Minimizing Transceivers in Optical Path Networks

Prashant Iyer, Rudra Dutta*, and Carla D. Savage†
Department of Computer Science
N. C. State University, Box 8206
Raleigh, NC 27695, USA
{piyer, dutta, savage}@csc.ncsu.edu

February 9, 2006

Abstract

We consider the problem of routing traffic on lightpaths in unidirectional, WDM path networks with the goal of minimizing the number of transceivers. We show that the problem is NP-hard, even in the special case that all traffic requests are destined for a single *egress* node. In the case of egress traffic, we give a simple heuristic that will never be worse than twice the optimal.

Keywords: Traffic Grooming, Approximation algorithms, NP-complete problems, Optical networks.

1 Introduction - The Physical Problem

Wavelength Division Multiplexing (WDM) allows an optical fiber to carry traffic on multiple channels by assigning to each channel a unique wavelength in which the corresponding traffic is transmitted. Each node in a WDM network is capable of optically routing traffic on any of the wavelengths supported by the network. This allows the creation of a *virtual topology* where the nodes are connected by *lightpaths* that may span multiple physical links. Each such lightpath between two nodes is assigned a fixed wavelength that will optically pass through the intermediate nodes. Lightpaths sharing a physical link will carry their traffic on distinct wavelengths. Traffic requests are thus satisfied by routing them on a sequence of lightpaths in the virtual topology.

Figure 1(a) shows a 5-node network connected by optical links. Each link can carry traffic over 3 wavelengths (Figure 1(b)). Figure 1(c) shows a possible lightpath setup that results in the virtual topology shown in Figure 1(d).

*Research supported by NSF grant ANI-0322107
†Research supported by NSF grants ANI-0322107 and INT-0230800

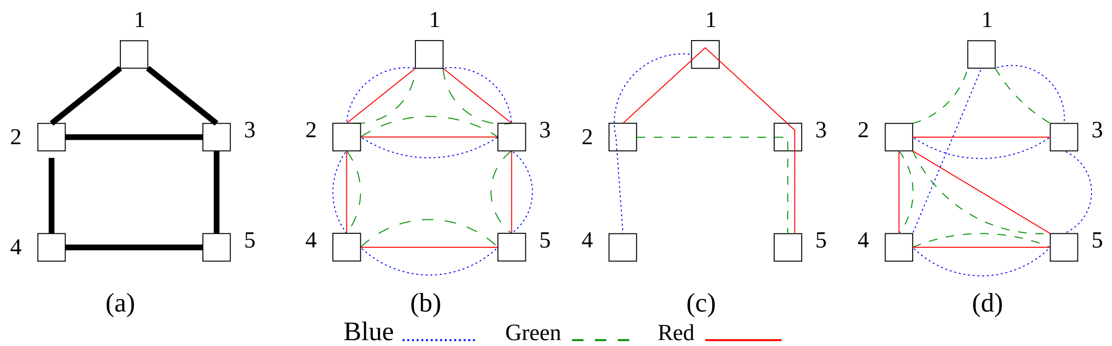

Figure 1: The virtual topology consisting of lightpaths. Traffic on blue passes through node 2. Traffic on green passes through node 3. Traffic on red passes through nodes 1 and 3.

Typically, the bandwidth available in each wavelength far exceeds the requirement per traffic request. For efficient utilization, multiple traffic streams must be multiplexed into each lightpath. However, the computation required at intermediate nodes to recognize these traffic streams and route them on different paths cannot (as yet) be performed directly on the optical signal.

This gives rise to the necessity of switches at lightpath end points that process traffic electronically and can convert it to and from the optical domain. Such electronic switching equipment is likely to be much costlier per Mbps than optical switching. The efficient design of lightpaths and traffic routing to minimize overall network costs is called *traffic grooming*. This problem has attracted a lot of attention recently [6, 2, 14, 10, 13, 8]. Current research has been surveyed in [3, 15, 11]. Traffic grooming is known to be computationally hard for a variety of cost models, such as the total amount of electronic switching of traffic [2, 7], number of lightpath endpoints in the network [12, 9], number of SONET Add-Drop Multiplexers (ADMs) [1], all of which represent the cost of the required electronic switching capability for the network.

In this paper, we examine the problem of traffic grooming with the goal of minimizing the total number of lightpath endpoints (equivalently, of lightpaths). That is, we attempt to minimize the total number of *transceivers*, the optical devices that originate or terminate a lightpath. This cost metric reflects the actual network cost more directly, and, in recent literature, has been recognized as of more immediate interest to the network designer. A heuristic algorithm to solve the traffic grooming problem in general networks under this cost model is presented in [9], but the question of whether the problem was NP-hard was left open.

We show that in a directed path network, routing traffic on lightpaths with the goal of minimizing the number of lightpaths is NP-hard. This holds even in the special case that all traffic requests are destined for a single *egress* node. For the case of egress traffic, we give a simple heuristic that will never be worse than twice the optimal.

In the next section, we give a precise definition of the problem of traffic grooming on a

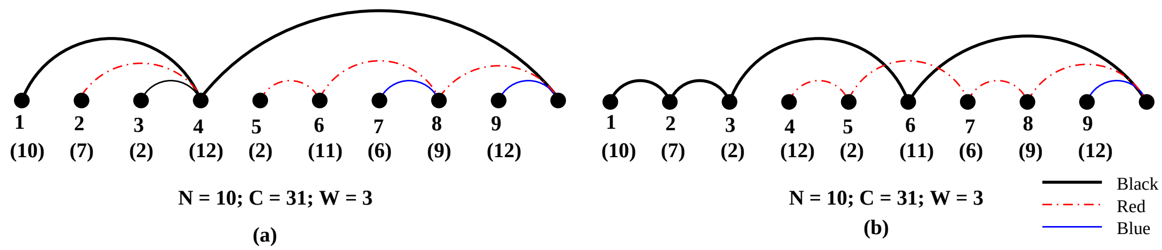

Figure 2: Two feasible solutions to $PG(10, 3, 32, [10, 7, 2, 12, 1, 11, 6, 9, 12])$. Both solutions use the minumum number of lightpaths (9), but (a) uses 15 ADMs and (b) uses 12.

path network with egress node. We contrast our goal of minimizing the number of lightpaths with the apparently similar goal of minimizing the number of ADMs. Section 3 contains the NP-completeness proof and the approximation algorithm is presented in Section 4. Some open questions are highlighted in Section 5.

2 The Mathematical Model and Cost Measure

We consider a WDM path network consisting of N nodes numbered 1 to N and physical optical links connecting node i to node $i + 1$ for $1 \leq i < N$. Each physical link is capable of carrying C units of traffic in each of W distinct wavelengths. For each node $i < N$, the demand $r_i > 0$ is the number of traffic requests that must be sent from node i to node N. Clearly, in order for this to be possible, we must have $r_1 + r_2 + \cdots + r_{N-1} \leq CW$.

Let $PG(N, W, C, \mathbf{r})$, $\mathbf{r} = [r_1, r_2, \ldots, r_{N-1}]$ be the problem of establishing a virtual topology of lightpaths and routing the traffic over lightpaths to satisfy the requests, while obeying the capacity and wavelength constraints.

We represent a virtual topology by the matrix $A = [a_{ij}]$ where a_{ij} is the number of lightpaths from i to j. The routing of traffic on the lightpaths can be specified as a matrix $B = [b_{ij}]$ where b_{ij} is the number of traffic requests which are routed on a lightpath from i to j in A. In specifying b_{ij} we do not distinguish between traffic originating at i and traffic routed through i. If A has multiple lightpaths between the same two nodes, we do not distinguish between such lightpaths when specifying the routing.

A feasible solution $\mathcal{S} = (A, B)$ to the problem $PG(N, W, C, \mathbf{r})$ is one that routes all the traffic using at most W wavelengths per link, with each lightpath carrying at most C units of traffic.

A complete solution to the problem would require an assignment of wavelengths to lightpaths. However, for path networks, A represents an interval graph, and it is well known that if the wavelength constraint is satisfied, we can, in linear time, assign wavelengths to the lightpaths of A in such a way that lightpaths traversing the same physical link are assigned different wavelengths [4].

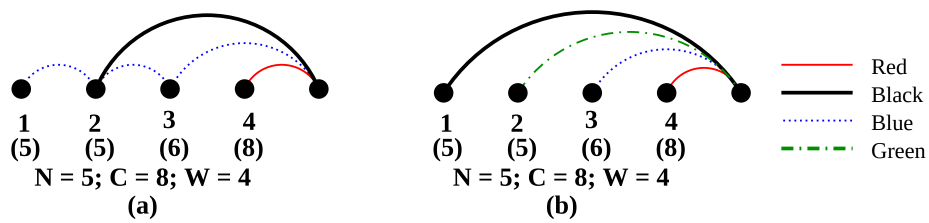

Figure 3: Two feasible solutions to $PG(5, 4, 8, [5, 5, 6, 8])$. Both solutions use the minumum number of ADMS (8), but (a) uses 5 lightpaths and (b) uses 4.

Our goal in this paper is, given the path grooming problem $PG(N, W, C, \mathbf{r})$, to find a feasible solution which minimizes the number of lightpaths (and thereby the number of transceivers). As an example, two feasible solutions to the problem $PG(10, 3, 32, \mathbf{r})$, with $\mathbf{r} = [10, 7, 2, 12, 2, 11, 6, 9, 12]$, are shown in Figure 2, with a possible assignment of wavelengths indicated. Figure 3 shows two feasible solutions to another instance $PG(5, 4, 8, [5, 5, 6, 8])$.

Like transceivers, wavelength specific ADMs are also used to terminate lightpahts, but ADMs have the property that they may be shared by two lightpaths of the same wavelength at a common endpoint. The problem of minimizing the ADM count in a network has been shown to be NP-complete [1] even when restricted to path networks with an egress node. However, the examples in Figures 2 and 3 show that minimizing the number of ADMs does not necessarily minimize the number of lightpaths and conversely. Next, we settle the question of computational complexity for the transceiver cost model.

3 The NP-Completeness Proof

Even though the problem of minimizing the number of lightpaths is different from that of minimizing the number of ADMs, we are able to use an approach very similar to that of [1] to show it is NP-hard, namely reduction from bin packing. We first note a lower bound.

Lemma 1 *In any feasible solution to $PG(N, W, C, \mathbf{r})$, the number of lightpaths must be at least $\sum_{i=1}^{N-1} \lceil r_i/C \rceil$. If $r_i > 0$ for $1 \le i \le N$, then $N - 1$ is a lower bound on the number of lightpaths.*

Proof. Each node i must have at least $\lceil r_i/C \rceil$ lightpaths originating from it in order to route all the requests from that node. Summing these gives the first result. If all r_i are positive, $\lceil r_i/C \rceil \ge 1$ so the sum is at least $N - 1$. ■

Theorem 1 *The problem of determining whether the path grooming problem $PG(N, W, C, \mathbf{r})$ has a feasible solution using $N - 1$ lightpaths is NP-complete.*

Proof. The problem is clearly in NP. We show that it is NP-hard by reduction from bin packing [5]. In the *bin packing problem*, we are given a set U of m items, an integer size

$s(u) > 0$ for each $u \in U$, and positive integers B, k. The decision to be made is whether U can be partitioned into k sets $U_1, \ldots, U_k$ in such a way that $\sum_{u \in U_j} s(u) \leq B$ for $1 \leq j \leq k$.

We transform an instance $BP(U, m, s, B, k)$, of the bin packing problem with $U = \{u_1, \ldots, u_m\}$ into the path grooming problem $PG(m+1, k, B, [s(u_1), \ldots, s(u_m)])$. We show that the answer to $BP(U, m, s, B, k)$ is "yes" if and only if $PG(m+1, k, B, [s(u_1), \ldots, s(u_m)])$ has a feasible solution with m lightpaths.

Suppose first that $U_1, \ldots, U_k$ is a solution to $BP(U, m, s, B, k)$. For $1 \leq i \leq m$, let $b(i)$ be the "bin" containing u_i, i.e., $b(i) = t$ iff $u_i \in U_t$. For $1 \leq i \leq m$, define $f(i)$ by

$$f(i) = \min\left(\{m+1\} \cup \{j \mid j > i,\ b(i) = b(j)\}\right).$$

Then a feasible solution to $PG(m+1, k, B, [s(u_1), \ldots, s(u_m)])$ is to create the m lightpaths

$$\{(i, f(i)) \mid 1 \leq i \leq m\},$$

assign lightpath $(i, f(i))$ to wavelength $\lambda_{b(i)}$, and route the $s(u_i)$ traffic requests originating at node i over the chain of lightpaths connecting the nodes

$$i, f(i), f(f(i)), \ldots, m+1.$$

This requires only k wavelengths and since $\sum_{\{i|b(i)=j\}} s(u_i) = \sum_{u \in U_j} s(u) \leq B$ for $1 \leq j \leq k$, the capacity constraint is satisfied on every wavelength λ_j.

Conversely, suppose $PG(m+1, k, B, [s(u_1), \ldots, s(u_m)])$ has a solution with m lightpaths. Since the request at each node i is $s(u_i) > 0$, there must be at least one lightpath originating at each node, hence exactly 1, since there are a total of m. So no request at any node is split onto more than one lightpath. On the other hand, at most k lightpaths can terminate at the egress node so all requests are ultimately routed to the egress node through these k lightpaths. This defines a partition of the nodes $\{1, \ldots, m\}$ into equivalence classes $V_1, \ldots, V_k$ such that two nodes i and j are equivalent if and only if their requests $s(u_i)$ and $s(u_j)$ use the same final lightpath to arrive at the egress node. Each of the lightpaths can carry at most B units of traffic. This means the sum of the number of requests from all nodes in an equivalence class cannot exceed B, so $V_1, \ldots, V_k$ is a solution to $BP(U, m, s, B, k)$. ■

4 Algorithm

In this section we present a greedy approximation algorithm `Setup-Lightpaths` to obtain a feasible solution to the path grooming problem $PG(N, W, C, \mathbf{r})$, $\mathbf{r} = [r_1, \ldots, r_{N-1}]$. We show that the number of lightpaths required by this solution exceeds the minimum number, L, by at most $\min\{W, N-2, L\}$, giving a 2-approximation.

Given an instance of the problem, `Setup-Lightpaths` initializes the available capacity c_i on wavelength λ_i to C for $1 \leq i \leq W$, initializes the number of unrouted requests r'_i from node i to be r_i for $1 \leq i < N$, and calls `Assign` with $i = 1, j = 1$.

Algorithm 1 `Setup-Lightpaths`$(N, W, C, \mathbf{r})$

1: **for** $i = 1$ to W **do** {Initialize $\boldsymbol{c}$}

2:     $c_i \leftarrow C$

3: **for** $i = 1$ to $N - 1$ **do** {Initialize $\mathbf{r}'$}

4:     $r'_i \leftarrow r_i$

5: `Assign`$(1, 1, \mathbf{r}', \boldsymbol{c}, N, W)$

6: **procedure** `Assign`$(i, j, \mathbf{r}', \boldsymbol{c}, N, W)$

7: **if** $i < N$ and $j \leq W$ **then**

8:     **if** $r'_i < c_j$ **then**

9:         **if** $r'_i > 0$ **then**

10:             Create lightpath$(i, i + 1)$

11:             $c_j \leftarrow c_j - r'_i; \quad r'_i \leftarrow 0$

12:         `Assign`$(i + 1, j, \mathbf{r}', \boldsymbol{c}, N, W)$

13:     **else**

14:         Create lightpath(i, N)

15:         $r'_i \leftarrow r'_i - c_j; \quad c_j \leftarrow 0$

16:         `Assign`$(i, j + 1, \mathbf{r}', \boldsymbol{c}, N, W)$

When procedure `Assign` is called with parameters $i < N, j \leq W, \mathbf{r}', \boldsymbol{c}, N$ and W it uses the capacities $c_j, \ldots, c_W$ available in wavelengths $\lambda_j, \ldots, \lambda_W$, respectively, to set up lightpaths to route the r'_k requests from each node k for $i \leq k < N$. Each time `Assign` is called, one of the following actions will occur.

1. If $r'_i < c_j$ (Lines 8 - 12), a lightpath of wavelength λ_j is created from node i to node $i + 1$ and it will carry all the r'_i requests (along with prior requests routed on wavelength λ_j) to the node $i + 1$. All the r'_i requests are deemed routed and the available capacity c_j of λ_j is decreased by r'_i. `Assign` is then called recursively to satisfy requests $r_{i+1}, \ldots, r_{N-1}$ on wavelengths $\lambda_j, \ldots \lambda_W$, with remaining available capacities.

2. If $r'_i \geq c_j$ (Lines 13- 15), a lightpath of wavelength λ_j is created from node i to the egress node N and this lightpath will carry c_j requests from node i (along with prior requests routed on wavelength λ_j) to the egress node. Node i will have $r'_i - c_j$ requests unrouted and the capacity on wavelength j will be 0. `Assign` is then called recursively to satisfy requests $r_i, \ldots, r_{N-1}$ on wavelengths $\lambda_{j+1}, \ldots \lambda_W$ with remaining available capacities.

The procedure terminates when all requests are routed ($i = N$) or none of the wavelengths have available capacity ($j = W + 1$). In the former case, we have a valid virtual topology which satisfies all the requests and in the latter case we can conclude that the available capacity was not sufficient to route all requests.

Theorem 2 *If* $\sum r_i \leq CW$*,* `Setup-Lighpaths` *produces a feasible solution to* $PG(N, W, C, \mathbf{r})$ *with at most* $N + W' - 2$ *lightpaths, where* $W' = \lceil (\sum r_i)/C \rceil$*.*

Proof. Note that `Assign`$(i, j, \mathbf{r}', c, N, W)$ preserves the invariant

$$S_i \triangleq \sum_{k=i}^{N-1} r_k \leq \sum_{k=j}^{W} c_k \triangleq L_j.$$

Let N^* and W^* be the values of i and j supplied to the final call of `Assign`. Each call to `Assign` (except the last, which does nothing) creates at most one lightpath and calls itself with i or j increased by 1. Thus it follows by induction that if $S_i \leq L_j$, then `Assign`$(i, j, \mathbf{r}', c, N, W)$ will route the remaining S_i requests using at most $N^* - i + W^* - j$ additional lightpaths.

The procedure `Setup-Lightpaths` solves the path grooming problem by calling `Assign` with $i = 1$ and $j = 1$, thus the solution produced uses at most $N^* - 1 + W^* - 1 = N^* + W^* - 2$ lightpaths. If $\sum r_i \leq CW$, traffic from all nodes will be routed and $N^* = N$. In addition, note that procedure `Assign` completely uses the capacity of wavelength λ_j before routing requests on wavelength λ_{j+1}, so the procedure will terminate with $W^* = W'$. ■

Corollary 1 *Procedure* `Setup-Lightpaths` *is a 2-approximation algorithm for minimizing the number of lightpaths for the path grooming problem* $PG(N, W, C, \mathbf{r})$*. In fact, the number of lightpaths created by* `Setup-Lightpaths` *exceeds the minumum by no more than* $\min\{W' - 1, N - 2\}$*, where* $W' = \lceil (\sum r_i)/C \rceil$*.*

Proof. Let L be the minimum number of lightpaths used in any feasible solution to $PG(N, W, C, \mathbf{r})$ and let L^* be the number created by `Setup-Lightpaths`. By Theorem 2, $L^* \leq N + W' - 2$ and by Lemma 1, $W' \leq \sum \lceil r_i/C \rceil \leq L$, so

$$\begin{aligned} L^* - L &\leq N + W' - 2 - L \\ &\leq N + L - 2 - L \\ &= N - 2. \end{aligned} \tag{1}$$

Since we assume that all requests r_i are positive, $L \geq N - 1$. Hence,

$$\begin{aligned} L^* - L &\leq N + W' - 2 - (N - 1) \\ &= W' - 1. \end{aligned} \tag{2}$$

From (1) and (2) we get $L^* - L \leq \min\{W' - 1, N - 2\}$. In addition, from (1), using $L \geq (N - 1)$, it is clear that $L^* < 2L$. ■

Note that the algorithm `Setup-Lightpaths` will achieve the lower bound of $\sum \lceil r_i/C \rceil$ lightpaths whenever the nodes can be partitioned into sets of consecutive nodes where the sum of the requests from nodes within a set is a multiple of the capacity C. This is illustrated

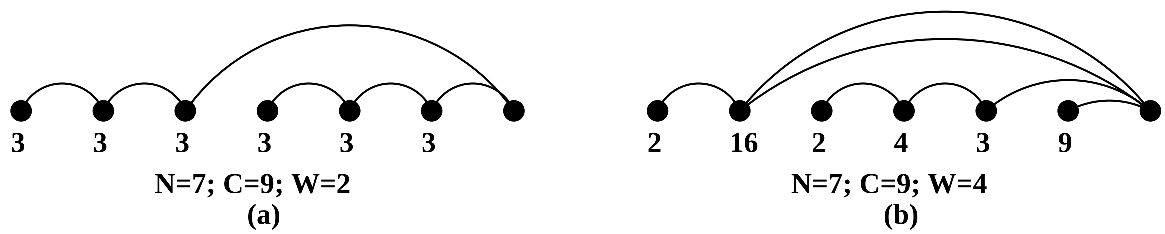

Figure 4: Best case inputs for `Setup-Lightpaths`

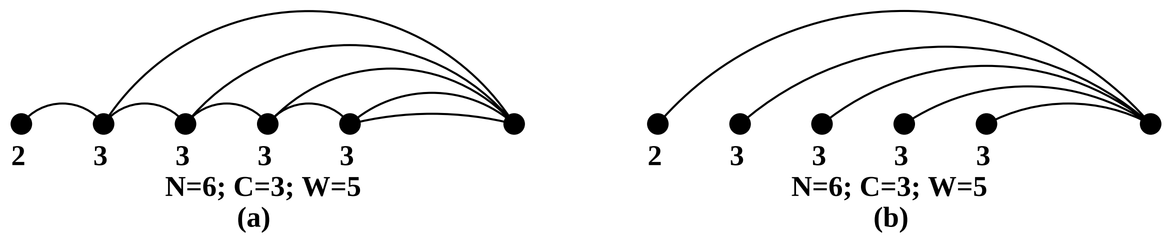

Figure 5: Worst case input for `Setup-Lightpaths`

in Figure 4(a) for uniform traffic $PG(7, 2, 9, [3, 3, 3, 3, 3, 3])$ and Figure 4(b) for non-uniform traffic $PG(7, 4, 9, [2, 16, 2, 4, 3, 9])$.

Figure 5 illustrates a problem instance for which the algorithm performs as badly as possible, achieving the upper bound of Theorem 2. Figure 5(a) shows the solution given by algorithm `Setup-Lightpaths`. It uses $L^* = N + W' - 2 = 9$ lightpaths whereas Figure 5(b) shows a solution achieving the lower bound of $N - 1 = 5$. This example can be generalized as follows. Given any $N > 1$ and $C > 1$, $PG(N, N-1, C, [C-1, C, C, \ldots, C])$, only $N - 1$ lightpaths are required, but algorithm `Setup-Lightpaths` will create $2N - 3 = N + (N-1) - 2 = N + W' - 2$.

In the case of uniform traffic, when $W = W'$, it can be shown that `Setup-Lightpaths` never uses more than one additional lightpath. However, when $W > W'$, the upper bound of Theorem 2 can be achieved even for uniform traffic. For example, if $N = C + 1$ and $W = C$ and $\mathbf{r} = [C-1, \ldots, C-1]$, we have $W' = \lceil \sum r_i / C \rceil = C - 1 < W$. The algorithm produces a solution with $2C - 2 = (C+1) + (C-1) - 2 = N + W' - 2$ lightpaths, but an optimal solution would create a lightpath from each node to the egress node for a total of C lightpaths.

5 Further Directions

Even in the very simple case when the network is a path with all traffic requests destined for a single egress node, there are several open questions regarding the complexity of traffic grooming. With uniform traffic in the egress model, is there a polynomial time strategy to minimize the number of lightpaths? Under what conditions can we simultaneously minimize both the ADM and the transceiver costs? Finally, can minimizing the electronic switching cost, as defined in [2], be done in polynomial time on the egress model [8, 7]?

References

[1] A.L. Chiu and E.H. Modiano. Traffic grooming algorithms for reducing electronic multiplexing costs in WDM ring networks. *Journal of Lightwave Technology*, 18(1):2–12, Jan 2000.

[2] R. Dutta and G.N. Rouskas. On optimal traffic grooming in WDM rings. *IEEE Journal on Special Areas in Communication*, 20(1):110–121, January 2002.

[3] R. Dutta and G.N. Rouskas. Traffic grooming in WDM networks: Past and future. *IEEE Network*, 16(6):46–56, Nov/Dec 2002.

[4] P.C. Fishburn. Interval graphs and interval orders. *Discrete Math.*, 55(2):135–149, 1985.

[5] M.R. Garey and D.S. Johnson. *Computers and Intractability A Guide to the Theory of NP-Completeness*. W.H.Freeman and Company, 1979.

[6] O. Gerstel, G. Sasaki, and R. Ramaswami. Cost effective traffic grooming in WDM rings. *IEEE/ACM Transactions on Networking*, 8(5):618–630, Oct 2000.

[7] S. Huang, R. Dutta, and G.N. Rouskas. Traffic grooming in path, star, and tree networks: Complexity, bounds, and algorithms. *IEEE Journal of Selected Areas in Communications*.

[8] P. Iyer, R. Dutta, and C.D. Savage. On the complexity of path traffic grooming. In *Proceedings of the Second International IEEE/Create-Net Workshop on Traffic Grooming*, pages 295–301, October 2005.

[9] V.R. Konda and T.Y. Chow. Algorithm for traffic grooming in optical network to minimize the number of transceivers. In *2001 IEEE Workshop on High Performance Switching and Routing*, pages 218–221. IEEE, 2001.

[10] X. Li, L. Liu, P Wan, and O. Frieder. Practical traffic grooming scheme for single hub SONET/WDM rings. In *Proceedings, 25th Annual IEEE Conference on Local Computer Networks*, pages 556–564. IEEE, 2000.

[11] E.H. Modiano and P.J. Lin. Traffic grooming in WDM networks. *IEEE Communications Magazine*, 39(7):124–129, Jul 2001.

[12] P. Wan, G. Calinescu, L. Liu, and O. Frieder. Grooming of arbitrary traffic in SONET WDM BLSRs. *IEEE Journal on Selected Areas in Communication*, 18(10):1995–2003, October 2000.

[13] X. Zhang and C. Qiao. An effective and comprehensive approach for traffic grooming and wavelength assignment in SONET WDM rings. *IEEE/ACM Transactions on Networking*, 8(5):608–617, October 2000.

[14] K. Zhu and B. Mukherjee. Traffic grooming in optical WDM mesh networks. *IEEE Journal on Selected Areas in Communication*, 20(1), January 2002.

[15] K. Zhu and B. Mukherjee. A review of traffic grooming in WDM optical networks: Architectures and challenges. *Optical Networks Magazine*, 4(2):55–64, March/April 2003.