

\[
v_i(t) = \sum_{k=-\infty}^{+\infty} \frac{B_k}{\pi} \sum_{n=1}^{N} \{ A_n \cos \left[ (\omega_n + k\omega_s) \left( t - \frac{n-1}{N}T_s \right) - \phi_n \right] \}
\] (6.6)

where \( B_k = \sin \left( \frac{k\pi}{N} \right) / k \). The value of the sinc function at \( k = 0 \) is \( \pi/N \). The time-multiplexed signal consists of the desired multitone signal weighted by \( 1/N \), plus a sum of aliased versions of the multitone signal centered at multiples of the switching frequency offset to the center frequency.

### 6.2.2 Intermodulation Cancellation

In order to compare the linearity of an amplified switched-tone input to an amplified steady-state multitone signal with equal amounts of power in the fundamental tones, \( v_i(t) \) from (6.6) is multiplied by \( N \) prior to the following analysis.

Assume that the output voltage of a memoryless amplifier may be written

\[
v_o(t) = \sum_{m=1}^{M} a_m v_i^m(t),
\] (6.7)

where \( a_m \) are the complex power series coefficients of the amplifier response. To keep the expansions which follow (6.7) tractable, let \( C_N \) represent the inner summation of (6.6):

\[
C_N = \sum_{n=1}^{N} A_n \cos \left[ (\omega_n + k\omega_s) \left( t - \frac{n-1}{N}T_s \right) + \phi_n \right].
\] (6.8)

The frequency terms of interest generated by (6.7) are those which fall closest to the fundamental tones \( \omega_1...\omega_N \). These terms are the result of a non-zero \( a_3 \):

\[
a_3 v_i^3(t) = a_3 \left\{ \sum_{k=-\infty}^{+\infty} \frac{B_k}{\pi} C_N \right\}^3
\]
When cubing (6.6), $k$ must be split into three different indices — $\alpha$, $\beta$, and $\delta$ — to account for all unequal-index permutations that are possible when multiplying three sums.

Consider $N = 2$. If the bandwidth of the amplifier is such that it encapsulates only the fundamental tones ($k = 0$), the third-order intermodulation products, i.e. the subset of tones from the triple-product that fall closest to $\omega_1$ and $\omega_2$, are

$$v_{\text{IM3}}(t) = \frac{3}{4} a_3 \left\{ A_1^2 A_2 \cos \left[ (2\omega_1 - \omega_2) t + \omega_2 T_s/2 + 2\phi_1 - \phi_2 \right] + A_1 A_2^2 \cos \left[ (2\omega_2 - \omega_1) t - \omega_2 T_s + 2\phi_2 - \phi_1 \right] \right\}.$$  \hfill (6.10)

The $3/4$ coefficient is the classical result for two-tone steady-state intermodulation. If, in addition to the fundamental tones ($k = 0$), the first set of switching harmonics $k = \pm 1$ is passed by the amplifier, additional terms are generated at integer multiples of the switching frequency $\omega_s$ away from $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$. The terms around $2\omega_1 - \omega_2$ fall at the frequencies

$$\omega_{\alpha,\beta,\delta} = (2\omega_1 - \omega_2) + (\alpha - \beta - \delta) \omega_s.$$  \hfill (6.11)

where $\alpha$, $\beta$, and $\delta$ are interchangeable. In any case, the sum of two indices is subtracted from the third. The combinations of $\alpha$, $\beta$, and $\delta$ which produce spectral content at the third-order intermodulation frequency $2\omega_1 - \omega_2$ for $k = -1...1$ are listed in Table 6.1, along with the amplitude and phase of each associated component. The “$A/a_3 A_1^2 A_2$” column is the amplitude of each term normalized by $a_3 A_1^2 A_2$. The “$\Delta \phi$” column is the phase of each IM3 component with respect to the phase of the $\alpha = \beta = \delta = 0$ non-multiplexed intermodulation distortion. The table for IMD products generated at $2\omega_2 - \omega_1$ is the same, except $A/a_3 A_1 A_2^2$ replaces $A/a_3 A_1^2 A_2$.

Some of the switching harmonics produce IM3 products which add to the non-multiplexed distortion (e.g. $\alpha = 0$, $\beta = -1$, $\delta = +1$), while others are $\pm \pi$ out-of-phase with the non-multiplexed products and subtract from the total distortion (e.g. $\alpha = +1$, $\beta = 0$, $\delta = +1$). If the amplifier passes the switching harmonics $k = \pm 1$ along with the fundamental
Table 6.1: IM3 Generated by Switching Harmonics, $N = 2$ and $k = -1...1$. The normalized amplitude ($A/a_3A_1A_2^2$) and phase ($\Delta \phi$) of each IM3 term at $2\omega_1 - \omega_2$ is listed by its triple-product indices $\alpha-\beta-\delta$. Some terms add to the non-multiplexed distortion with a phase of 0; other terms subtract from the distortion with a phase of $\pm \pi$.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$A/a_3A_1A_2^2$</th>
<th>$\Delta \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>$0$</td>
<td>$2/\pi^2$</td>
<td>$-\pi$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$0$</td>
<td>$-1$</td>
<td>$2/\pi^2$</td>
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</tr>
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<tr>
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<td>$0$</td>
<td>$+1$</td>
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<tr>
<td>$+1$</td>
<td>$+1$</td>
<td>$0$</td>
<td>$2/\pi^2$</td>
<td>$+\pi$</td>
</tr>
</tbody>
</table>

Tones at $k = 0$, the (normalized) amplitude of the third-order intermodulation is

$$V_{IM3} = \frac{3}{4} + 2 \frac{1}{\pi^2} - 4 \frac{2}{\pi^2} = \frac{3}{4} \left(1 - \frac{8}{\pi^2}\right),$$

(6.12)

which is a 14.5-dB reduction in intermodulation distortion from $k = 0$ only.

With higher-order switching harmonics ($|k| \geq 2$), the number of $\alpha-\beta-\delta$ combinations which add to the non-multiplexed distortion increases, but the number of combinations which are $\pm \pi$ out-of-phase with the distortion also increases. The subtraction always outweighs the addition while both converge to zero.

Upon writing the amplitude of the intermodulation distortion for when the amplifier passes the switching harmonics $k = -7...7$, a pattern emerges:

$$V_{IM3} = \left[\frac{3}{4} - \frac{6}{\pi^2} - \left(\frac{6}{\pi^2}\right) \left(\frac{1}{9}\right) \right] a_3A^3.$$

(6.13)

The intermodulation distortion as the amplifier passes the switching harmonics $-\eta...\eta$ is thus

$$V_{IM3} = \left[\frac{3}{4} - \frac{6}{\pi^2} \sum_{k=1}^{\eta} \frac{1}{k^2} \right] a_3A^3.$$

(6.14)
The sum may be rewritten using the relationship

$$\sum_{k=1, k\text{ odd}}^{\eta} \frac{1}{k^2} = \frac{3}{4} \sum_{k=1}^{\eta} \frac{1}{k^2}$$

(6.15)

such that (6.14) becomes

$$V_{IM3} = \frac{3}{4} \left[ 1 - \frac{6}{\pi^2} \sum_{k=1}^{\eta} \frac{1}{k^2} \right] a_3 A^3.$$

(6.16)

The reduction in IM3 from the non-multiplexed case, in decibels, is

$$\kappa = -20 \log_{10} \left[ 1 - \frac{6}{\pi^2} \sum_{k=1}^{\eta} \frac{1}{k^2} \right].$$

(6.17)

Evaluating (6.15) as $\eta \to \infty$ gives

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \zeta(2) = \frac{\pi^2}{6}$$

(6.18)

where $\zeta(2)$ is the Riemann Zeta Function evaluated at power 2. Substituting this result into (6.16) gives

$$\lim_{\eta \to \infty} (V_{IM3}) = \frac{3}{4} \left[ 1 - \frac{6}{\pi^2} \left( \frac{\pi^2}{6} \right) \right] a_3 A^3 = 0.$$

(6.19)

This result states that the theoretical third-order intermodulation distortion at the output of the amplifier, as the circuit passes all switching harmonics of the fundamental two tones, is zero. It can also be shown that cancellation of higher-order intermodulation distortion occurs by the same mechanism.

For $N = 4$ and $k = -4...4$, Table 6.1 is expanded to Table 6.2. The amplitudes and phases correspond to spectral content at the frequency $2\omega_4 - \omega_1$. As with $N = 2$, there are some $\alpha-\beta-\delta$ combinations which add to the non-multiplexed distortion. Some combinations
Table 6.2: IM3 Generated by Switching Harmonics, $N = 4$ and $k = -4 \ldots 4$. The normalized amplitude and phase of each IM3 term at $2\omega_4 - \omega_1$ is listed by its triple-product indices. Some terms add to the non-multiplexed distortion with a phase of 0, some terms subtract from the distortion with a phase of $\pm \pi$, and the rest of the terms cancel each other in pairs.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$A/a_3 A_1^4 A_1^4$</th>
<th>$\Delta \phi$</th>
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<td>0</td>
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<td>0</td>
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<td>$-\pi/2$</td>
</tr>
<tr>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>$4/3\pi^3$</td>
<td>$-\pi/2$</td>
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<tr>
<td>-3</td>
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<td>-2</td>
<td>$4/3\pi^3$</td>
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</tr>
<tr>
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<td>-3</td>
<td>$2/9\pi^2$</td>
<td>$-\pi/2$</td>
</tr>
<tr>
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<td>-3</td>
<td>+1</td>
<td>$4/3\pi^3$</td>
<td>$-\pi$</td>
</tr>
<tr>
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<td>-2</td>
<td>-1</td>
<td>-1</td>
<td>$4/\pi^3$</td>
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</tr>
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<td>$2/\pi^2$</td>
<td>$-\pi$</td>
</tr>
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<td>-3</td>
<td>$4/3\pi^3$</td>
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<td>-2</td>
<td>+2</td>
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<tr>
<td>0</td>
<td>-1</td>
<td>+1</td>
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<td>$1/\pi^2$</td>
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<td>-3</td>
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<tr>
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<td>-2</td>
<td>+3</td>
<td>$4/3\pi^3$</td>
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<td>+2</td>
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<td>$16/\pi^3$</td>
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<td>$4/3\pi^3$</td>
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<td>$+\pi$</td>
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<td>+1</td>
<td>$4/\pi^3$</td>
<td>$+\pi$</td>
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<td>+3</td>
<td>$2/9\pi^2$</td>
<td>$+\pi/2$</td>
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<tr>
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<td>+1</td>
<td>+2</td>
<td>$4/3\pi^3$</td>
<td>$+\pi/2$</td>
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<td>+2</td>
<td>+1</td>
<td>$4/3\pi^3$</td>
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</tr>
<tr>
<td>+3</td>
<td>+3</td>
<td>0</td>
<td>$2/9\pi^2$</td>
<td>$+\pi/2$</td>
</tr>
<tr>
<td>+4</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
subtract from the distortion with phases of ±π. The rest of the combinations are offset in phase from the α = β = δ = 0 distortion by multiples of π/2, but they cancel each other in pairs. The α = −1 terms are offset by +π/2, while the α = +1 terms are offset by −π/2. Because each set has the same series of amplitudes, they cancel. The α = −3 and α = +3 components add to zero the same way. The k = ±4 combinations do not add to or subtract from the distortion because at least one sinc function goes to zero if any of the three indices equals N.

Using k = −4...4, the total (normalized) distortion is

\[
V_{\text{IM3}} = \frac{3}{4} + \frac{58}{9\pi^2} - \left[ \frac{40}{3\pi^3} + \frac{8}{\pi^2} \right]
\]

(6.20)

\[
= \frac{3}{4} \left( 1 - \frac{56}{27\pi^2} - \frac{160}{9\pi^2} \right)
\]

(6.21)

which is a 13.3-dB reduction in intermodulation distortion from k = 0.

As with N = 2, for higher k values, the number of combinations which are ±π out-of-phase with the distortion outpaces the number of α-β-δ combinations which add to the non-multiplexed distortion, and the theoretical limit as the nonlinearity passes an infinite number of switching harmonics is \( V_{\text{IM3}} = 0 \).

For higher values of N and one full sinc-lobe of switching harmonics \( k = -N...N \), the span of phases −π...π becomes increasingly subdivided across these α-β-δ sets. With N tones, the terms are offset from each other by multiples of 2π/N. For even N, there will exist α-β-δ sets which cancel the original distortion at rotations of exactly −π and +π, while the other sets cancel themselves in pairs. For odd N, the distortion will not cancel with phase shifts of exactly ±π, but overall it will still be reduced.

The intuitive idea that intermodulation distortion is not generated, because only one tone at a time is presented to the nonlinearity, is shown to be true when using ideal switching with infinite bandwidth to pass all harmonics; however, distortion reductions of 13-14 dB are achieved when only the first ±k switching aliases are transmitted through the nonlinearity.
6.2.3 Summary

The theory behind LITMUS showed that the intermodulation distortion associated with amplitude modulation of a multitone signal can be reduced by time-multiplexing the signal’s tones before they are applied to a nonlinearity. When using ideal switching with infinite bandwidth, no intermodulation distortion is generated because only one tone is presented to the nonlinearity at any time. Still, by including only the first set of switching aliases (i.e. using a highly non-ideal sinusoidal switch waveform) reductions in IM3 from the non-multiplexed case between 13 and 14 dB are achieved.

6.3 Experimental Validation

Three sets of measurements are taken to validate the theory presented in Section 6.2. In this section, distortion reduction is demonstrated experimentally for (a) two tones, for a wide range of switching harmonics passed by the amplifier, (b) four tones, with several sets of non-zero phases and a range of power levels, and (c) up to 20 tones over a 3-MHz signal bandwidth.

6.3.1 Narrowband Measurements, 2 Tones

A measurement system which demonstrates the distortion cancellation for \( N = 2 \) is given in Fig. 6.3. The signal generator is the Agilent E8267C. The two tones are centered on \( f_0 = 465 \) MHz, spaced 100 kHz apart, and time-multiplexed with a switching frequency of \( f_s = 6 \) MHz. The Matlab script for programming the E8267C to output the multiplexed (and non-multiplexed) waveforms is given in Appendix A.2. The amplifier is the Amplifier Research 10W100C. A 20-dB coupler is used to tap off a portion of the amplifier’s output so as not to damage the spectrum analyzer. The spectrum analyzer is the Agilent E4445A. The results of this test are listed in Table 6.3.

The Agilent E8267C generator implements software filtering to control the number of switching harmonics applied to the amplifier. First, only the fundamental two tones are applied to the AR10W. The input power of these tones is adjusted to achieve a baseline distortion of \(-30.0\) dBc at the amplifier output. Next, the switching harmonics \( k = \pm 1,\)
Figure 6.3: Test setup for $N = 2$ distortion reduction. The Agilent E8267C generator has a digital-to-analog sampling rate of 90 MS/s. The input power $P_i$ is adjusted so that $P_a$ is 16.2 dBm at each of the two fundamental tones.

Table 6.3: Distortion Reduction by Amplifying Spectral Replicas, $N = 2$. The first column contains the $k$ values that are applied to the amplifier input; the second column contains the measured IM3 for these $k$’s that are active; the third column is the difference in distortion between the measured IM3 and -30.0 dBc, which is compared against the theoretical reduction calculated using (6.17) in the fourth column.

<table>
<thead>
<tr>
<th>Active $k$</th>
<th>Measured Distortion (dBc)</th>
<th>Measured Reduction (dB)</th>
<th>Theoretical Reduction (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-30.0</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>-1, 0, 1</td>
<td>-44.7</td>
<td>14.7</td>
<td>14.5</td>
</tr>
<tr>
<td>-3...3</td>
<td>-48.5</td>
<td>18.5</td>
<td>20.1</td>
</tr>
<tr>
<td>-5...5</td>
<td>-50.3</td>
<td>20.3</td>
<td>23.5</td>
</tr>
<tr>
<td>-7...7</td>
<td>-51.2</td>
<td>21.2</td>
<td>26.0</td>
</tr>
<tr>
<td>-9...9</td>
<td>-52.0</td>
<td>22.0</td>
<td>27.8</td>
</tr>
<tr>
<td>-11...11</td>
<td>-52.7</td>
<td>22.7</td>
<td>29.5</td>
</tr>
</tbody>
</table>

in addition to the fundamental tones at $k = 0$, are passed to the amplifier. The distortion drops to $-44.7$ dBc, a 14.7 dB reduction, which is in good agreement with the 14.5 dB reduction predicted by (6.12). Even for highly non-ideal (i.e. sinusoidal) switching, the IM3 is significantly reduced.

Passing additional switching harmonics further reduces the IM3 seen at the amplifier output. The measured reduction deviates from the theoretical reduction for higher values of $k$ because the frequency response of the amplifier is not flat over a wide bandwidth. When the frequency response is imbalanced, $a_0...a_p$ are not necessarily symmetric about $k = 0$, each term in the cancellation summation in (6.14) deviates from $1/k^2$, and
the distortion does not approach zero as indicated by (6.19).

The gain of the AR10W varies by several decibels over the span of $k = -11$ (corresponding to 399 MHz) to $k = +11$ (corresponding to 531 MHz). Thus, when all the harmonics between $k = -11$ and $k = +11$ are amplified, the distortion reduction deviates from the theoretical value by 6.8 dB. Still, the experiment indicates that a reduction in amplifier distortion of greater than 20 dB is possible by including spectral replicas of the fundamental tones at the switching harmonics $k = \pm 1, \pm 3, \pm 5$ in the amplification. A reduction of greater than 14 dB is possible by including only one switching harmonic ($k = \pm 1$).

Additional measurements performed with a commercial RFIC power amplifier demonstrate the improvement in effective linearity of a practical amplifier when using LITMUS. The RFMD RF2320 GaAs MESFET amplifier, with a specified OIP3 of 35 dBm and a P1dB output compression point of 23.5 dBm, replaces the AR10W [49]. The amplifier is then measured at several power levels with a time-multiplexed two-tone test signal for $k = 0$ (non-multiplexed) up through $k = \pm 13$. The tone spacing is again 100 kHz, centered at 465 MHz. The measurement results are summarized in Fig. 6.4. The results show a reduction in third-order intermodulation distortion ranging from 16 to 28 dB for $k = \pm 1$ up to $k = \pm 13$ at $P_o = 13$ dBm, and the improvement degrades by 4.7 dB as the output power increases to 17 dBm. The results demonstrate that LITMUS provides an improvement in linearity near compression and at backoff from compression.

### 6.3.2 Random Phases, 4 Tones

Uniform-amplitude random-phase multisines have previously proven useful for analyzing intermodulation products generated by nonlinear transfer functions and practical signal spectra [57]. Such multisines were chosen to represent Code-Division Multiple-Access communications signals under nonlinear amplification, for example [58]. In this section, a four-tone multisine whose phases are chosen randomly is used to mimic the behavior of a communication signal carrying information by amplitude and phase modulation. This signal is multiplexed before amplification to demonstrate the advantage of grafting LITMUS onto a pre-existing communication scheme.

A measurement system which demonstrates the distortion cancellation for four
Figure 6.4: IM3 measurement data for the RF2320 amplifier: time-multiplexed two-tone test signal with $k = 0$ through $k = \pm 13$. The third-order distortion produced by amplifying the multitone signal ($k = 0$) is reduced by also amplifying its spectral replicas ($k > 0$). A 10- to 14-dB drop in distortion from $k = 0$ (no switching) to $k = \pm 1$ (sinusoidal switching) is evident across a signal power of 13 to 17 dBm. Including higher switching harmonics (i.e. using more ideal switching) results in further distortion reduction.

tones ($N = 4$) and random initial phases ($\phi_1...\phi_4$) is given in Fig. 6.5. The baseband signal generator is the Agilent N6030A. The Matlab script for programming the N6030A to output the baseband waveforms is given in Appendix A.5. Here, the Agilent E8267C acts as the RF modulator, which upconverts the signal from the N6030A to a carrier of $f_0 = 850$ MHz. The four tones are spaced 391 kHz apart and time-multiplexed with a switching frequency of $f_s = 39$ MHz. Only the first switching lobe ($k = -4...4$) is passed by the E8267C to the input of the amplifier. The amplifier is the Ophir 5162, which has a small-signal gain of $a_1 = 46$ dB. The spectrum analyzer is again the Agilent E4445A.

For $N = 4$ and a single lobe of switching harmonics applied to the amplifier, the input signal given by (6.6) becomes

$$v_i(t) = \sum_{k=-4}^{4} \frac{B_k}{\pi} \sum_{n=1}^{4} A_n \cos \left[ (\omega_n + k\omega_s) \left( t - \frac{n-1}{4} T_s \right) + \phi_n \right].$$  \hspace{1cm} (6.22)

For this experiment, the phases of the four tones $\phi_1...\phi_4$ are randomized by the software controlling the Agilent N6030A. Each set of four random phases is independent of the oth-
Figure 6.5: Test setup for random-phase distortion reduction. The N6030A provides the baseband time-multiplexed signal; the E8267C upconverts the baseband to RF at 835 MHz. 

ers, and each of the four phases within each set is independent of the others. The phases are chosen from a uniform distribution between \(-\pi\) and \(+\pi\). Ten sets of four random phases are averaged from individual measurements to give the results of Table 6.4. A sample of the random-phase data for \(P_o = 7\) dBm is given in Fig. 6.6. On average, using a single lobe of switching harmonics to multiplex the signal, LITMUS reduces the distortion for four tones with random phases by 8.8 to 14.5 dB. These results demonstrate the feasibility of applying LITMUS to reduce intermodulation distortion generated by practical communication signals.

6.3.3 Wideband Measurements, 6+ Tones

Distortion reduction is also possible for wider signal bandwidths and higher peak to average power ratios (PAR) than reported so far. This represents more practical signal spectra. To demonstrate cancellation for wider bandwidths containing greater numbers of tones, a wideband multisine measurement system was constructed. A block diagram of this system is given in Fig. 6.7. The input to the amplifier comes from the Polyphase Microwave QM3337A quadrature modulator, into which is fed the baseband time-multiplexed waveform from the Agilent N6030A wideband generator and a local oscillator signal from the Agilent E8267C. The tones are centered on \(f_0 = 3562\) MHz, spaced 3.125/N MHz apart, and time-multiplexed with a switching frequency of \(f_s = 156.25\) MHz. The MiniCircuits ZHL-1042J is a high-linearity wideband amplifier which boosts the signal to a level sufficient to cause saturation in the Ophir 5162 amplifier, the device under test (DUT) in this setup. The Agilent E4445A again records the power spectra. The results of the wideband test are
Table 6.4: Measured Random-Phase Distortion Reduction, $N = 4$. IM3 is measured for four tones, four different power levels, and both non-multiplexed ($k = 0$) and single-lobe multiplexed ($k = -4...4$) cases.

<table>
<thead>
<tr>
<th>Output Power $P_o$ (dBm)</th>
<th>Measured Distortion, $k = 0$ (dBc)</th>
<th>Measured Distortion, $k = -4...4$ (dBc)</th>
<th>Measured Distortion Reduction (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>-46.5</td>
<td>-61.0</td>
<td>14.5</td>
</tr>
<tr>
<td>8</td>
<td>-49.2</td>
<td>-61.2</td>
<td>12.0</td>
</tr>
<tr>
<td>7</td>
<td>-52.4</td>
<td>-62.6</td>
<td>10.2</td>
</tr>
<tr>
<td>6</td>
<td>-56.4</td>
<td>-65.2</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Figure 6.6: Sample random-phase spectra; data taken using the system of Fig. 6.5, $N = 4$ and $P_o = 7$ dBm total. The distortion spectra produced by non-multiplexed ($k = 0$) and multiplexed ($k = -N...N$) inputs to the amplifier are contrasted. LITMUS is experimentally demonstrated to reduce distortion for uniform-amplitude random-phase multitone signals by 8 to 14 dB.
Figure 6.7: Test setup for wideband distortion reduction. The Agilent N6030A generator has a digital-to-analog sampling rate of 1.25 GS/s. The gain of the MiniCircuits ZHL-1042J amplifier is 27.6 dB at 3.562 GHz.

given in Table 6.5, and a sample of the wideband data for $N = 20$ is given in Fig. 6.8.

The Agilent N6030A implements software filtering to control the number of switching harmonics applied to the amplifier. Initially, for each value of $N$, only the fundamental tones are applied to the Ophir 5162. The power of the N6030A quadrature channels is adjusted to achieve a baseline distortion of approximately $-30.0$ dBc at the amplifier output, and the Adjacent Channel Power Ratios (ACPR) for the lower and upper sides of the 3.125-MHz-wide signal are recorded. Then a full set of switching harmonics up to $k = \pm N$ is added to the fundamental tones at the DUT input, and the reduction in ACPR values from the $k = 0$ case is tabulated.

The data shows that transmitting the full main lobe of the sinc waveform through the amplifier results in an ACPR reduction between 13.9 and 16.1 dB from the non-multiplexed case for multisines up to $N = 20$. The reduction varies by about 2 dB because the gain responses of the MiniCircuits and Ophir amplifiers are not entirely flat or phase-invariant over the 312.5 MHz bandwidth of the main multiplexed lobe, which produces deviation from the expected cancellation as in the two-tone case. Still, the experiment demonstrates that LITMUS can provide distortion reduction of at least 13.9 dB in ACPR for an amplified signal containing $N = 20$ simultaneous frequencies within a bandwidth of 3.125 MHz.
Table 6.5: Measured Distortion Reduction, Wideband, $N > 4$. The difference between the non-multiplexed ($k = 0$) and single-lobe multiplexed ($k = -N...N$) Adjacent Channel Power Ratios is recorded for up to 20 tones.

<table>
<thead>
<tr>
<th>Number of Tones $N$</th>
<th>PAR (dB)</th>
<th>$\Delta$ACPR, Low Side (dB)</th>
<th>$\Delta$ACPR, High Side (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>7.8</td>
<td>-14.4</td>
<td>-13.9</td>
</tr>
<tr>
<td>8</td>
<td>9.0</td>
<td>-16.1</td>
<td>-15.9</td>
</tr>
<tr>
<td>10</td>
<td>10.0</td>
<td>-14.8</td>
<td>-14.1</td>
</tr>
<tr>
<td>12</td>
<td>10.8</td>
<td>-14.5</td>
<td>-14.6</td>
</tr>
<tr>
<td>14</td>
<td>11.5</td>
<td>-16.0</td>
<td>-15.6</td>
</tr>
<tr>
<td>16</td>
<td>12.0</td>
<td>-15.3</td>
<td>-14.8</td>
</tr>
<tr>
<td>18</td>
<td>12.6</td>
<td>-14.7</td>
<td>-14.7</td>
</tr>
<tr>
<td>20</td>
<td>13.0</td>
<td>-15.3</td>
<td>-15.2</td>
</tr>
</tbody>
</table>

Figure 6.8: Sample traces; data taken using the system of Fig. 6.7, $N = 20$. The distortion spectra produced by non-multiplexed ($k = 0$) and multiplexed ($k = -N...N$) inputs to the amplifier are contrasted. An ACPR difference of greater than 15 dB is recorded for $N = 20$ between the non-multiplexed tones ($k = 0$) and using one full switching lobe ($k$ up to $\pm 20$).
6.3.4 Summary

Three sets of measurements validated the LITMUS distortion reduction concept. Two-tone measurements directly confirmed the intermodulation cancellation theory presented in Section 6.2 by showing approximately 14.5 dB of distortion reduction, from the non-multiplexed $k = 0$ case to the sinusoidally-multiplexed $k = -1...1$ case, across a signal power sweep. Four-tone measurements confirmed that time-multiplexing using the first $k = \pm 1$ set of switching harmonics still produces, on average, 9 to 15 dB of cancellation for tones with non-zero phases, i.e. signals containing real data. Wideband measurements showed that LITMUS may be extended to much higher numbers of tones; a distortion reduction of approximately 15 dB was recorded for 20 tones over a bandwidth of 3 MHz.

6.4 Linear Signal Recovery

If the amplified multiplexed signal is applied to a bandpass filter whose bandwidth is narrow enough to pass only the fundamental tones, the sampling aliases are eliminated, and the switched-tone amplified output is converted back to its non-multiplexed form. This reconverted RF signal contains less distortion than the directly-amplified multitone signal of equal tone power. This section presents theory and measurements which confirm these statements.

6.4.1 Theory

Let the transfer function of the bandpass filter which follows the amplifier be given by

$$H (\omega) = \frac{V_f (\omega)}{V_a (\omega)} = |H (\omega)| \exp \{j\theta (\omega) \}.$$  \hfill (6.23)

The passband of the filter encompasses the non-multiplexed signal spectrum with a bandwidth $B$. If the switching frequency is such that $\omega_s \gg B$, the filter removes all spectral components above and below the fundamental tones, leaving only $k = 0$ at its output. If the amplifier small-signal gain is assumed to be a constant $a_1$ over the fundamental tones, then
the spectrum of (6.4) after amplification and filtering (neglecting amplifier nonlinearity, for the moment) may be written

\[ V_f(\omega) = \frac{a_1}{N} \sum_{n=1}^{N} \left\{ A_n e^{i\phi_n} H(\omega_n) e^{-j\omega T_s[n-1]N} \times \pi \left[ \delta(\omega - \omega_n) + \delta(\omega + \omega_n) \right] \right\} \]  

(6.24)

whose time-domain equivalent is

\[ v_f(t) = \frac{a_1}{N} \sum_{n=1}^{N} A_n |H(\omega_n)| \cos \left[ \omega_n t + \phi_n + \theta(\omega_n) - \omega_n T_s [n-1] \right]. \]  

(6.25)

The output is thus a superposition of sinusoids at the fundamental tones without time-multiplexing. The amplitudes of the tones have been reduced by the switch duty cycle and the filter response, and increased by the amplifier gain. The phases of the tones depend upon the filter phase response and the time delay for each tone to be applied to the amplifier in the switching sequence.

If the amplifier passes all of the switching harmonics of \( V_i(\omega) \), then the output contains no distortion because the switching of the tones is instantaneous, so that only one tone is present in the amplifier at any one time, and no nonlinear mixing occurs. In practice, however, every amplifier has a finite bandwidth, and every filter is weakly nonlinear; these properties produce distortion in all outputs. Nonetheless, a switched-and-filtered signal will contain much less distortion than a signal that is amplified directly, for the same output tone power.

### 6.4.2 Measurement

A block diagram of the measurement system which demonstrates both the multiplexed to non-multiplexed conversion and the distortion reduction provided by this conversion is given in Fig. 6.9. The Agilent N6030A and E8267C units together form the signal generator. The signal consists of \( N = 4 \) tones, spaced 391 kHz apart, centered on \( f_0 = 835 \) MHz, and time-multiplexed with a switching frequency of \( f_s = 39 \) MHz. The filter is the transmitter channel of a Lorch Microwave WD-00003 basestation duplexer, cen-
Figure 6.9: Test setup for demonstrating linear signal recovery from an amplified time-multiplexed signal. The through-port of the 20-dB coupler is terminated in 50 Ω (not shown). The transmit-band of a duplexer acts as the bandpass filter. The data of Figs. 6.10(a) and 6.11(a) are recorded without the duplexer in the signal path; the data of Figs. 6.10(b) and 6.11(b) are recorded with the duplexer in the signal path.

The switched-tone to multisine conversion is demonstrated experimentally in Figs. 6.10 and 6.11. The switched-tone input to the amplifier \( v_a \) is shown in Fig. 6.10(a); its spectrum is displayed in Fig. 6.11(a). This time-domain waveform has a nearly-constant RF envelope and its spectrum is wideband. The residual modulation (i.e. the non-constant amplitude envelope) is due to the truncated switching spectrum; i.e. only \( k = -4...4 \) are active, so the switching is not ideal. The multitone output from the amplifier \( v_o \) is shown in Fig. 6.10(b); its spectrum is displayed in Fig. 6.11(b). This time-domain waveform resembles a steady-state four-tone interference pattern and its spectrum is narrowband. The switching harmonics present in the time-multiplexed signal (at \( |k| > 0 \)) are filtered, leaving only the four fundamental tones (at \( k = 0 \)). The result given by (6.25) is confirmed.

The data of Fig. 6.12 show the distortion reduction associated with the switched-to-multisine conversion. The spectrum of the filter output is plotted in the vicinity of the fundamental tones. The two traces are for (a) \( k = 0 \), which is the multisine without time-multiplexing, and (b) the main sinc lobe, which is the multisine produced by time-multiplexing and filtering. The filter is retained for the non-multiplexed case so that a comparison of distortion levels can be made for equal fundamental tone powers. The filtered time-multiplexed multisine contains 14.3 dB less distortion than its non-multiplexed
Figure 6.10: Measured time-domain comparison of switched-tone to multitone conversion: (a) amplified signal not filtered, (b) amplified signal filtered using a low-loss duplexer. The multiplexed signal in (a) is relatively flat across time because it contains multiple switching harmonics in addition to the four fundamental tones. The filtered signal in (b) displays a four-tone interference pattern because the switching harmonics fall outside of the filter passband and are attenuated.
Figure 6.11: Measured frequency-domain comparison of switched-tone to multitone conversion: (a) amplified signal not filtered, $k = -1,0,+1$ not attenuated; (b) amplified signal filtered, $k = -1,0,+1$ attenuated by the low-loss duplexer. There are four tones embedded in each visible peak. The spectral replicas evident in the time-multiplexed signal of (a) are attenuated by the duplexer to produce (b). The attenuation is greater on the higher-frequency side of the filter band.
Figure 6.12: Sample traces; data taken using the system of Fig. 6.9 with \( N = 4 \). The distortion spectra of the non-multiplexed \((k = 0)\) and single-lobe multiplexed \((k = -N...N)\) outputs from the filter are contrasted. A 14.3-dB reduction in distortion is recorded when extracting the desired amplitude-modulated four-tone signal from its time-multiplexed, amplified counterpart.

counterpart. Since the RF envelope variation at the input to the amplifier is considerably reduced in the time-multiplexed case, the intermodulation produced by variations in gain over the period of the switched-tone signal is significantly reduced. The linearization of an amplified signal by time-multiplexing it before the amplifier and filtering it after the amplifier is demonstrated.

6.4.3 Summary

The theory developed and data presented in this section validated the LITMUS linear-recovery concept. To remove the switching harmonics and extract the desired amplitude modulated signal from the time-multiplexed version, a filter with a bandwidth less than the switching frequency was placed after the amplifier.\(^1\) The time-multiplexing process reduced the distortion produced by the signal in the amplifier, and the filtering process restored the non-multiplexed signal. Between the non-multiplexed amplified signal and its multiplexed-amplified-filtered counterpart, a 14.3 dB distortion reduction was recorded.

\(^1\)Equivalently, the switching frequency was chosen to be greater than the filter bandwidth.
6.5 Conclusions

A novel signal processing technique for reducing intermodulation distortion associated with an amplitude-modulated RF signal was presented. LITMUS combines the time-multiplexing of a switch with the frequency-selectivity of a bandpass filter. LITMUS is unique because it requires no advanced knowledge of the nonlinearity in the signal path and no cancellation calibration in order to reduce AM distortion.

Theory showed that rapidly time-multiplexing the spectrum of an input signal reduces its peak-to-amplitude ratio at the expense of widening its bandwidth, but that the reduced PAR of the time-multiplexed version of the signal generates significantly less distortion than the original waveform when it is applied to a nonlinearity. With ideal switching and a memoryless nonlinearity, no intermodulation is generated because only a single frequency is present in the nonlinearity at any one time. With only sinusoidal switching, a 14.5-dB reduction in distortion is predicted for two tones. Theory predicts greater distortion cancellation when a greater number of switching harmonics are transmitted by the amplifier, assuming a flat gain characteristic.

The 14.5-dB reduction for two tones was measured with a real signal generator and amplifier. Reductions between 8 and 15 dB were recorded for four tones and random phases across a signal power sweep. A distortion reduction of 15 dB was recorded for 20 tones across a 3-MHz bandwidth.

Theory showed that a bandpass filter placed after the amplifier would remove the wideband time-multiplexing aliases and recover the desired amplified signal. Measurements using the transmitter-band of a low-loss duplexer confirmed this portion of the LITMUS theory. A 14.3-dB distortion reduction was recorded for four tones — between the non-multiplexed amplified signal and its multiplexed-amplified-filtered counterpart — which validated the complete LITMUS system.
Conclusions and Future Work

7.1 Summary of Research and Original Contributions

This dissertation has shed new light on the roles that time-frequency effects play in wireless communication systems. A new cause of linear and nonlinear co-site interference has been identified, a number of advances in linear and nonlinear metrology using a combination of time- and frequency-domain concepts have been made, and a new time-frequency technique for improving transmitter linearity has been developed.

Transient properties of narrowband circuits were found to last much longer than conventional time/bandwidth rules-of-thumb. The resonant structure of these circuits was recognized as the source of long-tail behavior, and a simulation method for observing pulse distortion produced by individual resonators was created. A simplification of the analytical solutions to the differential equations which govern a narrowband filter’s response was presented. The frequency-dependent transient properties of bandpass filters were shown to cause smearing of communications pulses in time, which generates intersymbol interference between adjacent pulses of the same frequency and intermodulation distortion between adjacent pulses of different frequencies. Evaluations of SNR degradation and IM3 generation were performed on frequency-hopping scenarios.

Three new time-domain techniques for the linear metrology of resonant circuits
were introduced. The first method extracts the loaded quality factor of a filter’s outermost resonator from the initial decay rate of its reflected-pulse response, the second method estimates the bandwidth of such a filter from the time-interval between the nulls of the same pulse response, and the third method extracts broadband S-parameters from the device’s short-pulse response. All three methods can be used in the non-destructive testing of filters on integrated circuits. The first two are particularly suited to narrowband filters, and the third may be extended to any linear device. Measurements are limited by the time-resolution of the available oscilloscope and the bandwidth of the available signal generator, but no spectrum analyzer is needed. Reflected two-tone pulse overlaps were captured wirelessly from an antenna-filter combination by probing it with a switched-tone frequency sweep, providing another way to characterize RF front-ends remotely.

A nonlinear method for extracting the transmission band of a filter was introduced. The filter’s two-port behavior can be extracted from a single measurement port; only the filter input must be accessible. By measuring intermodulation power traveling back through the filter, the ambiguity of the linear case between reflection and re-transmission is avoided. By exploiting the relatively-slow energy-storage properties of narrowband filters, a single relatively-fast frequency-switching source may be used to take the measurement instead of two independent synthesizers. Retransmitted IM3 power was captured wirelessly from an antenna-filter-amplifier combination; this measurement is the nonlinear analogue of the aforementioned linear method for characterizing RF front-ends remotely.

Linear Amplification by Time-Multiplexed Spectrum (LITMUS), a signal processing technique which combines time- and frequency-selectivity, was developed to reduce the intermodulation distortion associated with amplitude modulation. The signal processing technique improves the IMD of an amplified signal at the expense of momentarily widening the signal bandwidth before and after the nonlinear amplifier. Time-multiplexing the signal’s tones minimizes their interaction at the nonlinearity; in this way, no advanced knowledge of the nonlinearity is required in order to reduce distortion from the non-multiplexed case. LITMUS is ideally applied to phase-modulation with constant power across several signal tones, but the distortion reduction technique is applicable to any signal containing multiple simultaneous tones. Measurements were taken which validated the full LITMUS system: time multiplexing, distortion reduction, and linear recovery.
7.2 Future Research

There are several ways to build upon the results presented in this work. Chapters 3 through 6 each provide a framework from which new research initiatives may be drawn.

A key result from Chapter 3 was that narrow, sharp filters react much more slowly than previously thought. Systems with tight bandwidth constraints will need to be designed with this result in mind, otherwise their resonant components will produce intolerable ringing, intersymbol interference, and intermodulation distortion. The particular ways in which these systems should be designed in order to avoid such filtering effects is open to research. This information would be invaluable to those who engineer communication systems with high frequency selectivity in addition to high data rates.

In Chapter 4 it was discovered that the pulsed responses of circuits are rich with information about the circuits themselves. This work showed how quality factor can be extracted from the beginning of the pulse response and bandwidth can be extracted from the time-interval between the nulls of the pulse response. The rest of the pulse is expected to contain more information about individual segments within the filter. Methods for extracting circuit parameters for each of the filter’s individual resonators — from the same pulse response — have not yet been developed. Such methods would add to the existing library of non-destructive integrated-circuit test procedures for RF front-ends.

Chapter 5 showed that two-tone measurements could be performed using a switched-frequency synthesizer and a filter instead of two independent synthesizers. To characterize a nonlinear device more completely, e.g. to measure in-band distortion, higher numbers of input tones must be used. The extension of the switched-to-multitone measurements to more tones, as well as a determination of the accuracy of these measurements, is open to research.

LITMUS was presented in Chapter 6 as a stand-alone modulation format, but the concept of time-multiplexing a multitone signal to reduce its distortion at a nonlinearity is applicable to any modulation that contains multiple simultaneous tones. The LITMUS distortion improvement can be demonstrated on pre-existing communications formats such as OFDM. Such an implementation could improve nonlinear distortion in transmitters without requiring a change in the system’s modulation.
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APPENDICES
A

Instrument Control & Calibration

A.1 USB to GPIB Drivers

In order to control the Agilent and Tektronix units from Matlab, the following equipment is required: (a) at least one USB port on the computer running Matlab, (b) one USB-to-GPIB adapter, (c) at least 1 GPIB cable per RF generation/capture device, and (d) the following software installed on the computer:

1. GPIB-USB-A drivers from National Instruments–

2. I/O Libraries Suite from Agilent–
   http://www.home.agilent.com/agilent/editorial.jspx?nid=-35199.536882504.02&lc=eng&cc=US&ckey=1000000416%3Aepsg%3Asud&id=1000000416%3Aepsg%3Asud

3. Waveform Download Assistant (WDA) from Agilent–

GPIB cabling between each piece of hardware may be routed (a) from one central unit (e.g. the computer) to all of the others, or (b) in cascade. Any arrangement will suffice as long
as each device occupies a unique address on the GPIB bus.¹

¹In the following scripts, the E8267C occupies GPIB address 19, the TDS684B occupies address 24, and the E4445A occupies address 18.
A.2 Agilent E8267C Signal Generator

A.2.1 Single-Tone Pulses

This Matlab script directs the WDA to (a) upload the baseband IQ waveform of an RF pulse to the Agilent E8267C and (b) set the synthesizer’s power & RF carrier frequency. The E8267C performs the upconversion of the baseband signal to RF so that an RF pulse appears at its output port. The waveform plays and repeats indefinitely. A trigger pulse, synchronized to the start of each waveform segment, appears on the Event 1 port at the back-plane of the E8267C.

clear;

pulse_frequency = 465e6; % RF carrier frequency
pulse_pwr = -20; % RF pulse power (dBm)
ramp_up_time = .1e-6; % rise time for pulse (seconds)
pulse_duration = 1e-6; % pulse duration
ramp_down_time = .1e-6; % fall time for pulse
pulse_off_time = 2e-6; % time in-between pulses
sampclk = carrier/5; % ARB sample clock frequency

% length of each RF pulse (in samples)
n_ramp_up = floor( ramp_up_time * sampclk );
n_pulse = floor(pulse_duration * sampclk );
n_ramp_down = floor( ramp_down_time * sampclk );
n_between = floor( pulse_off_time * sampclk );

pulse_shape = [ linspace(0,1,n_ramp_up) ones(1,n_pulse) ...
    linspace(1,0,n_ramp_down) zeros(1,n_between) ];
i = pulse_shape;
q = zeros(1,length(pulse_shape));

% insert additional copies of the pulses into digital output
% (sometimes required for the Agilent PSG to produce proper waveforms)
i = [i i]; q = [q q];
figure(1)
subplot(2,1,1)
plot( (1:length(i))/sampclk, i)
axis([-Inf Inf -Inf Inf])
ylabel('In-Phase')

subplot(2,1,2)
plot( (1:length(q))/sampclk, q)
axis([-Inf Inf -Inf Inf])
ylabel('Quadrature')
xlabel('Time (sec)')

IQData = [i + (j * q)];

% make a new connection to the PSG over the GPIB interface
io = agt_newconnection('gpib',0,19);

% verify that communication with the PSG has been established
[status, status_description,query_result] = agt_query(io,'*idn?');
if (status < 0) return; end

% preset the instrument
[status, status_description] = agt_sendcommand(io,:STATus:PRESet');

% carrier_string = ...
carrier_string = strcat(['SOURce:FREQuency ' num2str(pulse_frequency)]);
[status, status_description] = agt_sendcommand(io,carrier_string);

% put the ALC into manual control and set the IQ real time scaling
[status, status_description] = ...
agt_sendcommand(io, 'POWer:ALC:STATe OFF');

% download the iq waveform the PSG baseband generator for playback
[status, status_description] = agt_waveformload(io, IQData, ...
  'pulse_1tone', sampclk, 'play', 'no_normscale', Markers);

% Turn on modulation
[status, status_description ] = ...
  agt_sendcommand( io, 'OUTPut:MODulation:STATe ON');

% Turn on RF output power
[status, status_description ] = ...
  agt_sendcommand( io, 'OUTPut:STATe ON' );
A.2.2 Switched-Tone Pulses

This Matlab script directs the WDA to (a) upload the baseband IQ waveform of a switched-tone signal to the Agilent E8267C and (b) set the synthesizer’s power & RF carrier frequency for balanced double-sideband generation (i.e. the carrier frequency is set halfway between the switched tones). The waveform plays and repeats indefinitely. A trigger pulse, synchronized to the start of each waveform segment, appears on the Event 1 port at the back-plane of the E8267C.

```matlab
clear;

RF_pwr = 0; % RF pulse power (dBm)

frequency1_freq = 895e6; % 1st frequency
frequency1_pulse = 1e-6; % duration of 1st pulse (seconds)
time_between = 1e-6; % (chirp) time between f1,f2

frequency2_freq = 905e6; % 2nd frequency
frequency2_pulse = 1e-6; % duration of 2nd pulse (seconds)
time_after = 1e-6; % (chirp) time between f2,f1

% balanced double-sideband signal generation
carrier = (1/2) * abs(frequency1_freq + frequency2_freq);
sampclk = carrier / 10; % ARB sample clock frequency

% length of each waveform piece (in samples)
n_frequency1_pulse = floor( frequency1_pulse * sampclk );
n_time_between = floor( time_between * sampclk );
n_frequency2_pulse = floor( frequency2_pulse * sampclk );
n_time_after = floor(time_after * sampclk );

% define an array which contains the the pulsed waveform
chirp_dev = abs(frequency1_freq - frequency2_freq); fm = (chirp_dev/2) * ([ (-1*ones(1,n_frequency1_pulse)) ...
    (linspace(-1,1,n_time_between)) ...
    (1*ones(1,n_frequency2_pulse)) ...
    (linspace(1,-1,n_time_after)) ]);```
if frequency2_freq < frequency1_freq
    fm = -1 * fm;
end

% use an integral to translate from fm to pm
pm = (2*pi/sampclk) * cumsum(fm); i = cos(pm); q = sin(pm);

% insert additional copies of the pulses into digital output
% (sometimes required for the Agilent PSG to produce proper waveforms)
i = [i i]; q = [q q]; fm = [fm fm];

% plot the waveform samples and scale the plot
figure(1)
subplot(2,1,1)
plot((1:length(i))/sampclk, i)
axis([-Inf Inf -Inf Inf])
ylabel('In-Phase')

subplot(2,1,2)
plot((1:length(q))/sampclk, q)
axis([-Inf Inf -Inf Inf])
ylabel('Quadrature')
xlabel('Time (sec)')

% define a composite iq matrix for download to the PSG
IQData = [i + (j * q)];

% activate a marker to indicate the beginning of the waveform
Markers = zeros(2,length(IQData)); Markers(1,1:10) = 1;

% make a new connection to the PSG over the GPIB interface
io = agt_newconnection('gpib',0,19);

% verify that communication with the PSG has been established
[status, status_description,query_result] = agt_query(io,'*idn?');
if (status < 0) return; end

% preset the instrument
[status, status_description] = agt_sendcommand(io,':STATus:PRESet');
% set carrier frequency and power on the PSG
carrier_string = strcat(['SOURce:FREQuency ' num2str(carrier)]);
[status, status_description] = ...
    agt_sendcommand(io,carrier_string);
[status, status_description] = ...
    agt_sendcommand(io, ['POWer ' num2str(RF_pwr)]);

% put the ALC into manual control and set the IQ real time scaling
[status, status_description] = ...
    agt_sendcommand(io, 'POWer:ALC:STATe OFF');
[status, status_description] = ...
    agt_sendcommand(io, 'RADio:ARB:RSCaling 70.7');

% download the iq waveform the PSG baseband generator for playback
[status, status_description] = agt_waveformload(io, IQData, ...
    'switched_tone', sampclk, 'play', 'no_normscale', Markers);

% Turn on modulation
[status, status_description ] = ...
    agt_sendcommand( io, 'OUTPut:MODulation:STATe ON');

% Turn on RF output power
[status, status_description ] = ...
    agt_sendcommand( io, 'OUTPut:STATe ON');
A.2.3 100 MS/s Time-Multiplexed Multi-Tones

This program generates time-multiplexed uniform-amplitude signals at a default sample rate of 100 MS/s for testing with an amplitude-modulation transmission system. The multitone signals are formulated as baseband I/Q waveforms and upconverted to the final carrier frequency within the E8267C.²

```matlab
clear;

N=4; % number of tones
fspace = 5e6/(6*N); % tone frequency spacing (Hz)
fsbb = 100e6; % baseband ARB sampling rate (Samples/s)
A = 1/2; % initial amplitude
fsw = 100e6/4; % time multiplex switch rate

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

fres = fspace/2; % frequency resolution for simulation
tstop = 1/fres; % stop time based on freq resolution
fs = 10e9; % sampling frequency
ts = 1/fs;
tm = 0:ts:tstop-ts; % sampling time vector
f = -fs/2:fres:(fs/2)-fres; % frequency grid

xi = zeros(1,length(ts)); % initialize time-sample vectors
xq = zeros(1,length(ts));
xmuxi = xi;
xmuxq = xq;

tsbb = 1/fsbb; tbb = 0:tsbb:tstop-tsbb;
tsw = 1/fsw; % multiplex switch period

sw = ones(1,(tsw/tsbb));
swl = length(sw);
sw = cat(2,sw,zeros(1,(N-1)*tsw/tsbb));
swt = sw;
for b = 1:(length(tbb)/(length(sw))-1)
    swt = cat(2,swt,sw);
end
```

²This script was developed by Kevin G. Gard.
end;
swup = swt;
swdn = circshift(swt',swl*(N/2))';
swk = swt;
swm = circshift(swt',swl*(N/2))';
for n = 1:N/2
    a = 1;
    xi = xi + (A*(a))*cos(2*pi*fspace*(abs(n)-1/2)*tbb) ...
         + (A/(a))*cos(2*pi*fspace*(abs(n)-1/2)*tbb);
    xq = xq + (A*(a))*sin(2*pi*fspace*(abs(n)-1/2)*tbb) ...
         - (A/(a))*sin(2*pi*fspace*(abs(n)-1/2)*tbb);
    xmuxi = xmuxi + N*(A*(a))*cos(2*pi*fspace*(abs(n)-1/2)*tbb+0).*swup ...
            + N*(A/(a))*cos(2*pi*fspace*(abs(n)-1/2)*tbb+0).*swdn;
    xmuxq = xmuxq - N*(A*(a))*sin(2*pi*fspace*(abs(n)-1/2)*tbb+0).*swup ...
            + N*(A/(a))*sin(2*pi*fspace*(abs(n)-1/2)*tbb+0).*swdn;
    swup = circshift(swt',swl*(n+0))';
    swdn = circshift(swt',swl*(n+N/2+0))';
    if n < N/2
        swk = swk + swup;
        swm = swm + swdn;
    end;
end;
nt = 1:length(tm);
ty = 1:1000;
fct = .95;
ff = [0 fct fct 1];
af = [1 1 0 0];
b = fir2(length(tbb)/4,ff,af);
xmuxi = filtfilt(b,1,xmuxi);
xmuxq = filtfilt(b,1,xmuxq);

xi = interp(xi,fs/fsbb);
xq = interp(xq,fs/fsbb);
xmuxi = interp(xmuxi,fs/fsbb);
xmuxq = interp(xmuxq,fs/fsbb);

i = xmuxi/N;
q = xmuxq/N;

% define an IQ matrix for download to the PSG using the WDA
IQData = [i + (j * q)];
activate a marker to indicate the beginning of the waveform
Markers = zeros(2,length(IQData));
Markers(1,1:10) = 1;

% make a new connection to the PSG over the GPIB interface
io = agt_newconnection('gpib',0,19);

% verify that communication with the PSG has been established
[status, status_description,query_result] = agt_query(io,'*idn?');
if (status < 0) return;
end

% preset the instrument
[status, status_description] = agt_sendcommand(io,':STATus:PRESet');

% put the ALC into manual control and set the IQ real time scaling
[status, status_description] = ... 
    agt_sendcommand(io, 'POWer:ALC:STATe OFF');
[status, status_description] = ... 
    agt_sendcommand(io, 'RADio:ARB:RSCaling 70.7');

% download the iq waveform the PSG baseband generator for playback
[status, status_description] = agt_waveformload(io, IQData, ...
    'TMMT_Ntone', fs, 'play', 'no_normscale', Markers);

% Turn on modulation
[status, status_description ] = ... 
    agt_sendcommand( io, 'OUTPut:MODulation:STATe ON');

% Turn on RF output power
[status, status_description ] = ... 
    agt_sendcommand( io, 'OUTPut:STATe ON');
A.3 Tektronix TDS684B Oscilloscope

This Matlab script presets the TDS684B to

- collect 15000 samples (i.e. the maximum data length),
- trigger on Channel 4,
- record $2.5 \times 10^9$ samples per second,
- set the port impedance to 50 Ω,
- average [a user-defined number of] traces,
- set the vertical scaling to [user-defined] volts/division,

and directs the oscilloscope to capture its data after a single set of traces has been averaged. Typically, the trigger pulse from the Event 1 port at the back-plane of the Agilent E8267C is fed to Channel 4 of the Tektronix TDS684B. The time-domain trace is stored as the vectors “time” and “voltage” in a MAT file whose name is specified by the user.

clear;

filename = ['v_out']; % set name for trace output file
v_scale = 20e-3; % set volts/division
avg_count = 100; % set number of traces to average

% set sample rate

io = agt_newconnection('gpib',0,24);

[status, status_description,query_result] = agt_query(io,'*idn?');
if (status < 0) return; end

time_step = 1/(2.5e9); % sample rate
time = (0:15000-1) * time_step; % time vector

[status, status_description] = ... 
agt_sendcommand(io,'DATa:ENCdg ASCIi');
[status, status_description] = ... 
agt_sendcommand(io,'DATa:WIDth 2');
[status, status_description] = ...
agt_sendcommand(io,'DATa:STArt 1');
[status, status_description] = ...
agt_sendcommand(io,'DATa:STOp 15000');

[status, status_description] = ...
agt_sendcommand(io,'TRIGger:MAIn:EDGE:SOURce CH4');
[status, status_description] = ...
agt_sendcommand(io,'TRIGger:MAIn:LEVel 1');
[status, status_description] = ...
agt_sendcommand(io,'HORizontal:TRIGger:POSition 0');
[status, status_description] = ...
agt_sendcommand(io,'HORizontal:RECORDLength 15000');
[status, status_description] = ...
agt_sendcommand(io,'HORizontal:MAIn:SCAle 20e-9');
[status, status_description] = ...
agt_sendcommand(io,'HORizontal:POSition 30');

[status, status_description] = ...
agt_sendcommand(io,'CH1:POSition 0');
[status, status_description] = ...
agt_sendcommand(io,'CH1:IMPedance FIFty');

[status, status_description] = ...
agt_sendcommand(io,'ACQuire:MODe AVErage');
[status, status_description] = ...
agt_sendcommand(io,['ACQuire:NUMAVg ' num2str(avg_count)]);
[status, status_description] = ...
agt_sendcommand(io,'ACQuire:STOPAfter SEQuence');
[status, status_description] = ...
agt_sendcommand(io,'CH1:POSition 0');
[status, status_description] = ...
agt_sendcommand(io,'CH1:OFFSet 0');
[status, status_description] = ...
agt_sendcommand(io,['CH1:SCALE ' v_scale]);

[status, status_description] = ...
agt_sendcommand(io,'ACQUIRE:STATE RUN');
collection_time = (21 / 100) * avg_count; % pause to
pause(ceil(collection_time)) % collect data

[status, status_description] = ...
agt_sendcommand(io,'DATa:SOURce CH1');
agt_query(io, 'CURVe?');

voltage_step = v_scale * 5 / 32768;

voltage = str2num(raw_data) * voltage_step;  \% voltage vector

save([filename '.mat'], 'time', 'voltage');

figure(2)
plot(time/(10^{-6}), voltage/(10^{-3}))
ylabel('Voltage (mV)')
xlabel('Time (\mu s)')
title('Tektronix TDS684B Recorded Trace')
A.4 Agilent E4445A Spectrum Analyzer

This Matlab script presets the E4445A to

- collect [a user-defined range and number of] frequency points,
- set a Resolution Bandwidth (RBW) of 10 kHz,
- set an amplitude reference level of 10 dBm,
- add 20 dB of front-end attenuation to the incoming signal,
- set the vertical scaling to 10 dB per division,

and directs the spectrum analyzer to capture a single frequency-domain trace as the vectors “frequency” and “power” in a MAT file whose name is specified by the user.

clear;

filename = ['v_out']; % file name for spectrum
start_freq = 850e6; % starting frequency
end_freq = 950e6; % ending frequency
points = 8001; % number of data points

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
frequency = linspace(start_meas_freq,end_meas_freq,points);

io = agt_newconnection('gpib',0,18);
[status, status_description,query_result] = agt_query(io,'*idn?');
if (status < 0) return; end

[status, status_description ] = ...
    agt_sendcommand( io, 'BAND 10 kHz' );
[status, status_description ] = ...
    agt_sendcommand( io, 'DISP:WIND:TRAC:Y:RLEV 10 dbm' );
[status, status_description ] = ...
    agt_sendcommand( io, 'POW:ATT 20' );
[status, status_description ] = ...
    agt_sendcommand( io, 'DISP:WIND:TRAC:Y:PDIV 10 DB' );
[status, status_description ] = ...
    agt_sendcommand( io,=['FREQ:STAR ' num2str(start_freq/10^6) ' MHz'] );
[status, status_description ] = ...
agt_sendcommand( io, ['FREQ:STOP ' num2str(end_freq/10^6) ' MHZ'] );
[status, status_description ] = ...
agt_sendcommand( io, ['SWE:POIN ' num2str(points)] );

[status, status_description,raw_data] = agt_query(io,'TRAC:DATA? TRACE1');
power = str2num(raw_data);

save([filename '.mat'], 'frequency', 'power'); % record frequency & power
A.5 Agilent N6030A Wideband Signal Generator

This program generates time-multiplexed uniform-amplitude signals at a default sample rate of 1.25 GS/s for testing with an amplitude-modulation transmission system. The multitone signals are formulated as baseband I/Q waveforms and upconverted to the final carrier frequency exterior to the N6030A.

clear

N = 4; % number of tones
rnd_phase = 0; % enable random phase or constant phase
back_off = 0; % attenuation factor for I&Q lines

fspace = 12.5e6/(8*N); % tone frequency spacing in Hz
fsw = 1.25e9/32; % multitone time multiplex switch rate
sw_lobes = 1; % active multiplexed switching lobes

rand_seed = 1; % random number generator seed value
fsbb = 1.25e9; % baseband ARB sampling rate
fc = 1e9; % RF center frequency
A = 1; % initial amplitude

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

fres = fspace/2; % frequency resolution for simulation
tstop = 1/fres; % stop time based on frequency resolution
fs = 1.25e9; % sampling frequency
ts = 1/fs;
tm = 0:ts:tstop-ts; % sampling time vector
f = -fs/2:fres:(fs/2)-fres; % frequency grid

xi = zeros(1,length(ts)); % initialize signal vectors
xq = zeros(1,length(ts));
xmuxi = xi;
xmuxq = xq;

tsbb = 1/fsbb;
tbb = 0:tsbb:tstop-tsbb; % baseband time sample vector

---

3This script was developed by Kevin G. Gard.
tsw = 1/fsw; % multiplex switch period
sw = ones(1,(tsw/tsbb)); % switch-on duration

swl = length(sw);
sw = cat(2,sw,zeros(1,round((N-1)*tsw/tsbb)));
swt = sw;
for b = 1:(length(tbb)/(length(sw)))-1
  swt = cat(2,swt,sw);
end;
swup = swt;
swdn = circshift(swt',swl*(N/2))';
swk = swt;
swm = circshift(swt',swl*(N/2))';

rand('seed',rand_seed); % random phases for the N tones
theta = 2*pi*rand(1,N);
if rnd_phase == 0
  theta = zeros(1,N);
end;

xi = zeros(1,length(ts));
xq = zeros(1,length(ts));
xmuxi = xi;
xmuxq = xq;

for n = 1:N/2
  a = 1;
  xi = xi + (A*(a))*(cos(2*pi*fspace*(abs(n)-1/2)*tbb+theta(n)) ... + cos(2*pi*fspace*(abs(n)-1/2)*tbb+theta(n+N/2)));
xq = xq + (A*(a))*(sin(2*pi*fspace*(abs(n)-1/2)*tbb+theta(n)) ... - sin(2*pi*fspace*(abs(n)-1/2)*tbb+theta(n+N/2)));

  xmuxi = xmuxi + (10^(1/20))*N*(A*(a)) * ... (cos(2*pi*fspace*(abs(n)-1/2)*tbb+theta(n)).*swup + ... cos(2*pi*fspace*(abs(n)-1/2)*tbb+theta(n+N/2)).*swdn);
xmuxq = xmuxq+(10^(1/20))*N*(A*(a))*... (sin(2*pi*fspace*(abs(n)-1/2)*tbb+theta(n)).*swup - ... sin(2*pi*fspace*(abs(n)-1/2)*tbb+theta(n+N/2)).*swdn);

  swup = circshift(swt',swl*(n+0))';
  swdn = circshift(swt',swl*(n+N/2+0))';
if n < N/2
    swk = swk + swup;
    swm = swm + swdn;
end;
end;

nt = 1:length(tm);
ty = 1:1000;
fct = .95;
ff = [0 fct fct 1];
af = [1 1 0 0];
b = fir2(length(tbb)/4,ff,af);
xmuxi = filtfilt(b,1,xmuxi);
xmuxq = filtfilt(b,1,xmuxq);

xmuxi = lowpass(tbb,xmuxi,sw_lobes*fsw);
xmuxq = lowpass(tbb,xmuxq,sw_lobes*fsw);

Xi = fftshift(fft(2*xi)/length(xi));
XidB = 20*log10(abs(Xi));
Ximux = fftshift(fft(2*xmuxq)/length(xmuxi));
XimuxdB = 20*log10(abs(Ximux));

xi = interp(xi,fs/fsbb);
xq = interp(xq,fs/fsbb);
xmuxi = interp(xmuxi,fs/fsbb);
xmuxq = interp(xmuxq,fs/fsbb);

x = xi .* cos(2*pi*fc*tm) - xq .* sin(2*pi*fc*tm);
xmux = xmuxi .* cos(2*pi*fc*tm) - xmuxq .* sin(2*pi*fc*tm);

X = fftshift(fft(2*x)/length(x));
XdB = 20*log10(abs(X));
Xmux = fftshift(fft(2*xmux)/length(x));
XmuxdB = 20*log10(abs(Xmux));

xi = 10^(-back_off/20)*xi;
xq = 10^(-back_off/20)*xq;
xmuxi = 10^(-back_off/20)*xmuxi;
xmuxq = 10^(-back_off/20)*xmuxq;

figure(1)
plot(f/(10^6),XmuxdB,'-x',f/(10^6), IDb)
grid on;
xlabel('Frequency (MHz)')
ylabel('Amplitude (dB)')

\[ i = \frac{x_{muxi}}{N}; \quad q = \frac{x_{muxq}}{N}; \]

% \[ i = \frac{x_i}{N}; \]
% \[ q = \frac{x_q}{N}; \]

\texttt{waveform = \[ i; q \];}

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%% code from Agilent to control the N6030A %%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% open a session to the instrument
[ instrumentHandle, errorN, errorMsg ] = agt_awg_open('pci','PXI*::*');
if( errorN ~= 0 )
    disp('Could not open a session to the instrument');
    return;
end

% enable the instrument output
[ errorN, errorMsg ] = agt_awg_setstate( ...
    instrumentHandle, 'outputenabled', 'true');
if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not enable the instrument output');
    return;
end

% set the instrument to ARB mode
[ errorN, errorMsg ] = agt_awg_setstate( ...
    instrumentHandle, 'outputmode', 'arb');
if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not set the instrument to ARB mode');
    return;
end

% set the output to single-ended amplified configuration
[ errorN, errorMsg ] = agt_awg_setstate( ...
instrumentHandle, 'outputconfig', 'amp');

if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not set the instrument to single-ended AMP mode');
    return;
end

% set the output to differential configuration
% [ errorN, errorMsg ] = agt_awg_setstate( ... %
%     instrumentHandle, 'outputconfig', 'diff');
% if( errorN ~= 0 ) %
%     agt_awg_close( instrumentHandle );
%     disp('Could not set the instrument to differential mode');
%     return;
% end

% modify the DC offset of Channel 1
[ errorN, errorMsg ] = agt_awg_setstate( ... %
    instrumentHandle, 'outputoffset', 0, 1);
if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not set the output offset');
    return;
end

% modify the DC offset of Channel 2
[ errorN, errorMsg ] = agt_awg_setstate( ... %
    instrumentHandle, 'outputoffset', 0, 2);
if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not set the output offset');
    return;
end

% set the output (active) gain
[ errorN, errorMsg ] = agt_awg_setstate( ... %
    instrumentHandle, 'outputgain', 0.500);
if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not set the output gain');
    return;
end
length_1 = floor(length(i)/64);
length_2 = length(i)/8 - length_1;
marker = [ones(1,length_1) zeros(1,length_2)];

% transfer the waveform to the instrument
[ waveformHandle,errorN,errorMsg ] =
agt_awg_storewaveformwithmarker( ...  
    instrumentHandle, waveform, marker, marker, marker, marker, marker, 0);
if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not transfer the waveform to the instrument');
    errorN
    errorMsg
    return;
end

% initiate playback of the waveform on the instrument
[ errorN, errorMsg ] = agt_awg_playwaveform( ...  
    instrumentHandle, waveformHandle );
if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not initiate playback of the waveform on the instrument');
    errorN
    errorMsg
    return;
end

[ errorN, errorMsg ] = agt_awg_setstate( ...  
    instrumentHandle, 'mkrsorce', 'ch1_wfm_mkri', 1);
if( errorN ~= 0 )
    agt_awg_close( instrumentHandle );
    disp('Could not set the marker.');
    return;
end

% close the session to the instrument
agt_awg_close( instrumentHandle );
A.6 Agilent N5230A Vector Network Analyzer

A.6.1 Two-Port Calibration

To calibrate the N5230 to take 2-port S-parameter measurements up to 4 GHz, follow these instructions:

1. Obtain the HP “85052A 3.5mm Calibration Kit” containing (at least)
   - one female short-circuit termination,
   - one female open-circuit termination,
   - one dielectric insert for the open-circuit termination,
   - one female 50-Ω load termination, and
   - one female/female through-port connector.

2. Boot the N5230A into Windows XP and load the “Network Analyzer” software.

3. Choose your desired frequency range. Enter this range using either “Start/Stop...” or “Center/Span” in the “Channel” menu.

4. Choose your desired frequency resolution. Enter the number of points at which to record amplitude and phase using “Number of Points” in the “Sweep” menu.

5. Begin the calibration by selecting “Calibration Wizard...” in the “Calibration” menu.

6. Choose “UNGUIDED Calibration” and click “Next”.

7. Change the calibration kit to the 85052A. Click “View/Select Cal Kit”, scroll to “85052A” under “Kit Name”, select this one, and click “OK”. Then click “Next”.

8. Connect the short-circuit termination to Port 1. Use a torque wrench to secure the connection. Click “SHORT” under “Port 1”. Click “APC 3.5 female short” and then “OK”.

9. Disconnect the short-circuit termination from Port 1 using a torque wrench and connect it to Port 2. Then click “SHORT” under “Port 2”. Click “APC 3.5 female short” and then “OK”. Remove the short-circuit termination from Port 2.
10. Connect the open-circuit termination to Port 1. Place the dielectric insert snugly inside the open-circuit termination. Click “OPEN” under “Port 1”. Click “APC 3.5 female open” and then “OK”. Remove the dielectric insert, then disconnect the open-circuit.

11. Connect the open-circuit termination with its dielectric insert to Port 2. Click “OPEN” under “Port 2”, “APC 3.5 female open” and then “OK”. Remove the dielectric insert and remove the open-circuit from Port 2.

12. Connect the 50-Ω load to Port 1, then click “LOADS” under “Port 1”. Click “APC female load” where “Min Freq” is “0 Hz” and “Max Freq” is “4.001 GHz”. Then click “OK”.

13. Remove the 50-Ω load from Port 1, connect it to Port 2, and record the lowband load setting as with Port 1. Then disconnect the load from Port 2.

14. Connect one end of the through-port connector to Port 1, and the other end to Port 2. Click “THRU”. Then remove the through-port connector from both Port 1 and Port 2.

15. To finish the calibration, click “Next” and then “Finish”. The network analyzer is now ready for 2-port measurements.

### A.6.2 Viewing & Saving S-Parameters

To add S-parameter traces to the N5230A display, do the following:

1. Select “New Trace” under the “Trace” menu.

2. Put a check-mark next to the particular S-parameter you wish to add to the display. Then click “OK”.

To save S-parameter data from the N5230A, do the following:

1. Select “Save As...” under the “File” menu.

2. Change “Save as type:” to “Trace (*.s2p)”.

3. Choose a file name and location for the save. Click “Save”.
The two-port S-parameters will be saved in S2P format. This file contains a header, after which the data appears in 9 columns. The first column is a vector of frequency points (in Hz). The next 8 columns are the S-parameters themselves, in pairs.

The second and third columns are $S_{11}$. The second is $|S_{11}|$ in dB. The third is $\arg(S_{11})$ in degrees. The rest of the columns are the magnitude & phase pairs for $S_{21}$, $S_{12}$, and $S_{22}$ in the same format (dB, degrees).

A.6.3 Viewing & Saving Group Delay

To view group delay (and smooth some of the noise of the measurement) on the N5230A display, do the following:

1. Add $S_{21}$ to the available traces.
2. Right-click on “S21”. Choose “Group Delay” from the “Format” menu.
3. Choose “Smoothing...” from the “Trace” menu.
4. Place a check-mark next to “Smoothing ON”. Choose a small percentage of the recorded data points over which to perform a moving-average. Then click “OK”.

The group delay data may be extracted from an S2P file (by taking the negative of the derivative of the phase trace with respect to frequency), but it may be recorded separately by saving the current trace in PRN format.
**B**

**fREEDA Elements**

**B.1 vlfmpulse — Linear FM Voltage Source**

![Diagram of vlfmpulse](image)

Figure B.1: Linear frequency-modulated source element.

**Netlist Entry Format**

\[ \text{vlfmpulse:<instance name> <n1> <n2> <parameter list>} \]

- \( n1 \) is the voltage output, above the reference terminal potential
- \( n2 \) is the reference terminal

This element creates a frequency-modulated voltage pulse. The amplitude of the pulse is \( va \) and its DC offset is \( vo \). The time delay (starting from zero) before the pulse is active is \( td \). The center of the linear FM sweep is \( f0 \) and the range of the sweep is \( deltaf \). The
Table B.1: VLFMpulse Parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Meaning</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>vo</td>
<td>DC offset (V)</td>
<td>0</td>
</tr>
<tr>
<td>va</td>
<td>pulse amplitude (V)</td>
<td>0</td>
</tr>
<tr>
<td>td</td>
<td>time delay (s)</td>
<td>0</td>
</tr>
<tr>
<td>f0</td>
<td>center frequency (Hz)</td>
<td>1.0</td>
</tr>
<tr>
<td>deltaf</td>
<td>frequency sweep range (Hz)</td>
<td>0.5</td>
</tr>
<tr>
<td>chirpdir</td>
<td>chirp direction (1 or -1)</td>
<td>1</td>
</tr>
<tr>
<td>phi</td>
<td>initial phase (degrees)</td>
<td>0</td>
</tr>
<tr>
<td>tau</td>
<td>pulse width (s)</td>
<td>0</td>
</tr>
<tr>
<td>per</td>
<td>pulse period (s)</td>
<td>0</td>
</tr>
</tbody>
</table>

chirp can be directed up or down by setting chirpdir to +1 or -1. The initial phase of the carrier wave is phi. The pulse is active for a time interval tau and the entire pulse repeats in time with a period per.

Netlist Example

vlfpulse:v1 1 0 vo=0 va=1 td=0e-9 f0=465e6 deltaf=5e6
    chirpdir=1 phi=90 tau=400e-9 per=2e-6

B.2 chebyshevbpf — Lumped Chebyshev Bandpass Filter

This section is adapted from the fREEDA v1.3 documentation for chebyshevbpf, “Lumped Chebyshev Bandpass Filter”, version 2008.05.11, written by Shawn Evans and Michael Steer.

Figure B.2: Chebyshev bandpass filter of order n.
Netlist Entry Format

chebyshevbpf:<instance name> <n1> <n2> <n3> <parameter list>

n1 is the input terminal
n2 is the reference terminal
n3 is the output terminal

Table B.2: ChebyshevBPF Parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>Meaning</th>
<th>Default Value</th>
<th>Required?</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>filter order</td>
<td>11</td>
<td>yes</td>
</tr>
<tr>
<td>f0</td>
<td>center frequency (Hz)</td>
<td>N/A</td>
<td>yes</td>
</tr>
<tr>
<td>bw</td>
<td>bandwidth (Hz)</td>
<td>N/A</td>
<td>yes</td>
</tr>
<tr>
<td>z0</td>
<td>port impedance (Ω)</td>
<td>50</td>
<td>yes</td>
</tr>
<tr>
<td>ripple</td>
<td>passband ripple (dB)</td>
<td>0.1</td>
<td>yes</td>
</tr>
<tr>
<td>q</td>
<td>resonator Q</td>
<td>10,000</td>
<td>no</td>
</tr>
</tbody>
</table>

This element designs a Cauer-2 Chebyshev bandpass filter, using pairs of parallel and series inductors and capacitors in a ladder structure. The filter can be of any order up to a maximum of 100. The filter has a center frequency $f_0$, a bandwidth $bw$, and a passband amplitude variation $ripple$. The input and output impedances $z_0$ are equal.

Netlist Example

chebyshevbpf: c1 2 0 3 n=5 f0=465e6 bw=10e6 z0=50 ripple=0.1
Matlab Helper Functions

C.1 RF Envelope Extraction

The following function removes the RF carrier(s) from a time-domain trace, outputting the amplitude envelope of the input waveform.

```matlab
function [ x_new, y_new ] = env_extract( x, y )

y = abs(y); % make all data positive

counter = 1; for n = 2:length(x)-1
    if ( y(n-1) < y(n) ) & ( y(n+1) < y(n) )
        x_new(counter) = x(n);
        y_new(counter) = y(n);
        counter = counter + 1;
    end
end

y_new = interp1(x_new,y_new,x,'spline'); % interpolation
```
C.2 Generalized Fourier Transform

The following function implements the Fourier Transform using the full Fourier Integral from (2.3) instead of using the Fast Fourier Transform algorithm.

```matlab
function [frequencies, spectrum] = general_ft(filename, delta_t, ...
    start_freq, end_freq, samples);

% Discrete Fourier Transform
%
% usage: general_ft(filename, delta_t, start_freq, end_freq, samples)
%
% inputs: filename = name of input data file
%          delta_t = time between data samples
%          start_freq = start of Fourier spectrum
%          end_freq = end of Fourier spectrum
%          samples = number of frequency points
%
% outputs: frequencies-- (1 x samples) matrix of frequency sample points
%          spectrum-- magnitude & phase of each frequency component

data = load(filename);
data = data';

N = length(data); n = 0:1:N-1;

total_time = N * delta_t;

frequencies = linspace(start_freq, end_freq, samples);
k = frequencies * total_time;

for i = 1:samples
    spectrum(i) = (1/N) * sum(data .* exp(-2*pi*1i*n*k(i)/N));
end
```
C.3 Bandpass Element Values from Low-Pass Prototypes

In order to design a bandpass low-ripple Chebyshev Type 1 filter from its lowpass prototype, the following procedure may be employed:

1. Decide the filter order (i.e. the number of poles at either passband edge in the bandpass case).

2. Decide on a Cauer 1 or Cauer 2 architecture (as shown in Fig. C.1).

3. Decide the acceptable passband ripple (e.g. 0.01 dB).

4. Refer to a table for the lowpass (normalized) filter prototype capacitance and inductances. Table C.1 gives these values for a passband ripple of 0.01 dB up to 9th order [59].

5. Decide the operating frequencies (e.g. $f_c = 465$ MHz, $B = 2\%$), and input/output port impedance (e.g. 50 Ω).

6. Transform the lowpass $C$ and $L$ values to bandpass $LC$ pairs using the appropriate scaling rules [60]. These are given in (2.23).

The following script performs the lowpass-to-bandpass transformation using the aforementioned scaling rules. The user inputs the lowpass $C$ and $L$ values, port impedance, and operating frequencies. The script outputs the bandpass element values for sets of parallel and series $LC$ pairs.
Table C.1: Normalized Chebyshev Filter Element Values, 0.01 dB Ripple

<table>
<thead>
<tr>
<th>Order</th>
<th>$C_1/L_1$</th>
<th>$L_2/C_2$</th>
<th>$C_3/L_3$</th>
<th>$L_4/C_4$</th>
<th>$C_5/L_5$</th>
<th>$L_6/C_6$</th>
<th>$C_7/L_7$</th>
<th>$L_8/C_8$</th>
<th>$C_9/L_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.4489</td>
<td>0.4078</td>
<td>0.9085</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.6292</td>
<td>0.6292</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.7129</td>
<td>1.2004</td>
<td>1.3213</td>
<td>0.6476</td>
<td>0.9085</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.7653</td>
<td>1.3049</td>
<td>1.5773</td>
<td>1.3049</td>
<td>0.7563</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.7814</td>
<td>1.3600</td>
<td>1.6897</td>
<td>1.5350</td>
<td>1.4970</td>
<td>0.7098</td>
<td>0.9085</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.7970</td>
<td>1.3924</td>
<td>1.7481</td>
<td>1.6331</td>
<td>1.7481</td>
<td>1.3924</td>
<td>0.7970</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.8073</td>
<td>1.4131</td>
<td>1.7824</td>
<td>1.6833</td>
<td>1.8529</td>
<td>1.6193</td>
<td>1.5555</td>
<td>0.7334</td>
<td>0.9085</td>
</tr>
<tr>
<td>9</td>
<td>0.8145</td>
<td>1.4271</td>
<td>1.8044</td>
<td>1.7125</td>
<td>1.9058</td>
<td>1.7125</td>
<td>1.8044</td>
<td>1.4271</td>
<td>0.8145</td>
</tr>
</tbody>
</table>

```matlab
clear;

% lowpass prototype values
C = [0.6292 0.6292];
L = [0.9703];

% input/output resistance, operating frequencies
R = 50;
f_center = 465e6;
BW = .02 * 465e6;

%f_upper = f_center + BW/2;
f_lower = f_center - BW/2;
gamma = (f_upper-f_lower)/f_center;

%f bandpass element values
L_series = R*L/(2*pi*(f_upper-f_lower));
C_series = gamma./(R*L*2*pi*f_center);
C_parallel = C/(R*2*pi*(f_upper-f_lower));
L_parallel = gamma*R. /(C*2*pi*f_center);
```
C.4 One-Sided Fourier Transform

The following function outputs the positive-frequency Fourier Transform (frequency and complex amplitude vectors) of the input time-domain trace (time and amplitude vectors).

function [ frequency, Y ] = fft_spectrum( x, y )

% [ Y, frequency ] = fft_spectrum( y, x )
% -- calculates the 1-sided FFT of 'y' as a function of 'x'
% -- spectrum samples of 'y' are stored in 'Y'
% -- frequency values of 'x' are stored in 'frequency'

T = x(length(x)) - x(1); % full period of x
tau = x(2) - x(1); % delta(x) sample length
Y_temp = fft(y); % Fast Fourier Transform
Y_temp = 2 / length(y) * Y_temp; % DFT scaling
Y = Y_temp(1:floor(length(Y_temp)/2)+1); % use only one side of spectrum
Y(1) = Y(1) / 2; % correction for true average value at f = 0

delta_f = 1/T;
frequency = ( (1:length(Y))-1 ) * delta_f;
C.5 Ideal Bandpass Filter

The following function outputs a band-limited version of an input time-domain trace.

```
function y_out = bandpass( x_in, y_in, start_freq, end_freq )

% y_out = bandpass( x_in, y_in, start_freq, end_freq )
% -- applies an ideal bandpass filter to 'y_in' as a function of 'x_in'
% -- start_freq = lower bandpass corner frequency
% -- end_freq = upper bandpass corner frequency
% -- y_out = filtered result, still a function of 'x_in'

T = x_in(length(x_in)) - x_in(1); % full period of x

tau = x_in(2) - x_in(1); % delta(x) sample length

Y_temp = fft(y_in); % Fast Fourier Transform

if mod(length(Y_temp),2) == 1
    frequency = [ (0:floor(length(Y_temp)/2))*(1/T) ...
                   fliplr((1:floor(length(Y_temp)/2))*(-1/T)) ];
else
    frequency = [ (0:length(Y_temp)/2)*(1/T) ...
                   fliplr((1:length(Y_temp)/2-1)*(-1/T)) ];
end

for n = 1:length(frequency)
    if ( frequency(n) >= -1*start_freq ) & ... 
        ( frequency(n) <= start_freq ) ...
    || ( frequency(n) <= -1*end_freq ) ... 
    || ( frequency(n) >= end_freq )
        Y_temp(n) = 0;
    end
end

y_out = real(ifft(Y_temp)); % inverse FFT
```
D

Cauer 1 vs. Cauer 2 Filter Designs

D.1 One Transfer Function, Two Reflection Coefficients

Chebyshev filters are designed for a particular magnitude characteristic. The general form of the Chebyshev transmission coefficient amplitude $|T(s)|$ is [61]

$$|T(s)| = \frac{1}{\sqrt{1 + \varepsilon^2 |K_n(s)|^2}}$$  \hspace{1cm} (D.1)

where the parameter $\varepsilon$ defines the passband ripple (PBR),

$$\text{PBR}_{\text{dB}} = 10 \log_{10} \left( \frac{1}{1 + \varepsilon^2} \right)$$  \hspace{1cm} (D.2)

and $K_n(s)$ is a Chebyshev polynomial of order $n$ given by

$$K_n(s) = \frac{1}{2} \left[ \left( s + \sqrt{s^2 - 1} \right)^n + \left( s - \sqrt{s^2 - 1} \right)^n \right].$$  \hspace{1cm} (D.3)

Assuming a lossless filter, conservation of energy states that all pulse power not
transmitted must be reflected:

\[ |T(s)|^2 + |\Gamma(s)|^2 = 1. \]  \hspace{1cm} (D.4)

Note that there is not necessarily a conservation of voltage:

\[ |T(s)| + |\Gamma(s)| \neq 1 \quad \text{and} \quad T(s) + \Gamma(s) \neq 1. \]  \hspace{1cm} (D.5)

Expanding and rearranging (D.4) gives

\[ \Gamma(s) \Gamma^*(s) = 1 - |T(s)|^2. \]  \hspace{1cm} (D.6)

For a particular reflection coefficient \( \Gamma_0(s) \) that satisfies (D.6), the complex conjugate of that particular reflection coefficient also satisfies (D.6):

\[ \Gamma_0(s) \Gamma_0^*(s) = 1 - |T_0(s)|^2 \]  \hspace{1cm} (D.7)

\[ \Gamma_0^*(s) \Gamma_0^{**}(s) = \Gamma_0^*(s) \Gamma_0(s) = 1 - |T_0(s)|^2 \]  \hspace{1cm} (D.8)

Thus, there exist two reflection coefficients, \( \Gamma_0(s) \) and \( \Gamma_0^*(s) \), that correspond to the same transmission coefficient amplitude \( |T_0(s)| \). In other words, it is possible to measure different reflected waveforms from two different filters having the same Chebyshev transfer function design.

D.2 Circuit Implementation & Simulation

Multiple realizations of the same transmission coefficient with different reflection coefficients are possible with Chebyshev (or Butterworth) filters, as in Fig. D.1. Circuit 1 is a Cauer type 1 topology, while circuit 2 is a Cauer type 2 topology.
Figure D.1: Third-order filter architectures with Cauer 1 and Cauer 2 realizations: Cauer 1 above, Cauer 2 below; lowpass prototypes on the left, bandpass designs on the right.

Figure D.2: ADS 2006 circuit to simulate Cauer 1 and Cauer 2 bandpass 5th-order Chebyshev designs: 465 MHz center frequency, 2% bandwidth, 0.1 dB passband ripple; Cauer 1 is above and Cauer 2 is below.
Figure D.3: Cauer 1 vs. Cauer 2 simulations: Comparison of steady-state transmission and reflection characteristics using the circuit of Fig. D.2. The transmission characteristics of the two designs are equal in magnitude and phase; the reflected characteristics are equal and magnitude and offset in phase by 180°.

Either form, Cauer 1 or Cauer 2, may implement an identical transfer function $T(s)$ [62]. The juxtaposition of the capacitors and inductors between the two forms gives a 180° phase shift in the reflected waves between Cauer 1 and Cauer 2, however. A simulation which compares the steady-state and transient characteristics for Cauer 1 and Cauer 2 forms in a 5th-order bandpass implementation is given in Fig. D.2. The results of the steady-state simulation are given in Fig. D.3; the results of the transient simulation are given in Fig. D.4.

The upper plots of Fig. D.3 show that the steady-state transmission characteristics are the same, both in magnitude and phase, over all frequencies, for both Cauer forms. The lower plots of Fig. D.3 show that the steady-state reflection characteristics are the same in magnitude, but 180° offset in phase.

The upper plots of Fig. D.4 show that the RF envelope and the phase of the RF carrier of the Cauer 1 transmission match that of the Cauer 2 transmission. The lower plots of Fig. D.4 show that the RF envelope of the reflections match, but the Cauer 2 reflection is out-of-phase from the Cauer 1 reflection by 180°.
Figure D.4: Cauer 1 vs. Cauer 2 simulations: Comparison of transient transmitted and reflected waveforms using the circuit of Fig. D.2 and an RF frequency of 462 MHz. Between the two designs, the transmitted and reflected voltage envelopes are the same. The phases of the transmitted waveforms have equal phases, but the phases of the reflected waveforms are offset by $180^\circ$. 
Filters Used in This Study

E.1 Filter Parameters from Manufacturer Datasheets

E.1.1 900-MHz Chebyshev Filters from Trilithic

Table E.1: 900-MHz Trilithic Chebyshev Filters, < 0.05 dB Ripple

<table>
<thead>
<tr>
<th>Trilithic Part Number</th>
<th>Design</th>
<th>Order</th>
<th>Center (MHz)</th>
<th>BW (MHz)</th>
<th>IL (dB)</th>
<th>Input/Output Ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>7BC900/27-3-KK</td>
<td>tubular</td>
<td>7</td>
<td>900</td>
<td>27</td>
<td>3.15</td>
<td>SMA-F/SMA-F</td>
</tr>
<tr>
<td>7BC900/36-3-KK</td>
<td>tubular</td>
<td>7</td>
<td>900</td>
<td>36</td>
<td>2.70</td>
<td>SMA-F/SMA-F</td>
</tr>
<tr>
<td>7BC900/45-3-KK</td>
<td>tubular</td>
<td>7</td>
<td>900</td>
<td>45</td>
<td>2.07</td>
<td>SMA-F/SMA-F</td>
</tr>
</tbody>
</table>

Additional Information for the 900-MHz Filters

- **7BC900/27-3-KK**: 3.0 dB down from passband at 915.6 and 883.9 MHz, 43.1 dB relative rejection at 870 MHz, 43.6 dB rejection at 930 MHz, ordered August 2007, quantity = 1, cost = $795.00

- **7BC900/36-3-KK**: 3.0 dB down from passband at 919.8 and 881.1 MHz, 46.0 dB relative rejection at 862 MHz, 44.5 dB rejection at 937 MHz,
ordered August 2007, quantity = 1, cost = $795.00

- **7BC900/45-3-KK**: 3.0 dB down from passband at 925.0 and 876.2 MHz, 47.0 dB relative rejection at 852 MHz, 46.6 dB rejection at 947 MHz, ordered August 2007, quantity = 1, cost = $795.00

### E.1.2 465-MHz Chebyshev Filters from Trilithic

Table E.2: 465-MHz Trilithic Chebyshev Filters, < 0.05 dB Ripple

<table>
<thead>
<tr>
<th>Part Number</th>
<th>Design</th>
<th>Order</th>
<th>Center (MHz)</th>
<th>BW (MHz)</th>
<th>IL (dB)</th>
<th>Input/Output Ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>7BC465/5-3-KK</td>
<td>tubular</td>
<td>7</td>
<td>465</td>
<td>5</td>
<td>9.86</td>
<td>SMA-F/SMA-F</td>
</tr>
<tr>
<td>5BC465/5-3-KK</td>
<td>tubular</td>
<td>5</td>
<td>465</td>
<td>5</td>
<td>5.68</td>
<td>SMA-F/SMA-F</td>
</tr>
</tbody>
</table>

Additional Information for the 465-MHz Filters

- **7BC465/5-3-KK**: 3.0 dB down from passband at 468.2 and 462.1 MHz, 41.6 dB relative rejection at 473 MHz, 46.8 dB rejection at 457 MHz, ordered June 2008, quantity = 1, cost = $695.00

- **5BC465/5-3-KK**: 3.0 dB down from passband at 469.1 and 461.4 MHz, 43.5 dB relative rejection at 477 MHz, 41.5 dB rejection at 453 MHz, ordered June 2008, quantity = 1, cost = $595.00

### E.1.3 4\textsuperscript{th}-Order Chebyshev Filters from Trilithic

Table E.3: 4\textsuperscript{th}-Order Trilithic Chebyshev Filters, < 0.05 dB Ripple

<table>
<thead>
<tr>
<th>Part Number</th>
<th>Design</th>
<th>Order</th>
<th>Center (MHz)</th>
<th>BW (MHz)</th>
<th>IL (dB)</th>
<th>Input/Output Ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>4BC500/20-3-KK</td>
<td>tubular</td>
<td>4</td>
<td>500</td>
<td>20</td>
<td>1.47</td>
<td>SMA-F/SMA-F</td>
</tr>
<tr>
<td>4BC1000/40-3-KK</td>
<td>tubular</td>
<td>4</td>
<td>1000</td>
<td>40</td>
<td>1.47</td>
<td>SMA-F/SMA-F</td>
</tr>
<tr>
<td>4BC2000/80-3-KK</td>
<td>tubular</td>
<td>4</td>
<td>2000</td>
<td>80</td>
<td>1.38</td>
<td>SMA-F/SMA-F</td>
</tr>
</tbody>
</table>
Additional Information for the 4th-Order Filters

- **4BC500/20-3-KK**: 3.0 dB down from passband at 511.5 and 489.0 MHz, 41.6 dB relative rejection at 539 MHz, 42.0 dB rejection at 457 MHz, ordered January 2007, quantity = 1, cost = $525.00

- **4BC1000/40-3-KK**: 3.0 dB down from passband at 1023 and 978 MHz, 44.0 dB relative rejection at 1079 MHz, 45.2 dB rejection at 912 MHz, ordered January 2007, quantity = 1, cost = $525.00

- **4BC2000/80-3-KK**: 3.0 dB down from passband at 2043.3 and 1956.4 MHz, 44.0 dB relative rejection at 2141 MHz, 44.2 dB rejection at 1845 MHz, ordered January 2007, quantity = 1, cost = $525.00

**E.1.4 5th-Order Chebyshev Filters from K&L Microwave**

<table>
<thead>
<tr>
<th>K&amp;L Microwave Part Number</th>
<th>Design</th>
<th>Order</th>
<th>Center (MHz)</th>
<th>BW (MHz)</th>
<th>IL (dB)</th>
<th>Input/Output Ports</th>
</tr>
</thead>
<tbody>
<tr>
<td>5MC10-500/T25-O/O</td>
<td>lumped</td>
<td>5</td>
<td>500</td>
<td>25</td>
<td>2.18</td>
<td>SMA-F/SMA-F</td>
</tr>
<tr>
<td>5DR30-1000/T25-O/O</td>
<td>dia res</td>
<td>5</td>
<td>1000</td>
<td>25</td>
<td>2.16</td>
<td>SMA-F/SMA-F</td>
</tr>
<tr>
<td>5DR30-2000/T25-O/O</td>
<td>dia res</td>
<td>5</td>
<td>2000</td>
<td>25</td>
<td>3.34</td>
<td>SMA-F/SMA-F</td>
</tr>
</tbody>
</table>

Additional Information for the 5th-Order Filters

- **5MC10-500/T25-O/O**: 3.0 dB down from passband at 513.5 and 486.5 MHz, ordered January 2007, quantity = 1, cost = $370.00

- **5DR30-1000/T25-O/O**: center frequency = 999.475 MHz, 3 dB bandwidth = 28.286 MHz, ordered January 2007, quantity = 1, cost = $370.00

- **5DR30-2000/T25-O/O**: center frequency = 1999.115 MHz, 3 dB bandwidth = 27.495 MHz, ordered January 2007, quantity = 1, cost = $370.00
E.2 S-Parameters from the Agilent N5230A

E.2.1 Magnitude & Phase

Figure E.1: S-parameters for the Trilithic 7BC900/27-3-KK filter.
Figure E.2: S-parameters for the Trilithic 7BC900/36-3-KK filter.
Figure E.3: S-parameters for the Trilithic 7BC900/45-3-KK filter.
Figure E.4: S-parameters for the Trilithic 7BC465/5-3-KK filter.
Figure E.5: S-parameters for the Trilithic 5BC465/5-3-KK filter.
Figure E.6: S-parameters for the Trilithic 4BC500/20-3-KK filter.
Figure E.7: S-parameters for the Trilithic 4BC1000/40-3-KK filter.
Figure E.8: S-parameters for the Trilithic 4BC2000/80-3-KK filter.
Figure E.9: S-parameters for the K&L Microwave 5MC10-500/T25-O/O filter.
Figure E.10: S-parameters for the K&L Microwave 5DR30-1000/T25-O/O filter.
Figure E.11: S-parameters for the K&L Microwave 5DR30-2000/T25-O/O filter.
E.2.2 Group Delay

Figure E.12: Group delay traces for the 900-MHz Trilithic filters.
Figure E.13: Group delay traces for the 4th-order Trilithic filters.
Figure E.14: Group delay traces for the 5th-order K&L Microwave filters.