The Impact of Granularity on Worked Examples and Problem Solving

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Abstract
In this paper, we explored the impact of two types of instructional interventions, worked examples and problem solving, at two levels of granularity: problems and steps. This study relied on an existing Intelligent Tutoring System (ITS) for Probability called Pyrenees and involved 266 students who were randomly assigned into five conditions. All students experienced the same procedure, studied the same training problems in the same order, and used the same ITS. The conditions differed only in how the training problems were presented. Our results showed that when the domain content and required steps are strictly equivalent, different granularities of pedagogical decisions can significantly impact students’ time on task. More specifically, the fine-grained step level decisions can have a stronger pedagogical impact than the problem-level ones.

Keywords: worked example, problem solving, faded worked example, granularity

Introduction
Much research has investigated the different impacts of worked examples (WE) and problem solving (PS) on student learning (Sweller & Cooper, 1985; McLaren, Lim, & Koedinger, 2008; McLaren & Isotani, 2011; McLaren, van Gog, Ganoe, Yaron, & Karabinos, 2014; Renkl, Atkinson, Maier, & Staley, 2002; Schwonke et al., 2009; Najar, Mitrovic, & McLaren, 2014; Salden, Aleven, Schwonke, & Renkl, 2010). In PS students are given tasks to complete either independently or with assistance while in WE, students are given detailed solutions. When comparing WE vs. PS, we often need to control content. For example, Sweller and Cooper compared the learning effects of WE-PS pairs with PS-only (Sweller & Cooper, 1985). In the WE-PS condition, students studied a worked example and then solved a practice problem. Their results showed that the WE-PS condition not only learned significantly more but spent significantly less time than the PS-only condition. However, it is possible that the primary benefit of the WE-PS training was that students received additional domain content that was not given to the PS-only ones. Therefore, in this paper we will focus on research that controlled for learning content across the conditions.

Several techniques have been employed to control for learning content. One approach is to use a tutor such as an Intelligent Tutoring System (ITS). ITSs are generally designed to give students on-demand hints, and to give immediate or delayed feedback on submitted solutions. In this paper we will focus on comparisons between WE and tutor-assisted PS, and we will explicitly state if this is not the case.

Tutoring in domains such as math and science can be viewed as a two-loop procedure (Vanlehn, 2006). The outer loop makes problem or task level decisions, such as deciding which problem or example to provide next, while the inner loop governs step level decisions during problem solving. In educational literature, the term “step” often refers to the application of a major domain principle or equation, such as Newton’s Third Law of Thermodynamics, during problem solving. Solving a whole problem generally involves carrying out many individual steps in a logical order. Based on this two-loop structure, we further divide the prior research into two levels of granularity: problem level and step level. Research on the impact of step level decisions has generally been focused on the impact of faded worked examples (FWE). FWEs interleave problem solving with step-level examples within a problem. In the remainder of this section we will describe prior work on on WE vs. PS at both levels of granularity and we will focus on two types of outcome measures: learning performance and time on task.

Problem Level Decisions
McLaren and colleagues compared problem-level WE-PS pairs with PS-only (McLaren et al., 2008). Every student was given a total of 10 training problems. Students in the PS-only condition were required to solve every problem while students in the WE-PS condition were given 5 example-problem pairs. Each pair consisted of an initial worked example problem followed by tutored problem solving. They found no significant difference in learning performance between the two conditions, however the WE-PS group spent significantly less time than the PS group.

McLaren and his colleagues found similar results in two subsequent studies (McLaren & Isotani, 2011; McLaren et al., 2014). In the former, the authors compared three conditions: WE, PS and WE-PS pairs, in the domain of high school chemistry. All students were given 10 identical problems. Students in the PS group were required to solve each problem in an ITS. Students in the WE group viewed them as examples, and students in the WE-PS group alternated worked examples with problem solving. As before, the authors found no significant difference among the three groups in terms of learning gains but the WE group spent significantly less time than the other two conditions; and no significant time on task difference was found between the PS and WE-PS conditions.

In a follow-up study, conducted in the domain of high school stoichiometry, McLaren and colleagues compared four conditions: WE, tutored PS, untutored PS, and Errorous Examples (McLaren et al., 2014). Students in the Errorous Examples condition were given incorrect worked examples containing between 1 and 4 errors and were tasked with correcting them. Again the authors found no significant differences among the conditions in terms of learning gains, and
as before the WE students spent significantly less time than the other groups. More specifically, they found: WE < Erroneous Examples < untutored PS < tutored PS on time on task. In fact, the WE students took only 30% of the time that the tutored PS students did. $M = 19.8$, $SD = 5.8$ and $M = 62.4$, $SD = 17.2$ respectively.

The advantages of worked examples also appeared in another study conducted in the domain of electrical circuits (Van Gog, Kester, & Paas, 2011). In this study, they compared four conditions: WE, WE-PS pairs, PS-WE pairs (problem-solving followed by an example problem), and PS only. Results showed that the WE and WE-PS students significantly outperformed the other two groups, and no significant differences was found among four conditions on time on task.

In short, prior research has shown that problem-level worked examples can be as or more effective than problem solving or alternating problems with examples, and the former can take significantly less time than the latter two (Sweller & Cooper, 1985; McLaren et al., 2008; McLaren & Isotani, 2011; McLaren et al., 2014; Renkl et al., 2002; Schwonke et al., 2009).

**Step Level Decisions**

With respect to step level decisions, the results from previous research are *mixed*. For example, Renkl et al. compared WE-PS pairs with FWE using a fixed fading policy (Renkl et al., 2002). For FEW with Fixed fading policy, the study designer predefined which steps to give examples and which steps to ask students to solve. The number of examples and unsolved steps provided was equal in both conditions. They found that FWE with the fixed fading policy significantly outperformed WE-PS pairs. No significant difference was found between the two groups on time on task.

Schwonke et al. compared FWE with a fixed fading policy to tutored PS (Schwonke et al., 2009). In two studies, they found no significant difference in terms of learning outcomes between the two conditions, however the FWE group spent significantly less time than tutored PS group.

Najar and colleagues (Najar et al., 2014) compared FWE with an adaptive fading policy to WE-PS pairs. They found that the FWE condition significantly outperformed the WE-PS condition in terms of learning outcomes and the former also spent significantly less time than the latter.

Finally, Salden et al. compared three conditions: FWE with a fixed fading policy, FWE with an adaptive fading policy, and PS-only (Salden et al., 2010). With respect to learning outcomes, they found that FWE with the adaptive fading policy outperformed FWE with the fixed fading policy, which in turn outperformed PS-only. They found no significant time on task differences among the groups.

In short, for step-level worked examples, while the results have been generally mixed, it has been shown that FWE with effective fading policies can outperform either PS or WE-PS pairs. It has also been shown that the former may require significantly less time than either of the latter two.

Our Approach

In this study, we focused on comparing five conditions:

1. **Worked Examples (WE):** where the tutor guides the student through a complete problem solution.
2. **Problem Solving (PS):** where the student is required to solve each problem with assistance of an ITS.
3. **Faded Worked Examples (FWE):** where problem solving steps are interspersed with step-level worked examples.
4. **WE/PS:** where students receive both WE and PS problems.
5. **ALL:** where students receive WE, FWE and PS problems.

In the prior research described above the authors compared the effectiveness of two or three conditions. To our knowledge, no prior study has compared all five conditions directly, especially WE vs. FWE.

For WE/PS, FWE, and ALL conditions, there are many different ways to decide when to provide students a WE, a PS, and for FWEs, it is necessary to decide which steps should be presented as worked example steps and which should be solved by a student. *Pedagogical strategies are policies* used to decide the next system action when there are multiple actions available.

Generally speaking, prior researchers studying problem-level decisions employed fixed pedagogical policies by giving set pairs: either WE-PS (a worked example first followed by problem solving) or PS-WE. Studies of step-level decisions (FWE) generally used a fixed fading policy or an adaptive fading policy. In the former case the order of steps was pre-specified and did not adapt to the students’ learning experience. In adaptive fading these decisions are made based on a real time evaluation of the student’s mastery of the subject knowledge. For example, once a student has demonstrated mastery of a problem-solving step the system will require them to complete it, otherwise it will give it as an example. Note that both fixed fading policies and adaptive fading policies in previous studies are defined by hand-coded rules.

In prior work we investigated the application of data-driven methodologies to induce pedagogical policies directly from student-system interaction data (Chi, Jordan, & VanLehn, 2014; M. Chi, VanLehn, Litman, & Jordan, 2012, 2011). In those studies we applied Reinforcement Learning (RL) to induce these policies directly from an exploratory corpus. The exploratory corpus was collected by having the ITS make random decisions when interacting with students. In prior work (M. Chi et al., 2012, 2011), we applied RL to induce pedagogical policies to decide when to provide an example step and when to require students to complete it. We found that, when students were all given the same FWEs, RL-induced policies significantly improved students’ learning gains compared to poor pedagogical policies and random decisions. On the other hand, we also found that students can still learn from these FWEs, even with poor policies. This was likely
due to the content exposure and available practice opportunities. In post-hoc comparisons, different versions of "poor" faded policies were compared, and no significant difference was found between them either in terms of learning outcomes or time on task.

In this study, we will investigate the impact of pedagogical policies on learning across two different granularities of decisions. For the purposes of this study we used a random pedagogical policy with WEs, PSs, and FWEs being selected via a random non-adaptive decision process. By making random decisions, we expect that the number of example steps would be equivalent among the FWE, WE/PS, and ALL conditions. Given that content is strictly controlled to be the same across conditions, we are interested in investigating on the impact of random pedagogical decisions on student learning across FWE, WE/PS, and ALL conditions, and moreover, how they will differ from the WE and PS-only groups.

We will examine students’ performance on pre- and posttest as well as their time on task. In light of prior research, we expect that there will be NO significant learning difference among the five conditions. However, WE group may even be more effective because of the ineffective random decisions being made in other conditions. For time on task, given the number of steps that students need to complete we expect: $WE < WE/PS = FWE = ALL < PS$.

**Methods**

**Participants**

The study was conducted in two sections of the Discrete Mathematics for Computer Science course at North Carolina State University in the Fall of 2014. 266 undergraduate students were assigned to complete the task as part of one of their regular homework assignments during the last two weeks of the class.

**Conditions**

The participants were randomly assigned to the five conditions. We used balanced random assignment stratified by course section and performance on a prior exam. The group sizes were as follows: $N = 31$ for WE, $N = 58$ for WE/PS, $N = 59$ for FWE, $N = 59$ for ALL, and $N = 59$ for PS. (Note that a smaller portion of students were assigned to the WE condition. That's because another purpose of this study is to collect exploratory data for subsequently applying RL to induce adaptive pedagogical policy.)

Due to the holiday break, preparations for final exams, and length of the experiment, 163 students completed the experiment. Four students were excluded from our subsequent analysis because they performed perfectly on the probability pretest. The remaining 159 students were distributed as follows: $N = 21$ for WE, $N = 38$ for WE/PS, $N = 37$ for FWE, $N = 34$ for ALL, and $N = 29$ for PS.

We performed a Chi-Squared independence test to examine the relation between completion rate and condition. We found no significant differences among five groups: $\chi^2(4, N = 266) = 4.12, p = 0.39$.

**Probability Tutor**

The ITS involved in this study is called Pyrenees, a web-based ITS for probability. Pyrenees teaches students 10 major principles of probability, such as the Complement Theorem and Bayes’ Rule. Prior studies have shown that Pyrenees is effective and have compared it to Andes, another well-evaluated ITS (VanLehn et al., 2005). Pyrenees has outperformed Andes in both physics (VanLehn et al., 2004) and probability (Chi & VanLehn, 2007; Chi & VanLehn, 2007). This improvement was observed in part because Pyrenees teaches students the domain-general problem-solving strategy skills, which draws students’ attention to the conditions under which each domain principle is applicable. The differences were apparent on all types of test problems: simple/complex problems and isomorphic/non-isomorphic problems, and the effects were large, with Cohen’s $d=1.17$ for overall posttest scores.

Figure 1 shows the interface of Pyrenees, which is divided into multiple windows. In the dialog window, Pyrenees can provide messages to the student, such as explaining a worked example step, or prompting them to complete the next step. The student can enter responses below such as writing an equation or giving the answer to a multiple-choice question. Any variables or equations that are defined through this process are displayed on left side of the screen for reference. Once students submit their answer, Pyrenees provides immediate feedback on whether or not it was correct.

In addition to providing immediate feedback, Pyrenees can also provide on-demand hints either explaining what is wrong with an incorrect step or prompting the student with what they should do next. Because Pyrenees requires students to follow the Target Variable Strategy, it knows exactly what step the student should be doing next so it gives specific hints. On Pyrenees, help was provided via a sequence of increasingly specific hint. The last hint in the sequence, the bottom-out hint, tells the student exactly what to do. For this study, Pyrenees had three basic modes. In the WE or PS modes, each step is performed either by the tutor or student throughout the whole course of problem solving. In FWE mode, there is a 50% chance at each step for either the student or the tutor to solve the step.

**Procedure**

The study was organized into four phases: 1) pre-training, 2) pretest, 3) training on Pyrenees, and 4) posttest.

During pre-training, all students studied the domain principles through a probability textbook. For each principle, they read a general description, reviewed some examples, and solved some single and multiple-principle problems. After solving each problem, the student’s answer was marked in green if it was correct and red if incorrect. They were also shown an expert solution at the same time. If the students failed to solve a single-principle problem then they were...
The test problems required students to derive an answer by writing and solving one or more equations. We used three scoring rubrics: binary, partial credit, and one-point-per-principle. Under the binary rubric, a solution was worth 1 point if it was completely correct or 0 if not. Under the partial credit rubric, each problem score was defined by the proportion of correct principle applications evident in the solution. A student who correctly applied 4 of 5 possible principles would get a score of 0.8. The One-point-per-principle rubric in turn gave a point for each correct principle application. All of the tests were graded in a double-blind manner by a single experienced grader. The results presented below were based upon the partial-credit rubric but the same results hold for the other two. For comparison purposes all test scores were normalized to fall in the range of [0,1].

Results
Several measures showed that the conditions were balanced in terms of students’ incoming competence. We found no significant difference across the five conditions before the intervention process, phase 3 training on Pyrenees, on various measures. The measures include (1) the probability pretest with respect to students’ test scores on three types of problems, single-principle, multiple-principle, and overall, across all 3 scoring rubrics, or (2) the probability pre-training on all three types of problems. Thus, despite attrition, the conditions remained balanced in terms of incoming competence. Next, we will compare students’ learning performance in the posttest and training time across the five conditions. We discuss each in turn.

Learning Performance
A repeated measures analysis using test type (pretest vs. isomorphic posttest) as a factor and test score as the dependent measure showed that there was a main effect for test \( F(4, 154) = 118.59, p < 0.0001 \). All five groups of students scored significantly higher on the posttest than on the pretest on the 14 isomorphic questions, \( F(1, 20) = 8.75, p = 0.008 \) for WE, \( F(1.37) = 25.66, p < 0.001 \) for WE/PS, \( F(1.36) = 29.34, p < 0.001 \) for FWE, \( F(1.33) = 20.61, p < 0.001 \) for ALL, and \( F(1.28) = 55.04, p < 0.001 \) for PS. Therefore all five conditions made significant gains from pretest to posttest. This suggests that the basic practices and problems, domain exposure, and interactivity of Pyrenees might help students to learn even when the problem- and step-level decisions are made randomly.

Table 1 compares the pretest, isomorphic posttest and overall posttest among the five conditions. The second column in Table 1 lists the number of students who completed the study in each condition, third, fourth and fifth columns in Table 1 list the mean and SD for the pretest, isomorphic posttest (14 isomorphic questions), and overall posttest scores. Overall, no significant differences was found among the five conditions on any of the learning outcome measures: \( F(4, 154) = 0.871, p = 0.483 \) for pretest, \( F(4, 154) = 1.25, p = 0.29 \) for isomorphic posttest questions, and \( F(4, 154) = 0.98, p = 0.42 \) for overall posttest respectively.

Additionally, we compared the adjusted posttest and NLG scores across the five conditions. The adjusted posttest scores were compared via an ANCOVA with the corresponding pretest score used as a covariate. The NLG score measures the students’ learning gains irrespective of their incoming competence: \( NLG = \frac{\text{posttest} - \text{pretest}}{1 - \text{pretest}} \). Here 1 is the maximum
score. Again, no significant difference was found among the conditions.

Table 1: Test Scores

<table>
<thead>
<tr>
<th>Cond</th>
<th># Stud</th>
<th>pretest</th>
<th>Iso Post</th>
<th>Overall Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>WE</td>
<td>21</td>
<td>.687(.160)</td>
<td>.789(.187)</td>
<td>.650(.197)</td>
</tr>
<tr>
<td>WE/PS</td>
<td>38</td>
<td>.658(.165)</td>
<td>.774(.130)</td>
<td>.630(.167)</td>
</tr>
<tr>
<td>FWE</td>
<td>37</td>
<td>.625(.134)</td>
<td>.736(.159)</td>
<td>.588(.145)</td>
</tr>
<tr>
<td>ALL</td>
<td>34</td>
<td>.664(.181)</td>
<td>.803(.136)</td>
<td>.651(.159)</td>
</tr>
<tr>
<td>PS</td>
<td>29</td>
<td>.618(.155)</td>
<td>.802(.118)</td>
<td>.645(.139)</td>
</tr>
</tbody>
</table>

Training Time

Table 2 shows the average amount of total training time (in minutes) students spent on Pyrenees for each condition. A one-way ANOVA showed significant differences among the five groups on time on task: \( F(4, 154) = 26.91, p = 0.000 \).

Subsequent pairwise t-tests showed that the WE group spent significantly less time than the other conditions: 
- \( t(57) = -5.22, p < .001, d = 1.33 \) for WE/PS condition,
- \( t(56) = -6.22, p = .000, d = 1.95 \) for FWE condition,
- \( t(53) = -6.26, p = .000, d = 1.70 \) for ALL condition, and
- \( t(48) = -8.93, p = .000, d = 2.55 \) for PS condition respectively.

Similarly, we found that the WE/PS condition spent significantly less time than FWE, ALL and PS conditions: 
- \( t(73) = -2.77, p = .007, d = 0.64 \) for WE/PS condition,
- \( t(70) = -2.49, p = .015, d = 0.58 \) for FWE condition,
- \( t(65) = -6.96, p = .000, d = 1.67 \) for ALL condition.

Finally, while we found no significant time on task differences between the FWE and ALL conditions, \( t(69) = 0.395, p = .69, d = 0.09 \), they both spent significantly less time than the PS condition: 
- \( t(64) = -3.60, p = .001, d = 0.89 \) for WE/PS condition,
- \( t(61) = -4.14, p < .001, d = 1.04 \) for FWE condition.

Overall, with respect to time on task. we found that: 
- WE < WE/PS < FWE = ALL < PS. In fact, WE group only spent about 43% of the training time as FWE and 32% of the training time as PS but reached the same learning gain as other conditions.

Table 2: Time on task

<table>
<thead>
<tr>
<th>Cond</th>
<th># Student</th>
<th>Time (in minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WE</td>
<td>21</td>
<td>47.96 (39.27)</td>
</tr>
<tr>
<td>WE/PS</td>
<td>38</td>
<td>92.23 (25.79)</td>
</tr>
<tr>
<td>FWE</td>
<td>37</td>
<td>112.80 (37.50)</td>
</tr>
<tr>
<td>ALL</td>
<td>34</td>
<td>109.48 (32.85)</td>
</tr>
<tr>
<td>PS</td>
<td>29</td>
<td>146.40 (37.88)</td>
</tr>
</tbody>
</table>

Finally, a one-way ANCOVA was conducted to determine a statistically significant difference among the five groups on their overall posttest scores controlling for both pretest and total training time as the covariates; no significant difference was found: \( F(4, 152) = 1.18, p = 0.32 \).

Discussion and Conclusion

In this study, we used an ITS called Pyrenees to compare five tutorial conditions: WE, PS, FWE, WE/PS, and ALL. For the WE/PS, FWE, ALL conditions, the tutor used a random policy to decide when to give students a worked example problem (or example step) or to ask them to solve the problem (or step). Our results showed that all five conditions learned significantly after training on Pyrenees, and no significant difference was found on all of our learning measures including the pretest, isomorphic posttest, and overall posttest scores.

This was despite the fact that the pedagogical strategies employed for the WE/PS, FWE, and ALL conditions were random and thus were rather ineffective. They were not adaptive to the students and thus were not able to make a positive impact on students’ performance beyond the baseline provided by content exposure in Pyrenees. Here the basic practices and problems, domain exposure, and interactivity of Pyrenees set a minimum bar of students’ learning that the pedagogical strategies, however poor, could not undercut. The lack of a significant difference among the five conditions supports our hypothesis and is consistent with the results from prior studies (M. Chi et al., 2012, 2011).

Previously, we found that students’ learning performance can be improved by employing effective pedagogical strategies (M. Chi et al., 2012, 2011). However, in that study no significant difference was found in terms of time on task between the students trained on the system with effective pedagogical policies and those with ineffective pedagogical policies. In this study, we showed that different granularities of pedagogical decisions can make a significant difference to students’ time on task.

Much of the prior research has shown that WE can be as effective as tutored PS but the former often take significantly less time than the latter. One potential explanation for this time difference is that the students in PS condition have to do more work. Given that the same amount of work was expected for students in the WE/PS, FWE, and ALL conditions, we hypothesized that: \( WE/PS = FWE = ALL \). However, our results suggest that for time on task \( WE/PS < FWE = ALL \). WE/PS spent significantly less time than both FWE and ALL.

There are many possible explanations why the FWE group took longer time than WE/PS group. Since both WE/PS and FWE groups get the same random decisions, we hypothesize that the granularity of the decision must therefore play an important role. Solving a problem in domains such as probability consists of applying domain principles in a valid logical order. Students’ later steps are directly dependent upon what they have done previously. This partial dependence may force students in the FWE condition to pay more attention to not only tutor-solved steps but also what their own steps.

Additionally, Pyrenees’ instructional methods may also explain some of the extra time taken by the FWE condition. If the tutor solves a problem in a way that is unexpected to the student, the student will require extra time to process the tu-
tor’s intentions and continue its progress. These tutor-solved steps may act as constraints on the student’s problem solving process. There are many possible strategies for solving a problem, and Pyrenees uses one specific strategy which may not be intuitive for the student. Thus these solved steps may lead students onto a different solution path which is outside of their expectations. We are currently in the process of analyzing our log files to determine why this occurred. Why did the same random pedagogical policy improve efficiency when applied at the problem level more than at the step level?

Our results from this study suggested that step-level decisions are more sensitive to ineffective pedagogical strategies than problem level decisions. With random decisions, the FWE group not only failed to learn more than students in any of the other conditions, they also spent significantly more time.

Overall, this study suggests that different granularities of pedagogical decisions can have a significant impact on students’ time on task. The fine-grained interaction steps can have a strong pedagogical impact. Our ultimate goal is to apply RL to induce effective pedagogical policies, at both the problem and step level, directly from our collected data. This raises an interesting question: with effective pedagogical strategies, will there be time on task difference and learning difference among the five conditions. This is an excellent question for future research.

ACKNOWLEDGMENTS
This work is supported by the National Science Foundation award 1432156. We would also like to thank the anonymous reviewers for their valuable feedback.

References


