HW solution for Lecture 8

51. Loading direction [001], σ = 75 MPa
(a) The slip system (111)[101]. The resolved shear stress is

\[ \tau_r = \sigma \cos \lambda \cos \phi = 75 \cos \lambda \cos \phi \]

See Fig. 6.34 for the definition of λ and φ.

\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 0 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{(-1)^2 + 0^2 + 1^2}}} = \frac{1}{\sqrt{2}}
\]

\[
\cos \phi = \frac{0 \cdot 1 + 0 \cdot 1 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{1^2 + 1^2 + 1^2}}} = \frac{1}{\sqrt{3}}
\]

\[ \tau_r = 75 \frac{1}{\sqrt{2} \sqrt{3}} = 30.6 \text{ MPa} \]

(b) The slip system (111)[\bar{1}10].

\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 1 + 1 \cdot 0}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{(-1)^2 + 1^2 + 0^2}}} = 0
\]

\[
\cos \phi = \frac{0 \cdot 1 + 0 \cdot 1 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{1^2 + 1^2 + 1^2}}} = \frac{1}{\sqrt{3}}
\]

\[ \tau_r = 75 \cdot 0 \cdot \frac{1}{\sqrt{3}} = 0 \]

52. Loading direction [001], σ = 55 MPa
(a) The slip system (101)[\bar{1}11]. The resolved shear stress is

\[ \tau_r = \sigma \cos \lambda \cos \phi \]

See Fig. 6.34 for the definition of λ and φ.

\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 1 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{(-1)^2 + 1^2 + 1^2}}} = \frac{1}{\sqrt{3}}
\]

\[
\cos \phi = \frac{0 \cdot 1 + 0 \cdot 0 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{1^2 + 0^2 + 1^2}}} = \frac{1}{\sqrt{2}}
\]

\[ \tau_r = 55 \frac{1}{\sqrt{3} \sqrt{2}} = 22.45 \text{ MPa} \]

(b) The slip system (110)[\bar{1}11].
\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 1 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2}} = \frac{1}{\sqrt{3}}
\]

\[
\cos \phi = \frac{0 \cdot 1 + 0 \cdot 1 + 1 \cdot 0}{\sqrt{0^2 + 0^2 + 1^2}} = 0
\]

\[
\tau_r = 55 \cdot \frac{1}{\sqrt{3}} \cdot 0 = 0
\]

53. Loading direction [00\bar{1}], \(\sigma = 4.75\) MPa

The resolved shear stress is \(\tau_r = \sigma \cos \lambda \cos \phi\). For the (11\bar{1}) slip plane,

\[
\cos \phi = \frac{0 \cdot 1 + 0 \cdot 1 + (-1) \cdot (-1)}{\sqrt{0^2 + 0^2 + (-1)^2}} = \frac{1}{\sqrt{3}}
\]

(a) for the [\bar{1}0\bar{1}] direction:

\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 1 + (-1) \cdot 1}{\sqrt{0^2 + 0^2 + (-1)^2}} = -\frac{1}{\sqrt{2}}, \text{ Remove the negative sign } \left(\frac{1}{\sqrt{2}}\right)
\]

\[
\tau_r = 4.75 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} = 1.94 \text{ MPa}
\]

(b) for the [0\bar{1}\bar{1}] direction:

\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 1 + (-1) \cdot 1}{\sqrt{0^2 + 0^2 + (-1)^2}} = -\frac{1}{\sqrt{2}}
\]

\[
\tau_r = 4.75 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{3}} = 1.94 \text{ MPa}
\]

(c) for the [\bar{1}10] direction:

\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 1 + (-1) \cdot 0}{\sqrt{0^2 + 0^2 + (-1)^2}} = 0
\]

\[
\tau_r = 4.75 \cdot 0 \cdot \frac{1}{\sqrt{3}} = 0
\]

54. Loading direction [001], \(\sigma = 85\) MPa, \(\tau_r = \sigma \cos \lambda \cos \phi\)

(a) The slip system (011)[\bar{1}1\bar{1}]. The resolved shear stress is

\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 1 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2}} = \frac{1}{\sqrt{3}}
\]
\[
\cos \phi = \frac{0 \cdot 0 + 0 \cdot 1 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{0^2 + 1^2 + 1^2}}} = \frac{1}{\sqrt{2}}
\]

\[
\tau_r = 85 \frac{1}{\sqrt{3} \sqrt{2}} = 34.7 \text{ MPa}
\]

(b) The slip system \((110)[111]\). The resolved shear stress is

\[
\cos \lambda = \frac{0 \cdot (-1) + 0 \cdot 0 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{(-1)^2 + 1^2 + 1^2}}} = \frac{1}{\sqrt{3}}
\]

\[
\cos \phi = \frac{0 \cdot 0 + 0 \cdot 0 + 1 \cdot 0}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{1^2 + 1^2 + 0^2}}} = 0
\]

\[
\tau_r = 85 \frac{1}{\sqrt{3}} 0 = 0 \text{ MPa}
\]

(c) The slip system \((0 \bar{1} 1)[111]\). The resolved shear stress is

\[
\cos \lambda = \frac{0 \cdot 0 + 0 \cdot 0 + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{1^2 + 1^2 + 1^2}}} = \frac{1}{\sqrt{3}}
\]

\[
\cos \phi = \frac{0 \cdot 0 + 0 \cdot (-1) + 1 \cdot 1}{\sqrt{0^2 + 0^2 + 1^2 \sqrt{0^2 + (-1)^2 + 1^2}}} = \frac{1}{\sqrt{2}}
\]

\[
\tau_r = 85 \frac{1}{\sqrt{3} \sqrt{2}} = 34.7 \text{ MPa}
\]

58.

(a) From Fig. 6.45, the cold work need to be 40% to attain a strength of 50 ksi.

(b) \(d_1 = 0.25''\), find \(d_0\)

\[
40\% = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1 = \frac{d_0^2}{d^2} - 1
\]

\[
d_0 = \sqrt{1.4d} = 0.296''
\]
60.
(a) \( \varepsilon_0 = ? \varepsilon_1 = ? \varepsilon_2 = ?, \ d_0 = ?, \ d_1 = 2.80 \text{ mm}, \ d_2 = 2.45 \text{ mm}, \ \varepsilon_1 = 20\% \)

\[
0.2 = \frac{l_1 - l_0}{l_0} = \frac{l_1}{l_0} - 1
\]

\[
\frac{l_1}{l_0} = 1.2 = \frac{A_0}{A_1} = \frac{d_0^2}{d_1^2}
\]

\[
d_0^2 = 1.2d_1^2
\]

\[
\varepsilon_2 = \frac{l_2 - l_0}{l_0} = \frac{l_2}{l_0} - 1 = \frac{A_0}{A_2} - 1 = \frac{d_0^2}{d_2^2} - 1
\]

\[
= \frac{1.2d_0^2}{d_2^2} - 1 = 56.7\%
\]

(b) From Fig. 6.44, for % cold work of approximately 57%, read

Ultimate strength = 54 ksi; Yield strength = 50 ksi,
Elongation = 3\%