HW solution for Lecture 12

8.21
(a) The phases present are the liquid and solid ($L + \alpha$).
   (ii) The chemical composition of liquid is $w_L = 62$ wt % Ni while that of the solid is $w_\alpha = 74$ wt % Ni.
   (iii) The weight percent of solid and liquid are:

\[
\text{Wt \% of liquid phase} = \frac{74 - 70}{74 - 62} \times 100\% = 33.3\%
\]
\[
\text{Wt \% of solid phase} = \frac{70 - 62}{74 - 62} \times 100\% = 66.67\%
\]

(b) At 1500°C, the alloy is 100% liquid.
(c) The microstructure of the alloy at these temperatures would look similar to the following sketches.

![Microstructure sketches](image)

1350°C                    1500°C

8.22
(a) At 1000°C:
   Phases present:
   Compositions of phases:
   liquid 100%

(b) At 800°C,
   Phases present:
   Compositions of phases:
   liquid 78% Ag in liquid phase
   beta 93% Ag in $\beta$
   Amounts of phases:

\[
\text{Wt \% liquid phase} = \frac{93 - 88}{93 - 78} \times 100\% = 33.3\%
\]
\[
\text{Wt \% beta phase} = \frac{88 - 78}{93 - 78} \times 100\% = 66.6\%
\]
(c) At $780^\circ\text{C} + \Delta T$,

| Phases present: | liquid | Wt % liquid phase = $\frac{91.2 - 88}{91.2 - 71.9} \times 100\% = 16.6\%$
|---|---|---|
| Compositions of phases: | 71.9% Ag in liquid phase | 91.2% Ag in $\beta$
| Amounts of phases: | beta | 83.4% |

(d) At $780^\circ\text{C} - \Delta T$,

| Phases present: | alpha | Wt % alpha phase = $\frac{91.2 - 88}{91.2 - 7.9} \times 100\% = 3.84\%$
<table>
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<tbody>
<tr>
<td>Compositions of phases:</td>
<td>91.2% Ag in $\beta$ phase</td>
<td>beta</td>
</tr>
<tr>
<td>Amounts of phases:</td>
<td>7.9% Ag in $\alpha$ phase</td>
<td>$\alpha$</td>
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8.23

(a) At $850^\circ\text{C}$,

Wt % liquid = $\frac{40 - 7.9}{52 - 7.9} \times 100\% = 72.8\%$

Wt % proeutectic $\alpha$ = $\frac{52 - 40}{52 - 7.9} \times 100\% = 27.2\%$

Weight of liquid phase = 500 g $\times$ 0.728 = 364 g

Weight of proeutectic $\alpha$ = 500 g $\times$ 0.272 = 136 g

(b) In the eutectic structure at $780^\circ\text{C} + \Delta T$,

Wt % liquid = $\frac{40 - 7.9}{71.9 - 7.9} \times 100\% = 50.2\%$

Wt % proeutectic $\alpha$ = $\frac{71.9 - 40}{71.9 - 7.9} \times 100\% = 49.8\%$

Weight of liquid phase = 500 g $\times$ 0.502 = 251 g

Weight of proeutectic $\alpha$ = 500 g $\times$ 0.498 = 249 g
(c) In the eutectic structure at $780^\circ$C $- \Delta T$, the number of grams of $\alpha$ present is,

\[ \text{Wt } \% \text{ total } \alpha = \frac{91.2 - 40}{91.2 - 7.9} \times 100\% = 61.5\% \]

Weight of total $\alpha = 500 \text{ g } \times 0.615 = 307.5 \text{ g}$

(d) In the eutectic structure at $780^\circ$C $- \Delta T$, the number of grams of $\beta$ present is,

\[ \text{Wt } \% \text{ total } \beta = \frac{40 - 7.9}{91.2 - 7.9} \times 100\% = 38.5\% \]

Weight of $\beta = 500 \text{ g } \times 0.385 = 192.5 \text{ g}$

8.24

Since the alloy contains 60 wt $\%$ proeutectic $\beta$, the wt $\%$ Sn must lie between 61.9 wt $\%$ and 97.5 wt $\%$:

\[ \% \text{ proeutectic } \beta = \frac{x - 61.9}{97.5 - 61.9} = 0.60 \]

\[ x = 0.6(35.6) + 61.9 = 83.3\% \]

Thus, the alloy consists of 83.3 $\%$ Sn and 16.7 $\%$ Pb.

8.25

At $50^\circ$C, the phase compositions are 100$\%$ Sn for $\beta$ and approximately 2$\%$ Sn for $\alpha$. Thus,

\[ \% \alpha = \frac{100.0 - x}{100.0 - 2.0} = 0.60, \quad x = 100 - 0.6(98.0) = 41.2\% \]

The alloy consists of 41.2 $\%$ Sn and 58.8 $\%$ Pb.

8.26

(a) This alloy is hypereutectoid; the composition lies to the right of the eutectic point.

(b) The first solid to form is solid solution $\beta$ containing approximately 98 $\%$ Sn.

(c) At $183^\circ$C $+ \Delta T$, the compositions of the phases present are 61.9$\%$ Sn in liquid phase and 19.2$\%$ Sn in beta phase. The amounts of the respective phases present are:

\[ \text{Wt } \% \text{ liquid} = \frac{97.5 - 70}{97.5 - 61.9} \times 100\% = 77.2\% \]

\[ \text{Wt } \% \beta = \frac{70 - 61.9}{97.5 - 61.9} \times 100\% = 22.8\% \]
(d) At $183^\circ$C $- \Delta T$, the compositions of the phases present are 19.2 % Sn in $\alpha$ phase and 97.5 % Sn in $\beta$ phase. The amounts of the respective phases present are:

$$\text{Wt \% total } \alpha = \frac{97.5 - 70}{97.5 - 19.2} \times 100\% = 35.1\%$$

$$\text{Wt \% total beta} = \frac{70 - 19.2}{97.5 - 19.2} \times 100\% = 64.8\%$$

(e) As the alloy is cooled below the eutectic temperature, the tin content in the alpha phase and the lead content in the beta phase are further reduced. However, at room temperature (20°C), equilibrium is not achieved because the diffusion rate is so slow. Referring to Fig. 8.13, if the solvs line is extrapolated to 20°C, the approximate composition of alpha and beta are 2.0% and 100.0 %, respectively. Thus,

$$\text{Wt \% total } \alpha = \frac{100 - 70}{100 - 2} \times 100\% = 30.6\%$$

$$\text{Wt \% total beta} = \frac{70 - 2}{100 - 2} \times 100\% = 69.4\%$$

8.27

(a) At 2600°C,

(i) Phases Present: Liquid and alpha phases
(ii) Compositions of Phases: 16% Os in liquid; 38% Os in alpha phase
(iii) Amounts of phases:

$$\text{Wt \% alpha} = \frac{30 - 16}{38 - 16} \times 100\% = 63.6\%$$

$$\text{Wt \% liquid} = \frac{38 - 30}{38 - 16} \times 100\% = 36.4\%$$

(b) At 2665°C $+ \Delta T$,

(i) Phases Present: Liquid and beta phases
(ii) Compositions of Phases: 23% Os in liquid; 61.5% Os in $\beta$ phase
(iii) Amounts of phases:

$$\text{Wt \% beta} = \frac{30 - 23}{61.5 - 23} \times 100\% = 18.2\%$$

$$\text{Wt \% liquid} = \frac{61.5 - 30}{61.5 - 23} \times 100\% = 81.8\%$$

(c) At 2665°C $- \Delta T$,

(i) Phases Present: Liquid and alpha phases
(ii) Compositions of Phases: 23% Os in liquid; 43% Os in $\alpha$ phase
(iii) Amounts of phases:

$$\text{Wt \% alpha} = \frac{30 - 23}{43 - 23} \times 100\% = 35.0\%$$

$$\text{Wt \% liquid} = \frac{43 - 30}{43 - 23} \times 100\% = 65.0\%$$
8.28
(a) At 2600°C,
(i) Phases Present: Alpha and beta phases
(ii) Compositions of Phases: 43% Os in alpha phase; 61.5% Os in beta phase
(iii) Amounts of phases:
\[
\text{Wt \% alpha} = \frac{61.5 - 60}{61.5 - 43} \times 100\% = 8.1\%
\]
\[
\text{Wt \% beta} = \frac{60 - 43}{61.5 - 43} \times 100\% = 91.9\%
\]

(b) At 2665°C + ΔT,
(i) Phases Present: Liquid and beta phases
(ii) Compositions of Phases: 23% Os in alpha phase; 61.5% Os in beta phase
(iii) Amounts of phases:
\[
\text{Wt \% liquid phase} = \frac{61.5 - 60}{61.5 - 23} \times 100\% = 3.9\%
\]
\[
\text{Wt \% beta phase} = \frac{60 - 23}{61.5 - 23} \times 100\% = 96.1\%
\]

(c) At 2665°C − ΔT,
(i) Phases Present: Alpha and beta phases
(ii) Compositions of Phases: 43% Os in liquid phase; 61.5% Os in beta phase
(iii) Amounts of phases:
\[
\text{Wt \% alpha} = \frac{61.5 - 60}{61.5 - 43} \times 100\% = 8.1\%
\]
\[
\text{Wt \% beta} = \frac{60 - 43}{61.5 - 43} \times 100\% = 91.9\%
\]

(d) At 2800°C,
(i) Phases Present: Alpha and beta phases
(ii) Compositions of Phases: 45% Os in liquid phase; 85% Os in beta phase
(iii) Amounts of phases:
\[
\text{Wt \% liquid} = \frac{85 - 60}{85 - 45} \times 100\% = 62.5\%
\]
\[
\text{Wt \% beta} = \frac{60 - 45}{85 - 45} \times 100\% = 37.5\%
\]
8.29

a) At 2600°C,
   (i) Phases Present: Liquid and alpha phases
   (ii) Compositions of Phases: 16% Os in liquid phase; 38% Os in alpha phase
   (iii) Amounts of phases:
         \[ \text{Wt} \% \text{ alpha} = \frac{30 - 16}{38 - 16} \times 100\% = 63.6\% \]
         \[ \text{Wt} \% \text{ liquid} = \frac{38 - 30}{38 - 16} \times 100\% = 36.4\% \]

b) At 2665°C + ΔT,
   (i) Phases Present: Liquid and beta phases
   (ii) Compositions of Phases: 23% Os in liquid phase; 61.5% Os in β phase
   (iii) Amounts of phases:
         \[ \text{Wt} \% \text{ beta} = \frac{30 - 23}{61.5 - 23} \times 100\% = 18.2\% \]
         \[ \text{Wt} \% \text{ liquid} = \frac{61.5 - 30}{61.5 - 23} \times 100\% = 81.8\% \]

c) At 2665°C − ΔT,
   (i) Phases Present: Liquid and alpha phases
   (ii) Compositions of Phases: 23% Os in liquid phase; 43% Os in α phase
   (iii) Amounts of phases:
         \[ \text{Wt} \% \text{ alpha} = \frac{30 - 23}{43 - 23} \times 100\% = 35.0\% \]
         \[ \text{Wt} \% \text{ liquid} = \frac{43 - 30}{43 - 23} \times 100\% = 65.0\% \]

8.30

In the copper-lead (Cu-Pb) system (Fig. 8.24) for an alloy of Cu-10 wt % Pb, determine the amounts and compositions of the phases present at (a) 1000°C (b) 955°C + ΔT, (c) 955°C − ΔT, and (d) 200°C.

(a) At 1000°C,
   Compositions of Phases: 100% Cu, 0% Pb in α phase;
   (b) 955°C + ΔT, (c) 955°C − ΔT, and (d) 200°C.
   Amounts of Phases:
   \[ \text{Wt} \% \alpha = \frac{19 - 10}{19 - 0} \times 100\% = 47.4\% \]
   \[ \text{Wt} \% L_1 = \frac{10 - 0}{19 - 0} \times 100\% = 52.6\% \]
(b) At 955°C + ΔT,
Compositions of Phases:
100% Cu, 0% Pb in α phase; 64% Cu, 36% Pb in L₁ phase

Amounts of Phases:

\[ \text{Wt } \% \alpha = \frac{36-10}{36-0} \times 100\% = 72.2\% \]

\[ \text{Wt } \% L_1 = \frac{10-0}{36-0} \times 100\% = 27.8\% \]

(c) At 955°C − ΔT,
Compositions of Phases:
100% Cu, 0% Pb in α phase; 13% Cu, 87% Pb in L₂ phase

Amounts of Phases:

\[ \text{Wt } \% \alpha = \frac{87-10}{87-0} \times 100\% = 88.5\% \]

\[ \text{Wt } \% L_2 = \frac{10-0}{87-0} \times 100\% = 11.5\% \]

(d) At 200°C,
Compositions of Phases:
99.995% Cu, 0.005% Pb in α phase; 0.007% Cu, 99.993% Pb in β phase

Amounts of Phases:

\[ \text{Wt } \% \alpha = \frac{99.99-10}{99.99-0} \times 100\% = 90\% \]

\[ \text{Wt } \% \beta = \frac{10-0}{99.99-0} \times 100\% = 10\% \]